ME 477/677
Problem Set 3 - Solutions

1. At what grain size would the yield stress of a steel be twice that at an ASTM grain size 3. Express the grain size in ASTM number and in μm. Use σ_i = 150MPa and k = 0.7MNm$^{-3/2}$. (*Dieter 6.5*)

Solution

Material 1: grain size = ASTM #3

D = 0.125 mm (from Table 6.1, Dieter)

Parameters for Hall-Petch relationship

σ_i = 150 MPa

k = 0.7 MN/m$^{3/2}$

$$(\sigma_0)_1 = \sigma_i + kD^{-0.5}$$

$$= 150\times10^6 \frac{N}{m^2} + 0.7\times10^6 \frac{N}{m^{3/2}} \frac{1}{\left(0.125\times10^{-3}\,m\right)^{1/2}}$$

$$= 212.6MPa$$

Material 2 has a yield str4ength twice that of Material 1. Therefore

$$(\sigma_0)_2 = 2\times(\sigma_0)_1$$

$$= 2\times212.6MPa$$

$$= 150\times10^6 \frac{N}{m^2} + 0.7\times10^6 \frac{N}{m^{3/2}} \frac{1}{(D_2)^{1/2}}$$

Therefore

$$D^{-1/2} = 393.2m^{-1/2}$$

$$D = 6.47\times10^{-6}\,m = 6.47\mu m$$

From Table 6.1, the ASTM grain size lies between 11 and 12.

$$D = \frac{1}{\sqrt{N_A}}$$

$$N_A = (1/D)^2 = 2.39\times10^{10}/m^2 \ \ at\ 1X$$

$$= 2.39\times10^4/mm^2 \ \ at\ 1X$$

$$= 2.39\times10^2/mm^2 \ \ at\ 100X$$

$$= 1542/in^2 \ \ at\ 100X$$

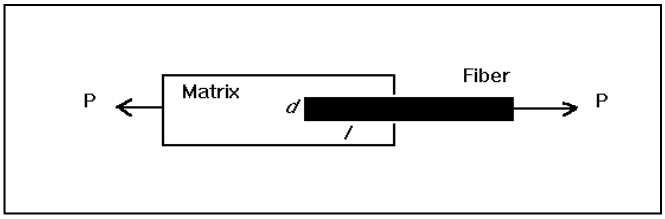
$$= 2^{n-1}$$

$$\log(1542) = (n-1)\log(2)$$

$$(n-1) = \frac{\log(1542)}{\log(2)} = 10.59$$

$$n = 11.59$$

2. The fracture strength, $\sigma_{f,}$, of the fiber in a fiber reinforced composite is eight times the maximum shear strength, τ_{max}, that the interface between the fiber and matrix can bear. Assuming the loading on an individual fiber can be modeled as shown in the figure, select the optimum fiber aspect ratio (d/l) at which the fiber would break rather than be pulled out of the matrix under the application of a tensile stress.*(based on Meyers&Chawla 12.3)*



Solution

$$\sigma_{f-fiber} = 8\tau_{\max-\text{interface}}$$

$$P = \sigma_{f-fiber} A_{fiber} = \tau_{\max-\text{interface}} A_{\text{interface}}$$

$$8 \times \tau_{\max-\text{interface}} \times \frac{\pi d^2}{4} = \tau_{\max-\text{interface}} \times \pi d l$$

$$\frac{d}{l} = 0.5$$

3. For a unit cube of a matrix containing spherical particles in a uniform cubic array
   (a) Show that the distance between particles is given by

$$\lambda = \frac{1}{N^{(1/3)}} - 2r$$

   where N is the number of particles.
   (b) Calculate λ for a volume fraction f of particles equal to 0.001, and r = 10^{-5} mm. *(Meyers&Chawla 11.4)*

Solution

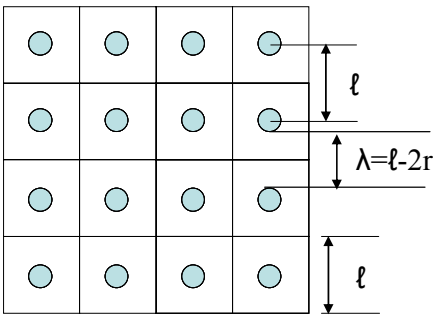
a) Assume there are N particles per unit volume, and each particle has a radius of r
   - If each particle is assumed to be at the center of a cube of length ℓ
   - Volume of each cube $V = \ell^3$
   - Number of such cubes per unit volume $N = \frac{1}{\ell^3}$
   - Length of the side of the cube $\ell = \frac{1}{N^{1/3}}$
   - Distance between the centers of the particles $\ell = \frac{1}{N^{1/3}}$
   - Distance between the edges of the particles $\lambda = \frac{1}{N^{1/3}} - 2r$

b) Assume the particles are uniformly distributed as described in part (a)
   - Volume fraction of particle = 0.001
   - Radius of particles = 10^{-5} mm
   - Number of particles/unit volume

$$N = \frac{volume\ \ fraction\ of\ \ particles}{volume\ of\ \ each\ \ particle} = \frac{0.001}{\left(\left(\frac{4}{3}\right)\pi r^3\right)\left(10^{-5}\,mm\right)^3} = 2.39\times10^{11}/mm^3$$

