ME 477/677
Problem Set 2

1. A nickel monocrystal has been sheared to a strain $\gamma = 0.1$ Assuming that the dislocation density is 10^8 cm^{-2}, and it remains constant, what is the average distance moved by each dislocation? If the shear strain rate $\dot{\gamma}$ is 10^{-4} s^{-1}, what is the mean velocity of the dislocations? *(Meyers&Chawla P8.1)*

Solution

(a)

$$\gamma = \rho b \bar{x} \qquad (eq.\ 4.11)$$

$$\therefore \quad \bar{x} = \frac{\gamma}{\rho b}$$

$$= \frac{0.1}{10^{12} m^{-2} (0.25 \times 10^{-9} m)}$$

$$= 4 \times 10^{-4} m = 4 \times 10^{-2} cm$$

(b)

$$\dot{\gamma} = \rho b \bar{v} \qquad (eq.\ 4.12)$$

$$\therefore \quad \bar{v} = \frac{\dot{\gamma}}{\rho b}$$

$$= \frac{1 \times 10^{-4} s^{-1}}{10^{12} m^{-2} (0.25 \times 10^{-9} m)}$$

$$= 4 \times 10^{-7} m/s = 4 \times 10^{-5} cm/s$$

2. Construct a two-dimensional drawing of the $(\bar{1}10)$ plane of a bcc lattice. By means of vector addition show that:

$$(a_0/2)[\bar{1}\bar{1}1] + (a_0/2)[111] \rightarrow a_0[001]$$

and

$$(a_0/2)[\bar{1}\bar{1}1] + (a_0/2)[\bar{1}\bar{1}\bar{1}] \rightarrow a_0[\bar{1}\bar{1}0]$$

Show that the last reaction is not energetically favorable.
(Dieter P5-4)

Solution

(a)

$$\vec{b}_1 = (a_0/2)[\bar{1}11]; \quad \vec{b}_2 = (a_0/2)[111]; \quad \vec{b}_3 = a_0[001];$$

$$b_1^2 = \frac{a_0^2}{4}((-1)^2 + (-1)^2 + 1^2) = \frac{3}{4}a_0^2$$

$$b_2^2 = \frac{a_0^2}{4}(1^2 + 1^2 + 1^2) = \frac{3}{4}a_0^2$$

$$b_3^2 = a_0(0^2 + 0^2 + 1^2) = a_0^2$$

$$b_1^2 + b_2^2 > b_3^2$$

Since the energies of dislocations are proportional to the square of their Burgers vectors, the reaction above constitutes a decrease in energy, and is, therefore, energetically favorable.

(b)

$$\vec{b}_1 = (a_0/2)[\bar{1}11]; \quad \vec{b}_2 = (a_0/2)[\bar{1}\bar{1}\bar{1}]; \quad \vec{b}_3 = a_0[\bar{1}\bar{1}0];$$

$$b_1^2 = \frac{a_0^2}{4}((-1)^2 + (-1)^2 + 1^2) = \frac{3}{4}a_0^2$$

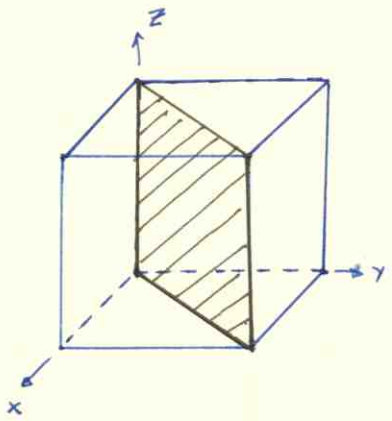
$$b_2^2 = \frac{a_0^2}{4}((-1)^2 + (-1)^2 + (-1)^2) = \frac{3}{4}a_0^2$$

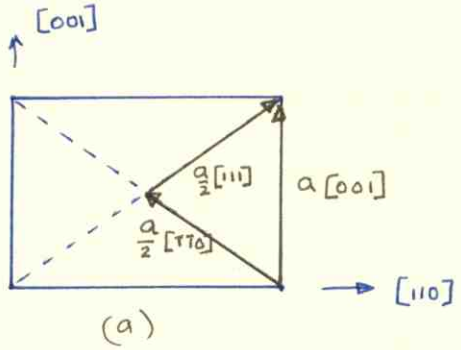
$$b_3^2 = a_0((-1)^2 + (-1)^2 + 0^2) = 2a_0^2$$

$$b_1^2 + b_2^2 < b_3^2$$

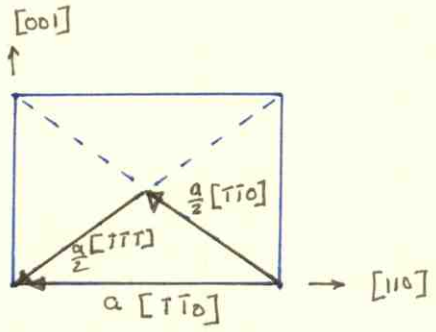
Since the above reaction constitutes an increase in energy, the reaction is not energetically favorable.

