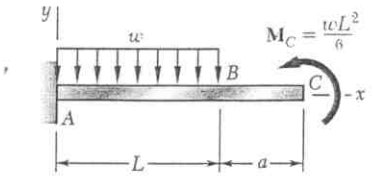
Problem 9.6

9.5 and 9.6 For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.



$[x=0, y=0]$

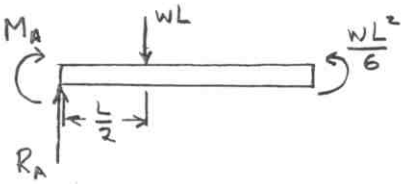
$[x=0, \frac{dy}{dx}=0]$

Using ABC as a free body,

$+\uparrow \Sigma F_y = 0$: $\quad R_A - wL = 0 \qquad R_A = wL$

$+\circlearrowleft \Sigma M_A = 0$: $\quad -M_A - (wL)(\frac{L}{2}) + \frac{wL^2}{6} = 0$

$M_A = -\frac{1}{3}wL^2$

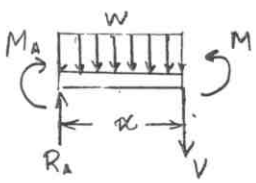


Using AJ as a free body, (Portion AB only)

$+\circlearrowleft M_J = 0$: $\quad M + (wx)(\frac{x}{2}) - R_A x - M_A = 0$

$M = -\frac{1}{2}wx^2 + R_A x + M_A$

$= -\frac{1}{2}wx^2 + wLx - \frac{1}{3}wL^2$



$EI\frac{d^2y}{dx^2} = -\frac{1}{2}wx^2 + wLx - \frac{1}{3}wL^2$

$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{2}wLx^2 - \frac{1}{3}wLx + C_1$

$[x=0, \frac{dy}{dx}=0]$: $\quad -0 + 0 - 0 + C_1 = 0 \qquad C_1 = 0$

$EIy = -\frac{1}{24}wx^4 + \frac{1}{6}wLx^3 - \frac{1}{6}wLx^2 + C_2$

$[x=0, y=0]$ $\quad -0 + 0 - 0 + C_2 = 0 \qquad C_2 = 0$

(a) Elastic curve over AB. $\quad y = \frac{w}{24EI}(-x^4 + 4Lx^3 - 4L^2x^2)$

$\frac{dy}{dx} = \frac{w}{6EI}(-x^3 + 2Lx^2 - L^2x)$

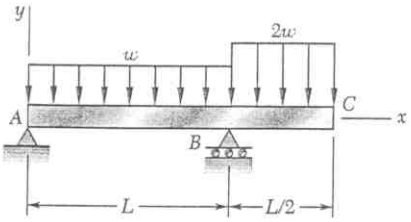
(b) y at $x = L$: $\quad y_B = -\frac{wL^4}{24EI} \qquad y_B = \frac{wL^4}{24EI} \downarrow$

(c) $\frac{dy}{dx}$ at $x = L$: $\quad \left.\frac{dy}{dx}\right|_B = 0 \qquad \theta_B = 0$

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Problem 9.8

9.8 For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the slope at *A*, (*c*) the slope at *B*.

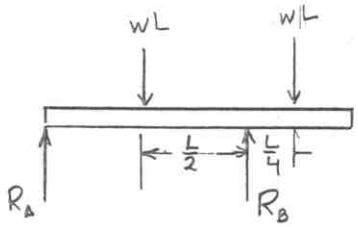


$[x=0, y=0]$ $[x=L, y=0]$

Using free body ABC,

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + (wL)\left(\frac{L}{2}\right) - (wL)\left(\frac{L}{4}\right) = 0$$

