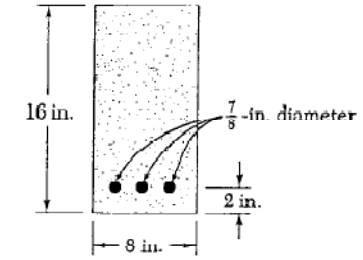
Problem 4.47

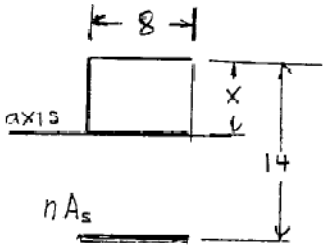
4.47 A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 30×10^6 psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.



$$n = \frac{E_s}{E_c} = \frac{30\times10^6}{3\times10^6} = 10$$

$$A_s = 3\,\frac{\pi}{4}d^2 = 3\left(\frac{\pi}{4}\right)\left(\frac{7}{8}\right)^2 = 1.8040 \text{ in}^2$$

$$nA_s = 18.040 \text{ in}^2$$



Locate neutral axis.

$$8x\,\frac{x}{2} - (18.040)(14-x) = 0$$

$$4x^2 + 18.040x - 252.56 = 0$$

Solve for x.

$$x = \frac{-18.040 + \sqrt{18.040^2 + (4)(4)(252.56)}}{(2)(4)} = 6.005 \text{ in.}$$

$$14 - x = 7.995 \text{ in}$$

$$I = \frac{1}{3}\,8x^3 + nA_s(14-x)^2 = \frac{1}{3}(8)(6.005)^3 + (18.040)(7.995)^2$$

$$= 1730.4 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $n = 1.0$, $|y| = 6.005$ in, $|\sigma| = 1350$ psi

$$M = \frac{(1350)(1730.5)}{(1.0)(6.005)} = 389\times10^3 \text{ lb-in} = 389 \text{ kip-in}$$

Steel: $n = 10$, $|y| = 7.995$, $\sigma = 20\times10^3$ psi

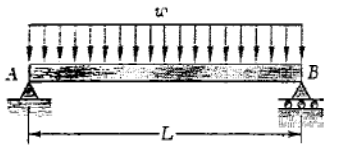
$$M = \frac{(20\times10^3)(1730.5)}{(10)(7.995)} = 433\times10^3 \text{ lb-in} = 433 \text{ kip-in}$$

Choose the smaller value. $M = 389$ kip-in $= 32.4$ kip-ft

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Problem 5.1

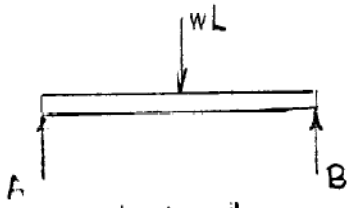
5.1 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.



Reactions.

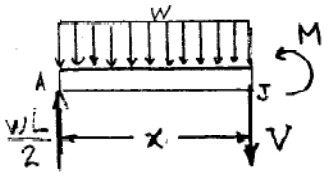
$\circlearrowleft \Sigma M_B = 0$: $\quad -AL + wL\cdot\frac{L}{2} = 0 \qquad A = \frac{wL}{2}$

$\circlearrowleft \Sigma M_A = 0$: $\quad BL - wL\cdot\frac{L}{2} = 0 \qquad B = \frac{wL}{2}$

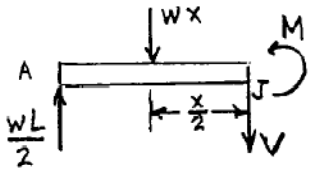


Free body diagram for determining reactions.

Over whole beam, $0 < x < L$



Place section at x.



Replace distributed load by equivalent concentrated load.

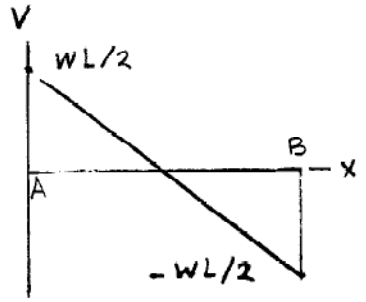
$+\uparrow \Sigma F_y = 0$: $\quad \frac{WL}{2} - wx - V = 0$

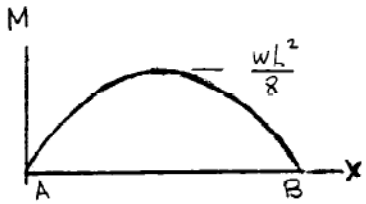
$$V = w\left(\frac{L}{2} - x\right)$$

$\circlearrowleft \Sigma M_J = 0$: $\quad -\frac{WL}{2}x + wx\frac{x}{2} + M = 0$

$$M = \frac{w}{2}(Lx - x^2)$$

$$M = \frac{w}{2}x(L - x)$$



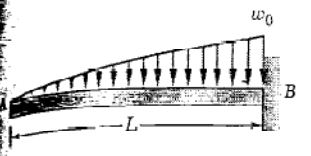


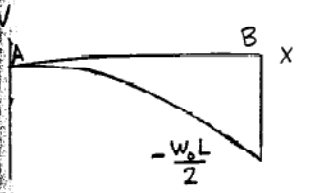
Maximum bending moment occurs at $x = \frac{L}{2}$.

