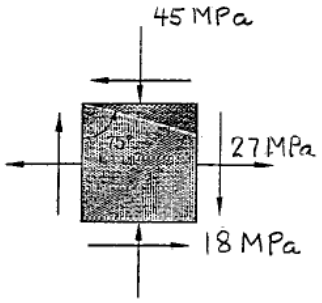
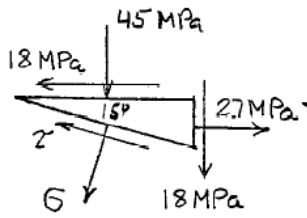
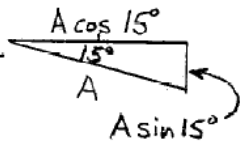
Problem 7.1

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

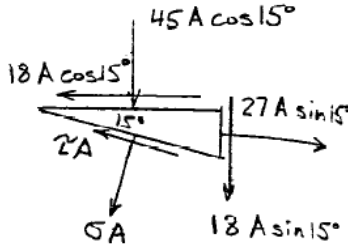




Stresses



Areas



Forces

$+\nearrow \Sigma F = 0$:

$$\sigma A + 18A\cos 15^\circ \sin 15^\circ + 45A\cos 15^\circ \cos 15^\circ - 27A\sin 15^\circ \sin 15^\circ + 18A\sin 15^\circ \cos 15^\circ = 0$$

$$\sigma = -18\cos 15^\circ \sin 15^\circ - 45\cos^2 15^\circ + 27\sin^2 15^\circ - 18\sin 15^\circ \cos 15^\circ$$

$$\sigma = -49.2 \text{ MPa} \quad \blacktriangleleft$$

$+\nwarrow \Sigma F = 0$:

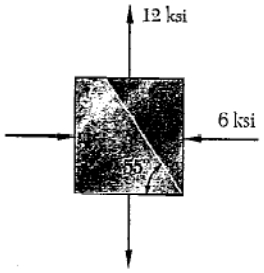
$$\tau A + 18A\cos 15^\circ \cos 15^\circ - 45A\cos 15^\circ \sin 15^\circ - 27A\sin 15^\circ \cos 15^\circ - 18A\sin 15^\circ \sin 15^\circ = 0$$

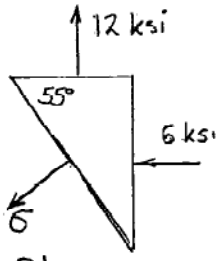
$$\tau = -18(\cos^2 15^\circ - \sin^2 15^\circ) + (45+27)\cos 15^\circ \sin 15^\circ$$

$$\tau = 2.41 \text{ MPa} \quad \blacktriangleleft$$

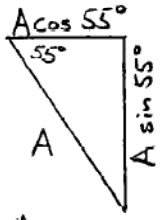
Problem 7.4

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

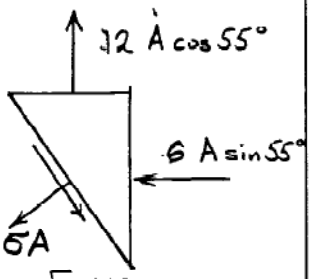




Stresses



Areas



Forces

$+\nearrow \Sigma F = 0$:

$$\sigma A - 12A\cos 55^\circ \cos 55^\circ + 6A\sin 55^\circ \sin 55^\circ = 0$$

$$\sigma = 12\cos^2 55^\circ - 6\sin^2 55^\circ$$

$$\sigma = -0.078 \text{ ksi} \quad \blacktriangleleft$$

$+\searrow \Sigma F = 0$:

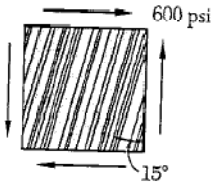
$$\tau A - 12A\cos 55^\circ \sin 55^\circ - 6A\sin 55^\circ \cos 55^\circ$$

$$\tau = 18\cos 55^\circ \sin 55^\circ$$

$$\tau = 8.46 \text{ ksi} \quad \blacktriangleleft$$

Problem 7.17

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical.. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.