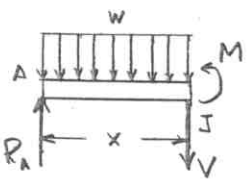
$$R_A = \frac{1}{4}wL$$



For portion AB, $(0 < x < L)$

$$+\circlearrowleft \Sigma M_J = 0 \qquad M - R_A x + (wx)\left(\frac{x}{2}\right) = 0$$

$$M = \frac{1}{4}wLx - \frac{1}{2}wx^2$$



$$EI\frac{d^2y}{dx^2} = \frac{1}{4}wLx - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = \frac{1}{8}wLx^2 - \frac{1}{6}wx^3 + C_1$$

$$EIy = \frac{1}{24}wLx^3 - \frac{1}{24}wx^4 + C_1x + C_2$$

$[x=0, y=0]$: $\quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

$[x=L, y=0]$ $\quad 0 = \frac{1}{24}wL^4 - \frac{1}{24}wL^4 + C_1L + 0 = 0 \qquad C_1 = 0$

(a) Elastic curve $(0 \le x \le L)$. $\quad y = \frac{w}{24EI}(Lx^3 - x^4)$ ◀

$$\frac{dy}{dx} = \frac{w}{24EI}(3Lx^2 - 4x^3)$$

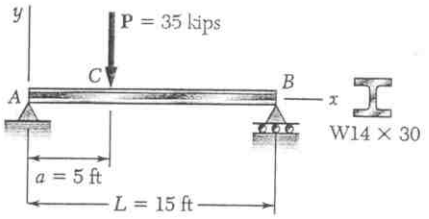
(b) $\frac{dy}{dx}$ at $x=0$. $\quad \left.\frac{dy}{dx}\right|_A = 0 \qquad \theta_A = 0$ ◀

(c) $\frac{dy}{dx}$ at $x=L$. $\quad \left.\frac{dy}{dx}\right|_B = -\frac{wL^3}{24EI} \qquad \theta_B = \frac{wL^3}{24EI}$ ◀

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Problem 9.13

9.13 For the beam and loading shown, determine the deflection at point C. Use $E = 29 \times 10^6$ psi.



$[x=0, y=0]$ $[x=L, y=0]$

$[x=a, y=y]$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$

Let $b = L - a$.

Reactions: $R_A = \frac{Pb}{L} \uparrow$, $R_B = \frac{Pa}{L} \uparrow$

Bending moments:

$0 < x < a$: $M = \frac{Pb}{L} x$

$a < x < L$: $M = \frac{P}{L}[bx - L(x-a)]$

$0 < x < a$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}(bx)$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left(\frac{1}{2}bx^2\right) + C_1 \quad (1)$$

$$EIy = \frac{P}{L}\left(\frac{1}{6}bx^3\right) + C_1 x + C_2 \quad (2)$$

$a < x < L$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}[bx - L(x-a)]$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left[\frac{1}{2}bx^2 - \frac{1}{2}L(x-a)^2\right] + C_3 \quad (3)$$

$$EIy = \frac{P}{L}\left[\frac{1}{6}bx^3 - \frac{1}{6}L(x-a)^3\right] + C_3 x + C_4 \quad (4)$$

$[x=0, y=0]$ Eq (2) $\quad 0 = 0 + 0 + C_2 \quad C_2 = 0$

$[x=a, \frac{dy}{dx}=\frac{dy}{dx}]$ Eqs. (1) and (3) $\quad \frac{P}{L}\left(\frac{1}{2}ba^2\right) + C_1 = \frac{P}{L}\left[\frac{1}{2}ba^2 + 0\right] + C_3 \therefore C_3 = C_1$

$[x=a, y=y]$ Eqs (2) and (4) $\quad \frac{P}{L}\left(\frac{1}{6}ba^3\right) + C_1 a + C_2 = \frac{P}{L}\left[\frac{1}{6}ba^3 + 0\right] + C_1 a + C_4 \quad C_4 = C_2 = 0$

$[x=L, y=0]$ Eq. (4) $\quad \frac{P}{L}\left[\frac{1}{6}bL^3 - \frac{1}{6}L(L-a)^3\right] + C_3 L = 0$

$$C_1 = C_3 = \frac{P}{L}\left[\frac{1}{6}(L-a)^3 - \frac{1}{6}bL^2\right] = \frac{P}{L}\left(\frac{1}{6}b^3 - \frac{1}{6}bL^2\right)$$