   - Distance between particles

$$\lambda = \frac{1}{N^{(1/3)}} - 2r = 1.412\times10^{-4}\,mm$$



4. For a dispersion strengthened aluminum alloy, the yield strength in shear of a matrix material about (G/1000), and its fracture strength is about (G/30), where G is the shear modulus of the matrix. For strengthening by dispersed particles to be effective, the yield strength in shear of the composite must be greater than that of the matrix, yet cannot exceed the fracture strength of the matrix. Ductility considerations limit the volume fraction of hard particles to 15%. Design this composite system, i.e., select a particle size or the range of particle sizes under these conditions for an aluminum alloy?
(based on Dieter 6.10)

Solution

Yield strength of matrix $\tau_0 = \frac{G}{1000}$

Fracture strength of matrix $\tau_f = \frac{G}{30}$

The strength of the dispersion strengthened material must lie between the above two stresses

$$\tau_0 \le \tau \le \tau_f$$

Assuming Orowan strengthening

$$\tau = \frac{Gb}{\lambda}$$

or

$$\frac{G}{1000} \le \frac{Gb}{\lambda} \le \frac{G}{30}$$

or

$$1000b \ge \lambda \ge 30b$$

If the burgers vector for aluminum is 2.5×10^{-10}m, then the inter-particle spacing must be in the range

$$0.25\mu m \ge \lambda \ge 0.0075\mu m$$

Ductility requirements limits volume fraction to be less than 0.15, then

$$\lambda = \frac{4(1-f)}{3f} r = 7.56r$$

$$0.25\mu m \ge 7.56r \ge 0.0075\mu m$$

$$0.033\mu m \ge r \ge 0.001\mu m$$

5. An aluminum alloy is strengthened by ordered, coherent particles. In such an alloy a moving dislocation has to either cut through the particles and create an anti-phase boundary in the particle, or bow between particles. Assume that the particle shearing stress due to order hardening is given by:

$$\tau_{ord} = \frac{\gamma_{APB} f}{2b}\left[\left(\frac{8\gamma_{APB} r}{\pi f G b^2}\right)^{\frac{1}{2}} - 1\right]$$

and the hardening due to dislocations bowing between particles is given by.

$$\tau_B = \frac{Gb}{(\lambda - 2r)}$$

The alloy is solutionized, quenched, and then aged at 200°C. At 200°C, the average particle radius (r) increases by a diffusion-controlled coarsening process for which