$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 600 \text{ psi}$

$\theta = -15° \qquad 2\theta = -30°$

(a) $\tau_{x'y'} = -\dfrac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$

$= -0 + 600 \cos(-30°)$

$\tau_{x'y'} = 520 \text{ psi}$ ◄

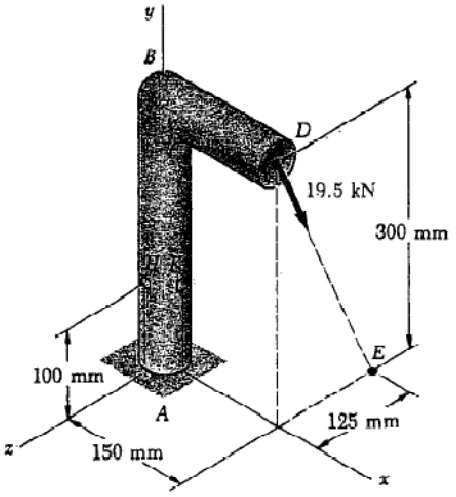
(b) $\sigma_{x'} = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$

$= 0 + 0 + 600 \sin(-30°)$

$\sigma_{x'} = -300 \text{ psi}$ ◄

Problem 7.23

7.23 A 19.5-kN force is applied at point D of the cast-iron post shown. Knowing that the post has a diameter of 60 mm, determine the principal stresses and the maximum shearing stress at point H.



$$\ell_{DE} = \sqrt{(300)^2 + (125)^2} = 325 \text{ mm}$$

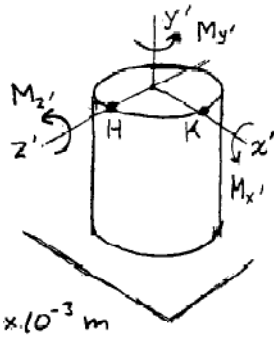
Resolve the 19.5 kN force F at point D into x, y, and z components.

$$F_x = 0$$

$$F_y = -\frac{300}{325}(19.5) = -18 \text{ kN} = -18\times10^3 \text{ N}$$

$$F_z = -\frac{125}{325}(19.5) = -7.5 \text{ kN} = -7.5\times10^3 \text{ N}$$

Determine the force-couple system at the point on the y-axis where it intersects the plane containing elements H and K.



$F_{x'} = 0$, $F_{y'} = -18\times10^3$ N, $F_{z'} = -7.5\times10^3$ N

$M_{x'} = -(7.5 \text{ kN})(300 \text{ mm} - 100 \text{ mm}) = -1.5\times10^3$ N·m

$M_{y'} = (7.5 \text{ kN})(150 \text{ mm}) = 1.125\times10^3$ N·m

$M_{z'} = -(18 \text{ kN})(150 \text{ mm}) = -2.7\times10^3$ N·m

Properties of section. (Circle) $c = \frac{1}{2}d = 30 \text{ mm} = 30\times10^{-3}$ m

$A = \pi c^2 = \pi(30)^2 = 2.8274\times10^3 \text{ mm}^2 = 2.8274\times10^{-3} \text{ m}^4$

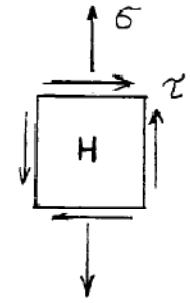
$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(30)^4 = 636.17\times10^3 \text{ mm}^4 = 636.17\times10^{-9} \text{ m}^4$

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(30)^4 = 1.27235\times10^6 \text{ mm}^4 = 1.27235\times10^{-6} \text{ m}^4$

(Semicircle) $Q = \frac{2}{3}c^3 = \frac{2}{3}(30)^3 = 18\times10^3 \text{ mm}^3 = 18\times10^{-6} \text{ m}^3$

$t = d = 60 \text{ mm} = 60\times10^{-3}$ m

Stresses at H.