Make $x = a$ in Eq. (2).

$$y_C = \frac{P}{EIL}\left[\frac{1}{6}ba^3 + \frac{1}{6}b^3a - \frac{1}{6}bL^2a\right] = \frac{P(ba^3 + b^3a - L^2ab)}{6EIL}$$

Data: $P = 35$ kips, $E = 29\times10^6$ psi $= 29\times10^3$ kips/in^2

$L = 15$ ft, $a = 5$ ft, $b = 10$ ft

$I = 291$ in^4, $EI = 8.439\times10^6$ kip·in^2 $= 58.604\times10^3$ kip·ft^2

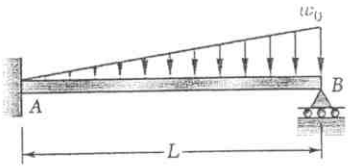
$$y_C = \frac{35}{(6)(58.604\times10^3)(15)}\left[(10)(5)^3 + (10^3)(5) - (15)^2(5)(10)\right]$$

$$= -33.179\times10^{-3} \text{ ft} = -0.398 \text{ in.}$$

$y_C = 0.398$ in. ↓

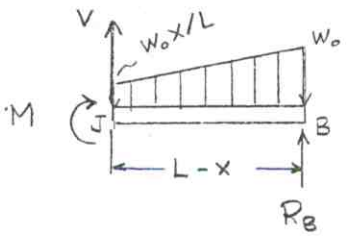
Problem 9.21

***9.19* through 9.22** For the beam and loading shown, determine the reaction at the roller support.



$[x=0, y=0]$ $[x=L, y=0]$

$\left[x=0, \frac{dy}{dx}=0\right]$



Reactions are statically indeterminate.

Boundary conditions are shown at left.

Using free body JB, $\circlearrowleft + \Sigma M_J = 0$:

$$-M + R_B(L-x) + \tfrac{1}{2}w_0(L-x)\tfrac{2}{3}(L-x) + \tfrac{1}{2}\frac{w_0x}{L}(L-x)\tfrac{1}{3}(L-x) = 0$$

$$M = R_B(L-x) - \frac{w_0}{6L}\left[2L(L-x)^2 + x(L-x)^2\right]$$

$$= R_B(L-x) - \frac{w_0}{6L}\left[2L^3 - 4L^2x + 2Lx^2 + xL^2 - 2Lx^2 + x^3\right]$$

$$= R_B(L-x) - \frac{w_0}{6L}\left(x^3 - 3L^2x + 2L^3\right)$$

$$EI\frac{d^2y}{dx^2} = R_B(L-x) - \frac{w_0}{6L}\left(x^3 - 3L^2x + 2L^3\right)$$

$$EI\frac{dy}{dx} = R_B\left(Lx - \tfrac{1}{2}x^2\right) - \frac{w_0}{6L}\left(\tfrac{1}{4}x^4 - \tfrac{3}{2}L^2x^2 + 2L^2x\right) + C_1$$

$$EIy = R_B\left(\tfrac{1}{2}Lx^2 - \tfrac{1}{6}x^3\right) - \frac{w_0}{6L}\left(\tfrac{1}{20}x^5 - \tfrac{1}{2}L^2x^3 + L^2x^2\right) + C_1x + C_2$$

$[x=0, y=0] \rightarrow C_2 = 0$

$\left[x=0, \frac{dy}{dx}=0\right] \rightarrow C_1 = 0$

$$[x=L, y=0] \quad 0 = R_BL^3\left(\tfrac{1}{2} - \tfrac{1}{6}\right) - \frac{w_0L^4}{6}\left(\tfrac{1}{20} - \tfrac{1}{2} + 1\right)$$

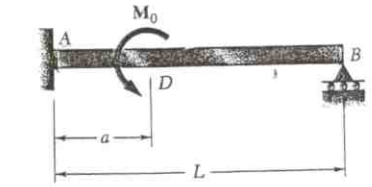
$$\tfrac{1}{3}R_B = \left(\tfrac{1}{6}\right)\left(\tfrac{11}{20}\right)w_0L$$