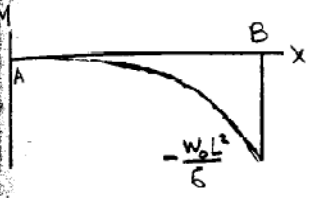
$$M_{max} = \frac{WL^2}{8}$$

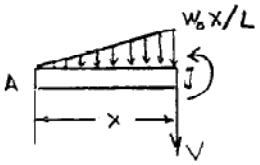
Problem 5.4

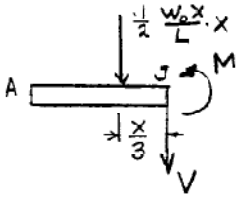
5.4 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.











$+\uparrow \Sigma F_y = 0 \qquad -\frac{1}{2}\frac{w_0 x}{L}\cdot x - V = 0$

$$V = -\frac{w_0 x^2}{2L}$$

$\circlearrowright \Sigma M_J = 0 \qquad \frac{1}{2}\frac{w_0 x}{L}\cdot x\cdot\frac{x}{3} + M = 0$

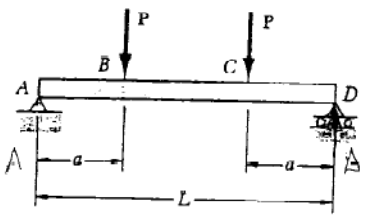
$$M = -\frac{w_0 x^3}{6L}$$

At $x = L$, $\quad V = -\frac{w_0 L}{2} \qquad |V|_{max} = \frac{w_0 L}{2}$

$$M = -\frac{w_0 L^2}{6} \qquad |M|_{max} = \frac{w_0 L^2}{6}$$

Problem 5.5

5.1 through 5.6 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.



Using entire beam ABCD as a free body,

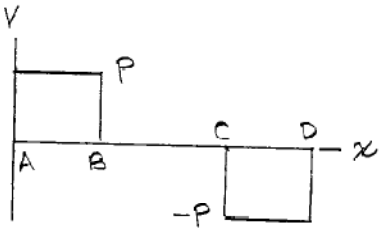
$+\circlearrowleft M_D = 0: \quad -AL + P(L-a) + Pa = 0$

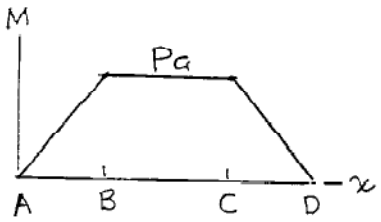
$A = P$

$+\circlearrowleft M_A = 0: \quad DL - Pa - P(L-a) = 0$

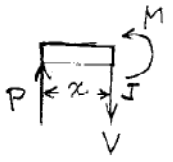
$D = P$

Check: $+\uparrow \Sigma F_y = P - P - P + P = 0$





From A to B: $0 < x < a$



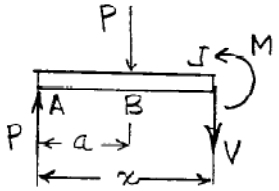
$+\uparrow \Sigma F_y = 0: \quad P - V = 0$

$V = P$ ◀

$+\circlearrowleft \Sigma M_J = 0: \quad -Px + M = 0$

$M = Px$ ◀

From B to C: $a < x < L-a$



$+\uparrow \Sigma F_y = 0$

$P - P - V = 0$

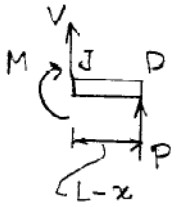
$V = 0$ ◀

$+\circlearrowleft \Sigma M_J = 0$

$-Px + P(x-a) + M = 0$

$M = Pa$ ◀

From C to D: $L-a < x < L$



$+\uparrow \Sigma F_y = 0: \quad V + P = 0$

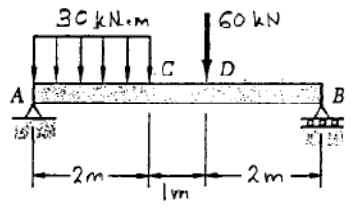
$V = -P$ ◀

$+\circlearrowleft \Sigma M_J = 0: \quad -M + P(L-x) = 0$

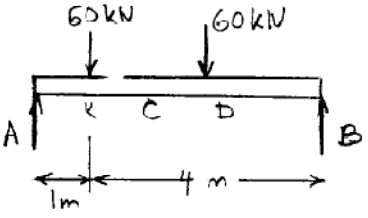
$M = P(L-x)$ ◀

Problem 5.9

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



Calculate reactions at A and B. Replace distributed load by an equivalent concentrated load after drawing the free body ACDB.