$$\sigma = \frac{F_x}{A} - \frac{M_x c}{I} = \frac{-18\times10^3}{2.8274\times10^{-3}} - \frac{(-1.5\times10^3)(30\times10^{-3})}{636.17\times10^{-9}}$$

$$= 64.370\times10^6 \text{ Pa} = 64.37 \text{ MPa}$$

$$\tau = \frac{F_x Q}{It} + \frac{M_y c}{J} = 0 + \frac{(1.125\times10^3)(30\times10^{-3})}{1.27235\times10^{-6}}$$

$$= 26.526\times10^6 \text{ Pa} = 26.526 \text{ MPa}$$

Continued

Problem 7.23 continued

Principal stresses $\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$\sigma_{max,min} = \frac{0 + 64.37}{2} \pm \sqrt{\left(\frac{0 - 64.37}{2}\right)^2 + (26.526)^2}$$

$$= -32.185 \pm 41.707 \quad \text{MPa}$$

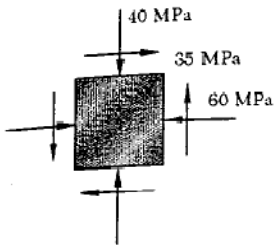
$\sigma_{max} = 73.9$ MPa

$\sigma_{min} = -9.52$ MPa

Maximum shearing stress $\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$ $\tau_{max} = 41.7$ MPa

Problem 7.31

7.31 Solve Probs. 7.5 and 7.9, using Mohr's circle.



7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.

$\sigma_x = -60$ MPa, $\sigma_y = -40$ MPa; $\tau_{xy} = 35$ MPa

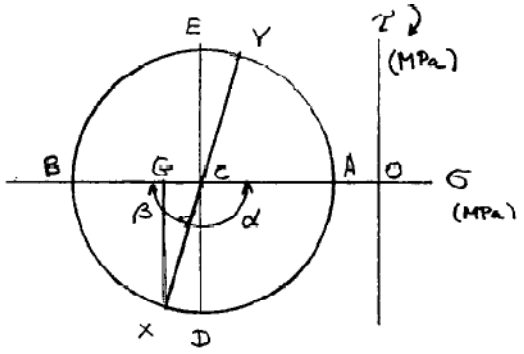
$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -50$ MPa

Plotted points for Mohr's circle.

X: $(\sigma_x, -\tau_{xy}) = (-60\text{ MPa}, -35\text{ MPa})$

Y: $(\sigma_y, \tau_{xy}) = (-40\text{ MPa}, 35\text{ MPa})$

C: $(\sigma_{ave}, 0) = (-50\text{ MPa}, 0)$



(a) $\tan\beta = \frac{GX}{CG} = \frac{35}{10} = 3.500$

$\beta = 74.05°$

$\theta_B = -\frac{1}{2}\beta = -37.03°$ ◀

$\alpha = 180° - \beta = 105.95°$

$\theta_A = \frac{1}{2}\alpha = 52.97°$ ◀

$R = \sqrt{\overline{CG}^2 + \overline{GX}^2} = \sqrt{10^2 + 35^2} = 36.4$ MPa

(b) $\sigma_{min} = \sigma_{ave} - R = -50 - 36.4$ $\quad \sigma_{min} = -86.4$ MPa ◀

$\sigma_{max} = \sigma_{ave} + R = -50 + 36.4$ $\quad \sigma_{max} = -13.6$ MPa ◀

(a') $\theta_D = \theta_B + 45° = 7.97°$ $\quad \theta_D = 8.0°$ ◀

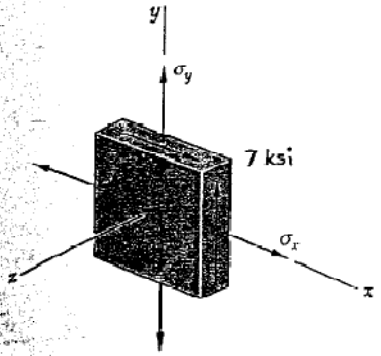
$\theta_E = \theta_A + 45° = 97.97°$ $\quad \theta_E = 98.0°$ ◀