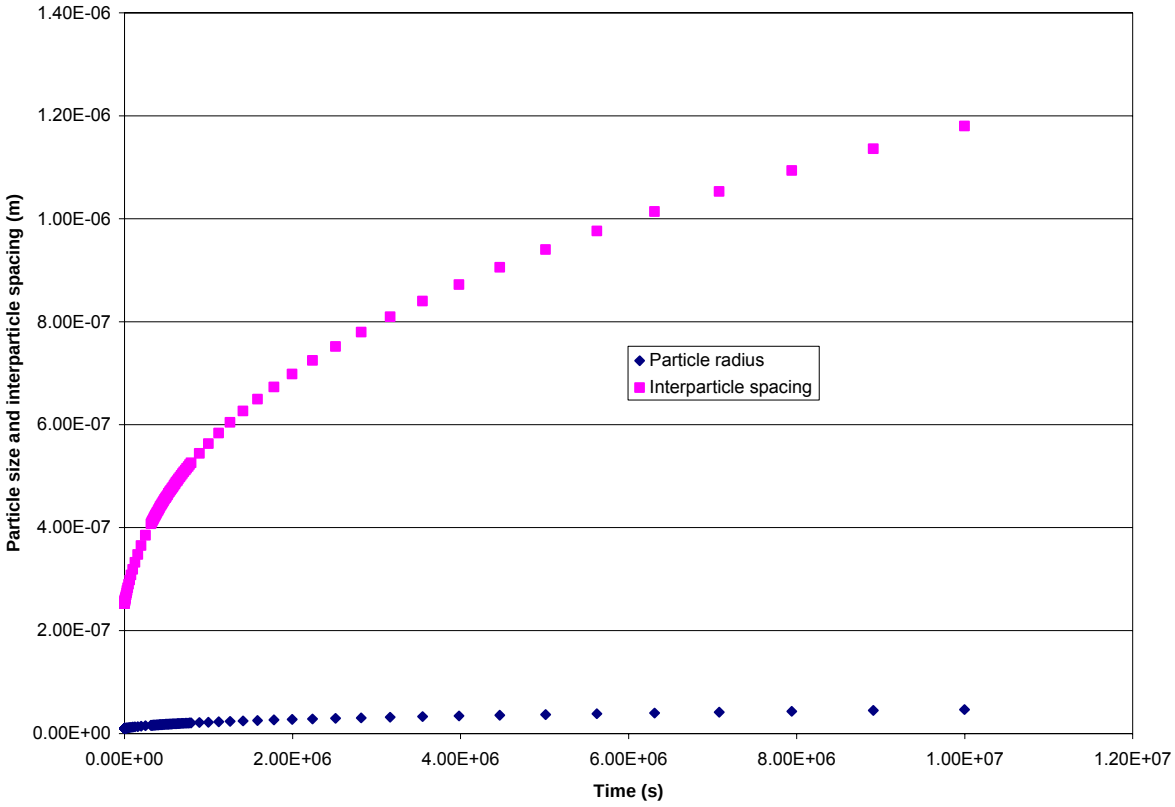
$$r^3 = r_0^3 + Kt$$

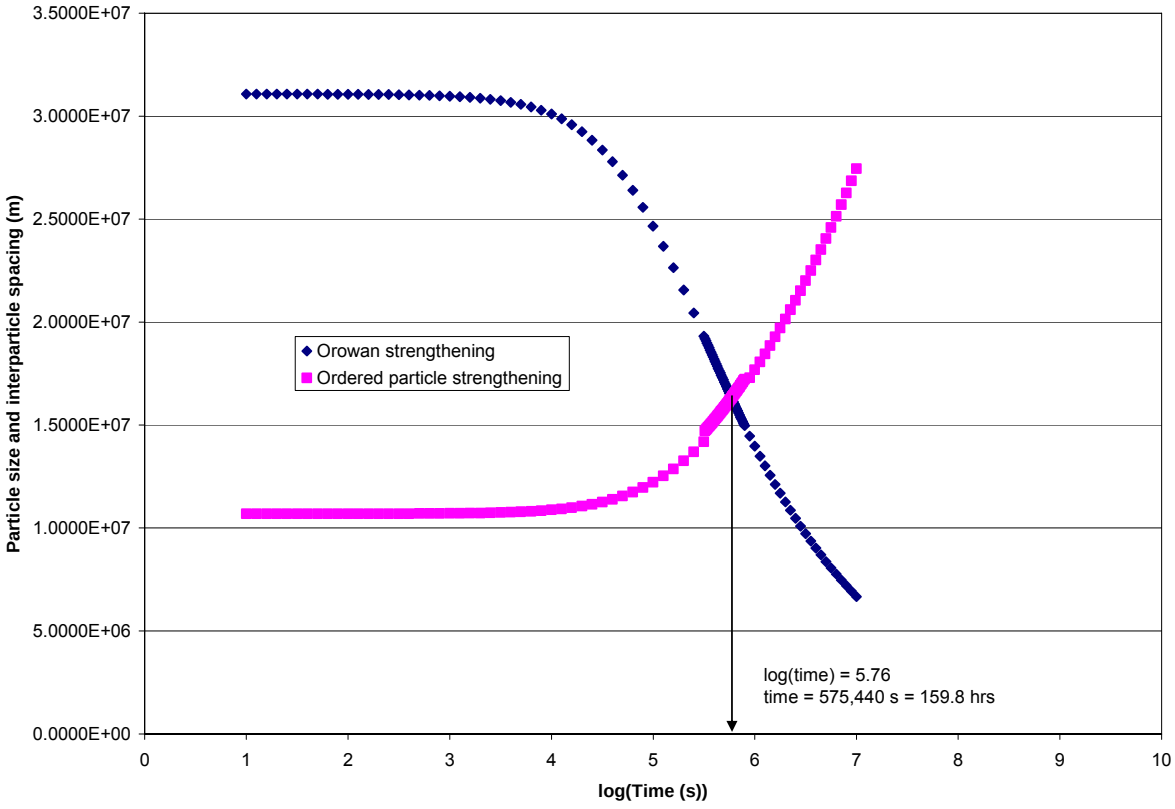
where r_0 is the initial particle radius and K is a constant (= 10^{-29} m^3/s).

How does the strength of the alloy change as a function of time. What is the optimum aging treatment for maximizing the strength of the alloy.

Data:	Shear Modulus	G	= $25x10^9$ N/m^3
	Burger's Vector	b	= 0.29 nm
	Anti-Phase Boundary Energy	γ_{APB}	= 50 mJ/m^2
	Volume Fraction of Particles	f	= 0.05
	Initial Radius of Particles	r_0	= 10 nm

Hint: Perform these calculations on a spread sheet.
(based on Courtney 5.20)


1.40E-06
1.20E-06
1.00E-06
8.00E-07
6.00E-07
4.00E-07
2.00E-07
0.00E+00
Particle size and interparticle spacing (m)
Particle radius
Interparticle spacing
0.00E+00
2.00E+06
4.00E+06
6.00E+06
8.00E+06
1.00E+07
1.20E+07
Time (s)


3.5000E+07
3.0000E+07
2.5000E+07
2.0000E+07
1.5000E+07
1.0000E+07
5.0000E+06
0.0000E+00
Particle size and interparticle spacing (m)
Orowan strengthening
Ordered particle strengthening
log(time) = 5.76
time = 575,440 s = 159.8 hrs
0
1
2
3
4
5
6
7
8
9
10
log(Time (s))