ME 477/677
Problem Set 4-Solutions

1. Estimate the theoretical fracture strength of iron if the surface energy is 1.2 J/m^2. How does this compare with the highest observed strength of commercially produced high strength steels? (D7.1, M&C3.2)

Solution
Elastic modulus of iron = 210 GPa; lattice constant for bcc iron = 0.287nm

$$\sigma_f = \sqrt{\frac{E\gamma_s}{a_0}} = \sqrt{\frac{210\times10^9(N/m^2)\times1.2(J/m^2)}{0.287\times10^{-9}m}} = 2.96\times10^{10}\,\frac{N}{m^2} = 29.6\ GPa$$

The fracture strength of high strength steel is approximately 3.5 GPa

2. Consider a brittle material with γ = 1 J/m^2 and E = 100GPa. What is the breaking strength of this material if it contains crack-like defects as long as 1 mm. If it were possible to increase γ to 3000 J/m^2, what would be the breaking strength for 1 mm long cracks? (M&C 3.5)

Solution
Since we are dealing with a brittle material, Griffith's equation can be used to determine the fracture strength. The problem does not specify edge or interior crack. So the problem is solved both ways.

Assuming interior cracks

For γ = 1 J/m^2 and interior cracks of length 2c = 1mm

$$\sigma_f\sqrt{\frac{2E\gamma_s}{\pi c}} = \sqrt{\frac{2\times100\times10^9(N/m^2)\times1(Nm/m^2)}{\pi\times0.5\times10^{-3}m}} = 11.28\times10^6(N/m^2) = 11.28MPa$$

For γ = 1 J/m^2 and interior cracks of length 2c = 1mm

$$\sigma_f\sqrt{\frac{2E\gamma_s}{\pi c}} = \sqrt{\frac{2\times100\times10^9(N/m^2)\times3000(Nm/m^2)}{\pi\times0.5\times10^{-3}m}} = 61.80\times10^6(N/m^2) = 618MPa$$

Assuming edge cracks

For γ = 1 J/m^2 and edge cracks of length c = 1mm

$$\sigma_f\sqrt{\frac{2E\gamma_s}{\pi c}} = \sqrt{\frac{2\times100\times10^9(N/m^2)\times1(Nm/m^2)}{\pi\times1\times10^{-3}m}} = 7.98\times10^6(N/m^2) = 7.98MPa$$

For γ = 1 J/m^2 and edge cracks of length c = 1mm

$$\sigma_f\sqrt{\frac{2E\gamma_s}{\pi c}} = \sqrt{\frac{2\times100\times10^9(N/m^2)\times3000(Nm/m^2)}{\pi\times1\times10^{-3}m}} = 43.70\times10^6(N/m^2) = 437MPa$$

3. Using Griffith's equation for plane stress, determine the critical crack length for
(a) Iron, (b) Zinc, and (c) NaCl
if the following constants apply: (D7.2)

Solution:
The flaw size c is calculated using Griffith's equation and interior flaws

	σ_c (dyne/cm^2)	γ (erg/cm^2)	E(dyne/cm^2)	c(cm)
Iron	$9x10^9$	1,200	$20.5x10^{11}$	$1.9x10^{-5}$
Zinc	$2x10^7$	800	$3.5x10^{11}$	0.447
NaCl	$2x10^7$	150	$5.0x10^{11}$	0.119

4. For a titanium alloy having an elliptical surface flaw, calculate the deepest crack length that can be tolerated if catastrophic failure is to be avoided when the alloy is exposed to an operating stress equal to 75 percent of its yield strength. For this material K_{Ic} = 115 MN/m$^{3/2}$, σ_y = 900 MN/m^2. The following equation is applicable:

$$\sigma_F = \frac{\left(1.38 - 0.212\left(\frac{\sigma_F}{\sigma_y}\right)^2\right)^{\frac{1}{2}}}{(1.20\pi a)^{\frac{1}{2}}} K_{Ic}$$

(Courtney 10.7)

Solution

Inserting the given values into the above equation

$$0.75\sigma_y = \frac{\left(1.38 - 0.212\left(\frac{0.75\sigma_y}{\sigma_y}\right)^2\right)^{\frac{1}{2}}}{(1.20\pi a)^{\frac{1}{2}}} K_{Ic}$$

$$0.75\times 900\times 10^6 (N/m^2) = \left(\sqrt{\frac{\left(1.38 - 0.212\times 0.75^2\right)}{(1.20\pi a)}}\right)\times 115\left(Nm^{0.5}\right) = \frac{66.5\times 10^6}{\sqrt{a}}\frac{N}{m^{3/2}}$$

$$\sqrt{a} = \frac{66.5\times 10^6}{675\times 10^6}\sqrt{m} = 0.0985\times 10^{-2}\sqrt{m}$$

$$a = 9.7\times 10^{-3} m = 9.7 mm$$