$\tau_{max} = R = 36.4$ MPa $\quad \tau_{max} = 36.4$ MPa ◀

(b') $\sigma' = \sigma_{ave} = -50$ MPa $\quad \sigma' = -50.0$ MPa ◀

Problem 7.66

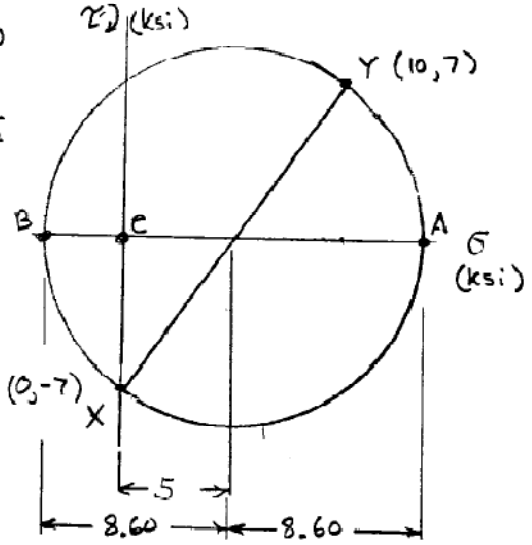
7.66 For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 0$ and $\sigma_y = 10$ ksi, (*b*) $\sigma_x = 18$ ksi and $\sigma_y = 8$ ksi. (*Hint*: Consider both in-plane and out-of-plane shearing stresses.)



(a) $\sigma_x = 0$, $\sigma_y = 10$ ksi, $\tau_{xy} = 7$ ksi

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{5^2 + 7^2} = 8.60 \text{ ksi}$$



$\sigma_a = \sigma_{ave} + R = 13.60$ ksi

$\sigma_b = \sigma_{ave} - R = -3.60$ ksi

$\sigma_c = 0$

$\sigma_{max} = 13.60$ ksi, $\sigma_{min} = -3.60$ ksi

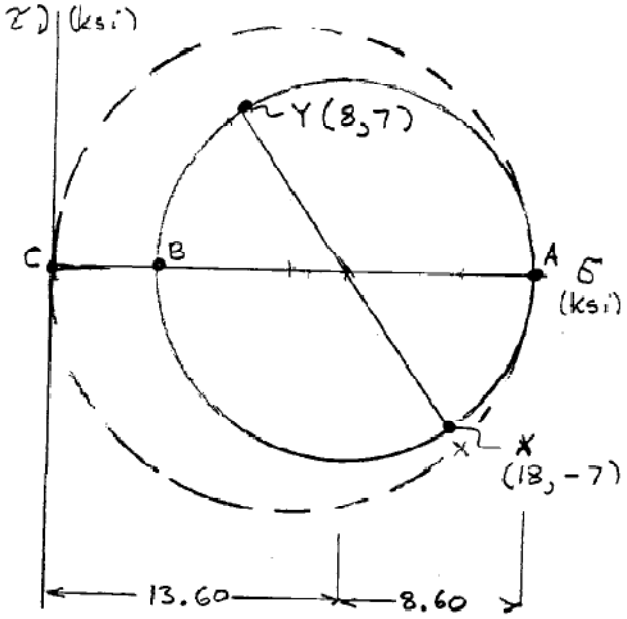
$\tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = \tfrac{1}{2}(13.60 + 3.60)$

$\tau_{max} = 8.60$ ksi

(b) $\sigma_x = 18$ ksi, $\sigma_y = 8$ ksi, $\tau_{xy} = 7$ ksi

$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 13$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{5^2 + 7^2} = 8.60$ ksi



$\sigma_a = \sigma_{ave} + R = 21.60$ ksi

$\sigma_b = \sigma_{ave} - R = 4.40$ ksi

$\sigma_c = 0$

$\sigma_{max} = 21.60$ ksi

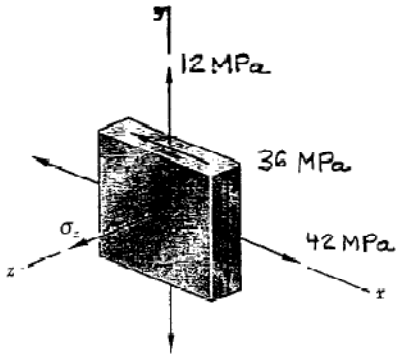
$\sigma_{min} = 0$

$\tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = \tfrac{1}{2}(21.60 - 0)$