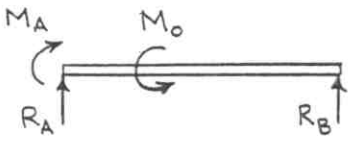
$$R_B = \tfrac{11}{40}w_0L \uparrow$$

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Problem 9.32

***9.31* and 9.32** Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.





$+\uparrow \Sigma F_y = 0$: $R_A + R_B = 0$ $\qquad R_A = -R_B$

$+\circlearrowleft \Sigma M_A = 0$: $M_0 - M_A + R_B L = 0$ $\qquad M_A = R_B L + M_0$

$0 \le x \le a$: $M = M_A + R_A x$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x$$

$$EI\frac{dy}{dx} = M_A x + \tfrac{1}{2}R_A x^2 + C_1$$

$$EIy = \tfrac{1}{2}M_A x^2 + \tfrac{1}{6}R_A x^3 + C_1 x + C_2$$

$a \le x \le L$: $M = M_A + R_A x - M_0$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - M_0$$

$$EI\frac{dy}{dx} = M_A x + \tfrac{1}{2}R_A x^2 - M_0 x + C_3$$

$$EIy = \tfrac{1}{2}M_A x^2 + \tfrac{1}{6}R_A x^3 - \tfrac{1}{2}M_0 x^2 + C_3 x + C_4$$

$\left[x = 0, \frac{dy}{dx} = 0\right]$: $0 + 0 + C_1 = 0$ $\qquad \therefore C_1 = 0$

$[x = 0, y = 0]$: $0 + 0 + 0 + C_2 = 0$ $\qquad \therefore C_2 = 0$

$\left[x = a, \frac{dy}{dx} = \frac{dy}{dx}\right]$: $M_A a + \tfrac{1}{2}R_A a^2 = M_A a + \tfrac{1}{2}R_A a^2 - M_0 a + C_3$ $\qquad \therefore C_3 = M_0 a$

$[x = a, y = y]$: $\tfrac{1}{2}M_A a^2 + \tfrac{1}{6}R_A a^3 = \tfrac{1}{2}M_A a^2 + \tfrac{1}{6}R_A a^3 - \tfrac{1}{2}M_0 a^2 + (M_0 a)(a) + C_4$ $\qquad \therefore C_4 = -\tfrac{1}{2}M_0 a^2$

$[x = L, y = 0]$: $\tfrac{1}{2}M_A L^2 + \tfrac{1}{6}R_A L^3 - \tfrac{1}{2}M_0 L^2 + (M_0 a)(L) - \tfrac{1}{2}M_0 a^2 = 0$

$$\tfrac{1}{2}(R_B L + M_0)L^2 + \tfrac{1}{6}(-R_B)L^3 - \tfrac{1}{2}M_0 L^2 + M_0 aL - \tfrac{1}{2}M_0 a^2 = 0$$

$$R_B = \frac{3M_0 a}{2L^3}(a - 2L) = \frac{3M_0}{2L^3}\left(\frac{L}{3}\right)\left(\frac{L}{3} - 2L\right) = -\frac{5M_0}{6L} \qquad R_B = \frac{5M_0}{6L} \downarrow$$

DEFLECTION AT D. (y AT $x = a = \frac{L}{3}$)

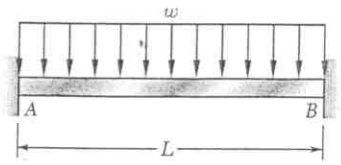
$$y_D = \frac{1}{EI}\left\{\tfrac{1}{2}M_A x^2 + \tfrac{1}{6}R_A x^3\right\} = \frac{1}{EI}\left\{\tfrac{1}{2}\left(-\frac{5M_0}{6L}L + M_0\right)\left(\frac{L}{3}\right)^2 + \tfrac{1}{6}\left(+\frac{5M_0}{6L}\right)\left(\frac{L}{3}\right)^3\right\}$$

$$= \frac{7M_0 L^2}{486EI} \qquad y_D = \frac{7M_0 L^2}{486EI} \uparrow$$

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Problem 9.34

9.33 and 9.34 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.