PROBLEM 2



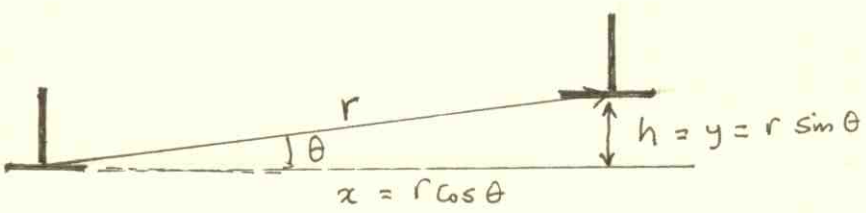


(a)



(b)

PROBLEM 4



22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS
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3. Typical values for the dislocation density in annealed and deformed nickel are 10^7 cm^{-2} and 10^{11} cm^{-2}. Calculate the average distance between dislocations. Also calculate the energy of the dislocations per atomic plane assuming the dislocations are (a) edge dislocations, and (b) screw dislocations. Express the energy in electron volts (eV). Also what is the energy (in J) per cm of dislocation in the above two cases. (E = 210 GPa, ν = 0.3, closest distance between atoms = 0.25 nm) *(Meyers&Chawla P6.9)*

Solution

$$Modulus\ of\ rigidity\ G = \frac{E}{2(1+\nu)} = \frac{210\ GPa}{2(1+.3)} = 80.77 GPa$$

Let ρ (number per unit area) = density of dislocations.
d = average distance between dislocations

If the dislocations in unit area of the crystal are assumed be be arranged in a square array, with, **n** dislocations per side, then,

$$\rho = n^2$$
$$or$$
$$n = \sqrt{\rho}$$
$$d = \frac{1}{n} = \frac{1}{\sqrt{\rho}}$$

For the given data:

$d_{annealed}$ = 3.162 x 10^{-4} cm
$d_{deformed}$ = 3.162 x 10^{-6} cm

In the "Volterra" model for a dislocation, the radius of the circle of influence for a dislocation is one half of the distance between dislocations.

$$Alternatively;$$
$$Area\ per\ dislocation = \frac{1}{\rho}$$
$$Assuming\ a\ circle\ of\ influence\ of$$
$$r_1 = \sqrt{\frac{1}{\rho\pi}}$$

The energy per unit length of a dislocation is

$$U_{edge} = \frac{Gb^2}{4\pi(1-\nu)} \ln\frac{r_1}{r_0}$$
$$U_{screw} = \frac{Gb^2}{4\pi} \ln\frac{r_1}{r_0}$$

The energy per atomic plane is the energy per unit length x distance between planes. For this problem, assume $r_0 \approx b$ and distance between planes $\approx$ b. The Burgers vector b is the closest distance between atoms. Using these values the energy of a dislocation is:

	Edge			Screw		
	(J/m)	per plane (J)	per plane (eV)	(J/m)	per plane (J)	per plane (eV)
Annealed	5.0227e-9	1.2557e-18	7.8381	3.5159e-9	8.7897e-19	5.5867
Deformed	2.3799e-9	5.9496e-19	3.7139	1.6659e-9	4.1647e-19	2.5997

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4. (a) Show that the shear stress produced on a slip plane by and edge dislocation on a parallel plane at coordinates (r,θ) is given by:

$$\tau_{xy} = \frac{Gb}{8\pi(1-\nu)} \cdot \frac{1}{h} \cdot \sin 4\theta$$

where h is the perpendicular distance between two parallel planes.