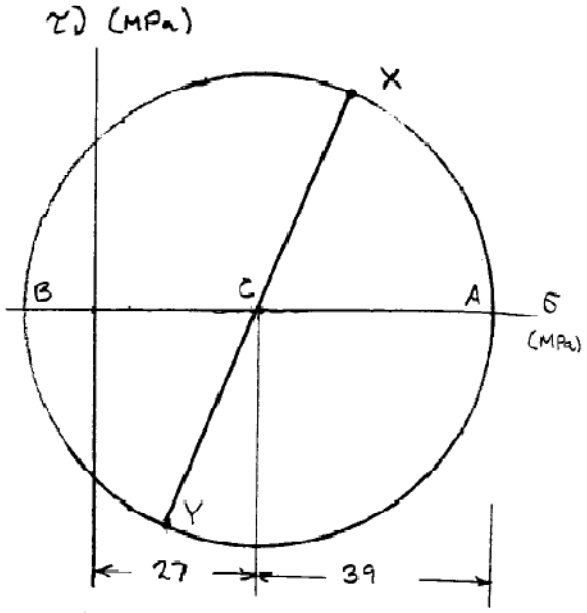
$\tau_{max} = 10.80$ ksi

Problem 7.71

7.70 and 7.71 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_z = +24$ MPa, (b) $\sigma_z = -24$ MPa, (c) $\sigma_z = 0$.



$\sigma_x = 42$ MPa, $\sigma_y = 12$ MPa, $\tau_{xy} = -36$ MPa



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 27 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(15)^2 + (-36)^2} = 39 \text{ MPa}$$ G.S

$$\sigma_a = \sigma_{ave} + R = 66 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -12 \text{ MPa}$$

(a) $\sigma_z = +24$ MPa $\quad \sigma_a = 66$ MPa $\quad \sigma_b = -12$ MPa

$\sigma_{max} = 66$ MPa $\quad \sigma_{min} = -12$ MPa $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 39$ MPa ◀

(b) $\sigma_z = -24$ MPa $\quad \sigma_a = 66$ MPa $\quad \sigma_b = -12$ MPa

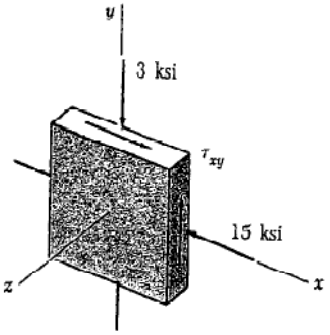
$\sigma_{max} = 66$ MPa $\quad \sigma_{min} = -24$ MPa $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 45$ MPa ◀

(c) $\sigma_z = 0$ $\quad \sigma_a = 66$ MPa $\quad \sigma_b = -12$ MPa

$\sigma_{max} = 66$ MPa $\quad \sigma_{min} = -12$ MPa $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 39$ MPa ◀

Problem 7.75

7.75 For the state of plane stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (*a*) 10 ksi, (*b*) 8.25 ksi.



$\sigma_x = -15$ ksi $\sigma_y = -3$ ksi $\sigma_z = 0$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -9$ ksi

(a) $\tau_{max} = 10$ ksi.

If τ_{max} is the in-plane maximum shearing stress, then $R = \tau_{max} = 10$ ksi

$\sigma_{max} = \sigma_{ave} + R = 1$ ksi

$\sigma_{min} = \sigma_{ave} - R = -19$ ksi

σ_z is the intermediate principal stress, the assumption $R = \tau_{max}$ is correct.

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{6^2 + \tau_{xy}^2} = 10$$

$$\tau_{xy} = \sqrt{10^2 - 6^2} \qquad \tau_{xy} = 8 \text{ ksi}$$

(b) $\tau_{max} = 8.25$ ksi.

If τ_{max} is the in-plane maximum shearing stress, then $R = \tau_{max} = 8.25$ ksi.

$\sigma_{max} = \sigma_{ave} + R = -9 + 8.25 = -0.75$ ksi

$\sigma_{min} = \sigma_{ave} - R = -9 - 8.25 = -17.25$ ksi

$\sigma_z = 0$ is <u>not</u> the intermediate principal stress.

Let $\sigma_z = 0 = \sigma_{max}$.

$\sigma_{min} = \sigma_{max} - 2\tau_{max} = 0 - (2)(8.25) = -16.5$ ksi

$R = \sigma_{ave} - \sigma_{min} = -9 - (-16.5) = 7.5$ ksi

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{6^2 + \tau_{xy}^2} = 7.5 \text{ ksi}$$

$$\tau_{xy} = \sqrt{7.5^2 - 6^2} = 4.5 \text{ ksi} \qquad \tau_{xy} = 4.5 \text{ ksi}$$