Reactions are statically indeterminate.

By symmetry, $R_B = R_A$; $M_B = M_A$

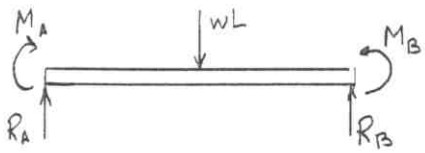
$\frac{dy}{dx} = 0$ at $x = \frac{L}{2}$

$[x=0, y=0]$ $[x=L, y=0]$

$[x=0, \frac{dy}{dx}=0]$ $[x=L, \frac{dy}{dx}=0]$

$+\uparrow \Sigma F_y = 0$: $R_A + R_B - wL = 0$

$R_B = R_A = \frac{1}{2}wL \uparrow$



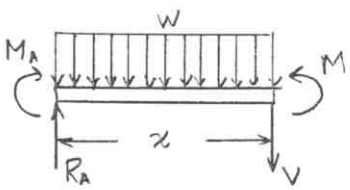
Over entire beam, $M = M_A + R_A x - \frac{1}{2}wx^2$

$EI\frac{d^2y}{dx^2} = M_A + \frac{1}{2}wLx - \frac{1}{2}wx^2$

$EI\frac{dy}{dx} = M_A x + \frac{1}{4}wLx^2 - \frac{1}{6}wx^3 + C_1$

$[x=0, \frac{dy}{dx}=0]$ $0 + 0 - 0 + C_1 = 0$

$C_1 = 0$



$[x=\frac{L}{2}, \frac{dy}{dx}=0]$ $\frac{1}{2}M_A L + \frac{1}{16}wL^3 - \frac{1}{48}wL^3 + 0 = 0$

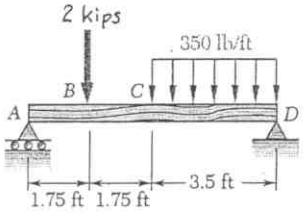
$M_A = -\frac{1}{12}wL^2$

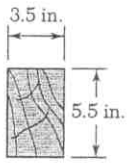
$M = -\frac{1}{12}wL^2 + \frac{1}{2}wLx - \frac{1}{2}wx^2$ $M = w\cdot[6x(L-x) - L^2]/12$

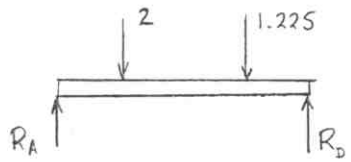
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Problem 9.45

9.45 For the timber beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 1.6 \times 10^6$ psi.







Units: Forces in kips; lengths in ft.

$+\circlearrowleft \Sigma M_D = 0$:

$-7R_A + (2)(5.25) + (1.225)(1.75) = 0$

$R_A = 1.80625$ kips

$w(x) = 0.350\langle x - 3.5\rangle^0$

$\frac{dV}{dx} = -w = -0.35\langle x - 3.5\rangle^0$

$\frac{dM}{dx} = V = 1.05625 - 1\langle x - 1.75\rangle^0 - 0.35\langle x - 3.5\rangle^1$

$EI\frac{d^2y}{dx^2} = M = 1.80625x - 2\langle x - 1.75\rangle^1 - 0.175\langle x - 3.5\rangle^2$ kip ft

$EI\frac{dy}{dx} = 0.903125x^2 - 1\langle x - 1.75\rangle^2 - 0.05833\langle x - 3.5\rangle^3 + C_1$ kip ft^2

$EIy = 0.301042x^3 - \frac{1}{3}\langle x - 1.75\rangle^3 - 0.014583\langle x - 3.5\rangle^4 + C_1x + C_2$ kip ft^3

$[x = 0, y = 0]$ $\quad C_2 = 0$

$[x = 7, y = 0]$ $\quad (0.301042)(7)^3 - \frac{1}{3}(5.25)^3 - 0.014583(3.5)^4 + C_1(7) + 0 = 0$