Solution

$$\tau_{xy} = \tau_0 \frac{bx(x^2-y^2)}{(x^2+y^2)^2}$$

$$\text{where} \quad \tau_0 = \frac{G}{2\pi(1-\nu)}$$

$$x = r\cos\theta$$

$$\text{and} \quad y = r\sin\theta = h$$

$$\tau_{xy} = \tau_0 \frac{b\, r\cos\theta(r^2\cos^2\theta - r^2\sin^2\theta)}{r^4}$$

$$= \tau_0 \frac{b\, r\cos\theta\cos 2\theta}{r^2}$$

$$= \tau_0 \frac{br}{r^2}\cos 2\theta\cos\theta \times \frac{y}{y}$$

$$= \tau_0 \frac{br}{r^2}\cos 2\theta\cos\theta \frac{r\sin\theta}{h}$$

$$= \frac{\tau_0}{2h} b\cos 2\theta\sin 2\theta$$

$$= \frac{\tau_0}{4h} b\sin 4\theta$$

$$= \frac{Gb}{8\pi(1-\nu)h}\sin 4\theta$$

(b) What is the shear stress required for two edge dislocations to pass on parallel slip planes that are 1000 Å apart in an annealed copper crystal?
(G = $5\text{x}10^{11}$ dyne cm^{-2}, ν = 1/3, b = 0.25 nm) *(Dieter P5-13)*

The maximum shear stress due to a dislocation is at a position where **sin4θ=1**. Using the equation derived above, and the data provided

$$(\tau_{xy})_{max} = \frac{Gb}{8\pi(1-\nu)h}$$
$$= \frac{5\times10^{11}dynes/cm^2\times2.5\times10^{-8}cm}{8\pi(1-1/3)\times1000\times10^{-8}cm}$$
$$= 7.460\times10^{7}dynes/cm^2$$
$$= 7.460MPa$$

This is the stress to be overcome if two dislocations are to glide past each other.

5. An edge dislocation in a nickel crystal stops upon encountering an obstacle. Upon the application of a shear stress of 140 MPa a second edge dislocation with identical Burgers vector and moving on the same plane approaches the first dislocation.
(a) How close can the second dislocation approach the first?
(b) What is the separation between the dislocations if they were both screw dislocations?
(E = 210 GPa, ν = 0.3, r_0 = 2.49 Å) *(Meyers&Chawla P6.14)*

Solution

(a) The shear stress at a distance r along the slip from an edge dislocation is given by

$$\tau_{r\theta} = \frac{\tau_0 b}{r}\cos\theta$$

along the slip plane $\theta = 0$. A second identical dislocation moving under a shear stress of 140 MPa will stop at a distance where the stress exerted by the first dislocation is equal to the applied shear stress. Therefore,

$$\tau_{r\theta} = \frac{G}{2\pi(1-\nu)}\frac{b}{r}$$
$$\therefore\ r = \frac{Gb}{2\pi(1-\nu)\tau_{r\theta}}$$
$$= \frac{80,77\times10^{9}Pa\times0.249\times10^{-9}m}{140\times10^{6}Pa\times2\pi\times0.7}$$
$$= 3.266\times10^{-8}m = 32.66nm$$

(b) The shear stress at a distance r along the slip from an screw dislocation is given by

$$\tau_{\theta z} = \frac{Gb}{2\pi r}$$

A second identical dislocation moving under a shear stress of 140 MPa will stop at a distance where the stress exerted by the first dislocation is equal to the applied shear stress. Therefore,

$$\tau_{\theta z} = \frac{Gb}{2\pi r}$$
$$\therefore\ r = \frac{Gb}{2\pi(\tau_{\theta z}}$$
$$= \frac{80,77\times10^{9}Pa\times0.249\times10^{-9}m}{140\times10^{6}Pa\times2\pi}$$
$$= 2.286\times10^{-8}m = 22.86nm$$

Alternate Solution:

The force acting per unit length on a dislocation under an applied shear stress of τ is

$$F = \tau\, b$$

The force per unit length between identical dislocations on the same slip plane, separated by a distance r is

$$F_r = \frac{Gb^2}{2\pi(1-\nu)r} \qquad \textit{for edge dislocations}$$

$$F_r = \frac{Gb^2}{2\pi r} \qquad \textit{for screw dislocations}$$

When one dislocation is held up at a barrier, the second dislocation will stop at a distance r from the first at which the force due to the applied stress is equal to the force between dislocations. Therefore:

$$r = \frac{Gb^2}{2\pi(1-\nu)F_r}$$
$$= \frac{Gb^2}{2\pi(1-\nu)\tau b}$$
$$= \frac{Gb}{2\pi(1-\nu)\tau} \qquad \textit{for edge dislocations}$$

$$r = \frac{Gb^2}{2\pi F_r}$$
$$= \frac{Gb^2}{2\pi\tau b}$$
$$= \frac{Gb}{2\pi\tau} \qquad \textit{for screw dislocations}$$