$C_1 = -7.54779$ kip·ft^2

Data: $E = 1.6 \times 10^6$ psi $= 1.6 \times 10^3$ ksi

$I = \frac{1}{12}(3.5)(5.5)^3 = 48.526$ in^4

$EI = (1.6 \times 10^3)(48.526) = 77.6417$ kip·in^2 $= 539.18$ kip·ft^2

(a) Slope at A. ($\frac{dy}{dx}$ at $x = 0$)

$EI\frac{dy}{dx} = 0 - 0 - 0 - 7.54779$ kip·ft^2

$\theta_A = -\frac{7.54779}{539.18} = -14.00 \times 10^{-3}$ rad $\qquad \theta_A = 14.00 \times 10^{-3}$ rad ↺

(b) Deflection at C. (y at $x = 3.5$ ft.)

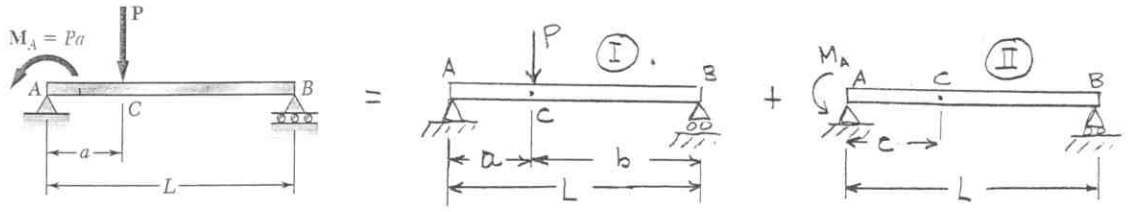
$EIy_C = (0.301042)(3.5)^3 - \frac{1}{3}(1.75)^3 - 0 - (7.54779)(3.5) + 0$

$= -15.297$ kip·ft^3

$y_C = -\frac{15.297}{539.18} = -28.37 \times 10^{-3}$ ft $\qquad y_C = 0.340$ in. ↓

Problem 9.66

For the beam and loading shown, determine (*a*) the deflection at the *C*, (*b*) the slope at end *A*.



Loading I: Case 5 of Appendix D. $b = L-a$

For $x = a$: $(y_c)_1 = -\dfrac{Pa^2b^2}{3EIL} = -\dfrac{Pa^2(L-a)^2}{3EIL}$

For end A: $(\theta_A)_1 = -\dfrac{Pb(L^2-b^2)}{6EIL} = -\dfrac{P(L-a)(2La-a^2)}{6EIL}$

$$= -\frac{Pa(2L^2-3La+a^2)}{6EIL}$$

Loading II: Mirror image of Case 5 of Appendix D.

Replace x by $(L-x)$ in expression for y.

$$y = -\frac{M_A}{6EIL}\left[(L-x)^3 - L^2(L-x)\right] = -\frac{M_A x}{6EIL}\left(3Lx - 2L^2 - x^2\right)$$

With $x = a$, and $M_A = Pa$, $(y_c)_2 = -\dfrac{Pa^2}{6EIL}(3La - 2L^2 - a^2)$

$$(\theta_A)_2 = \frac{M_A L}{3EI} = \frac{PaL}{3EI}$$

(a) Deflection at C. $y_c = (y_c)_1 + (y_c)_2$

$$y_c = -\frac{Pa^2}{6EIL}\left[2(L-a)^2 + 3La - 2L^2 - a^2\right] \qquad y_c = \frac{Pa^3(L-a)}{6EIL} \uparrow$$

(b) Slope at end A. $\theta_A = (\theta_A)_1 + (\theta_A)_2$

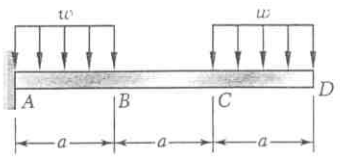
$$\theta_A = -\frac{Pa}{6EIL}\left[2L^2 - 3La + a^2 - 2L^2\right] = +\frac{Pa^2(3L-a)}{6EIL}$$

$$\theta_A = \frac{Pa^2(3L-a)}{6EIL} \curvearrowright$$

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Problem 9.71

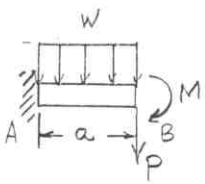
9.71 and 9.72 For the cantilever beam and loading shown, determine the slope and deflection at point *B*.



Consider portion AB as a cantiver beam subjected to the three loadings shown.

By statics: $P = wa$

$$M = (wa)\left(a + \frac{a}{2}\right) = \frac{3}{2}wa^2$$



Slope at B. $\theta_B = (\theta_B)_w + (\theta_B)_P + (\theta_B)_M$

Case 2 of App. D. $(\theta_B)_w = -\dfrac{wa^3}{6EI}$

Case 1 of App. D. $(\theta_B)_P = -\dfrac{(wa)a^2}{2EI} = -\dfrac{wa^3}{2EI}$

Case 3 of App. D $(\theta_B)_M = -\dfrac{\left(\frac{3}{2}wa^2\right)a}{EI} = -\dfrac{3wa^3}{2EI}$

$$\theta_B = -\frac{13wa^3}{6EI} \qquad \theta_B = \frac{13wa^3}{6EI} \circlearrowleft$$

Deflection at B. $y_B = (y_B)_w + (y_B)_P + (y_B)_M$

Case 2 of App. D. $(y_B)_w = -\dfrac{wa^4}{8EI}$

Case 1 of App. D. $(y_B)_P = -\dfrac{(wa)a^3}{3EI} = -\dfrac{wa^4}{3EI}$

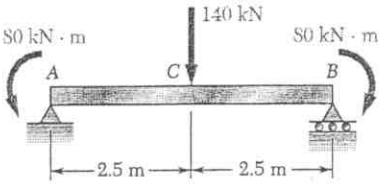
Case 3 of App D. $(y_B)_M = -\dfrac{\left(\frac{3}{2}wa^2\right)a}{2EI} = -\dfrac{3wa^3}{4EI}$

$$y_B = -\frac{29wa^4}{24EI} \qquad y_B = \frac{29wa^4}{24EI} \downarrow$$

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Problem 9.77

9.77 and 9.78 For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.



Units: Forces in kN, lengths in m.

Loading I: Moment at B.

Case 7 of Appendix D. $M = 80$ kN·m, $L = 5.0$ m, $x = 2.5$ m

$$\theta_A = \frac{ML}{6EI} = \frac{(80)(5.0)}{6EI} = \frac{66.667}{EI}$$

$$y_C = -\frac{M}{6EIL}(x^3 - L^2x) = -\frac{80}{6EI(5.0)}\left[2.5^3 - (5.0)^2(2.5)\right] = \frac{125}{EI}$$

Loading II: Moment at A. Case 7 of Appendix D.

$M = 80$ kN·m, $L = 5.0$ m, $x = 2.5$ m

$$\theta_A = \frac{ML}{3EI} = \frac{(80)(5.0)}{3EI} = \frac{133.333}{EI}$$

$$y_C = \frac{125}{EI} \quad \text{(Same as loading I)}$$

Loading III: 140 kN concentrated load at C. $P = 140$ kN

$$\theta_A = -\frac{PL^2}{16EI} = -\frac{(140)(5.0)^2}{16EI} = -\frac{218.75}{EI}$$

$$y_C = -\frac{PL^3}{48EI} = -\frac{(140)(5.0)^3}{48EI} = -\frac{364.583}{EI}$$

Data: $E = 200\times10^9$ Pa, $I = 156\times10^6$ mm^4 $= 156\times10^{-6}$ m^4

$$EI = (200\times10^9)(156\times10^{-6}) = 31.2\times10^6 \text{ N·m}^2 = 31200 \text{ kN·m}^2$$

(a) Slope at A. $\theta_A = \dfrac{67.667 + 133.333 - 218.75}{31200} = -0.601\times10^{-3}$ rad

$\theta_A = 0.601\times10^{-3}$ rad ↺

(b) Deflection at C. $y_C = \dfrac{125 + 125 - 364.583}{31200} = -3.67\times10^{-3}$ m

$y_C = 3.67$ mm ↓

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