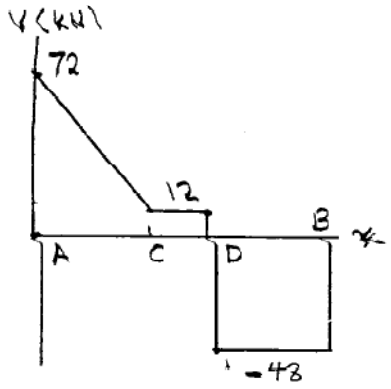
$+\circlearrowleft \Sigma M_B = 0$: $\quad -5A + (4)(60) + (2)(60) = 0$

$A = 72$ kN ↑

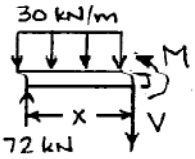
$+ \Sigma M_A = 0$: $\quad 5B - (1)(60) - (3)(60) = 0$

$B = 48$ kN ↑

Check $+\uparrow \Sigma F_y = 0$: $\quad 72 - 60 - 60 + 48 = 0$



A to C $\qquad 0 < x \leq 2$m



$+\uparrow \Sigma F_y = 0$:

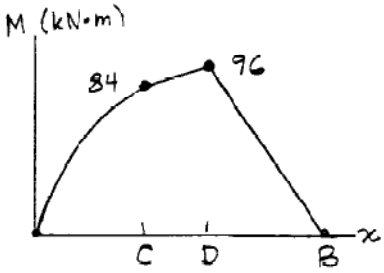
$72 - 30x - V = 0$

$V = (72 - 30x)$ kN

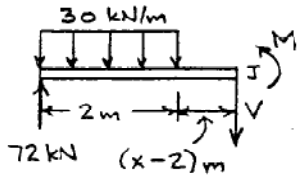
$+\circlearrowleft \Sigma M_J = 0$:

$-72x + (30x)\frac{x}{2} + M = 0$

$M = (72 - 15x^2)$ kN·m



C to D. $\qquad 2\text{m} \leq x < 3\text{m}$



$+\uparrow \Sigma F_y = 0$

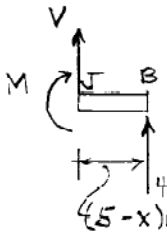
$72 - (30)(2) - V = 0$

$V = 12$ kN

$+\circlearrowleft \Sigma M_J = 0$:

$-72x + (30)(2)(x-1) + M = 0$

$M = (12x + 60)$ kN·m



$+\uparrow \Sigma F_y = 0$: $\quad V + 48 = 0$

$V = -48$ kN

$+\circlearrowleft \Sigma M_J = 0$: $\quad -M + 48(5-x)$

$M = 240 - 48x$

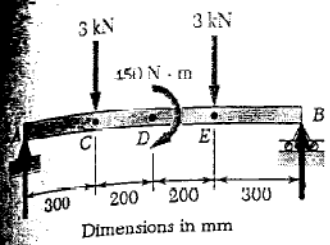
From the diagrams

(a) $|V|_{max} = 72.0$ kN

(b) $|M|_{max} = 96.0$ kN·m

Problem 5.12

5.11 and 5.12 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



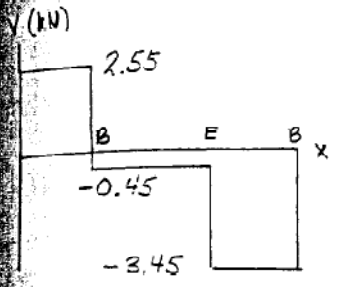
Dimensions in mm

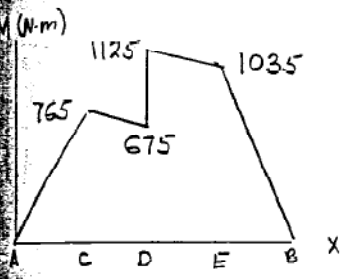
$\circlearrowleft \Sigma M_B = 0$: $(700)(3) - 450 + (300)(3) - 1000A = 0$

$A = 2.55$ kN ↑

$\circlearrowleft \Sigma M_A = 0$: $-(300)(3) - 450 - (700)(3) + 1000B = 0$

$B = 3.45$ kN ↑

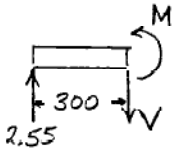




At A. $V = 2.55$ kN $\quad M = 0$

A to C. $V = 2.55$ kN

At C.



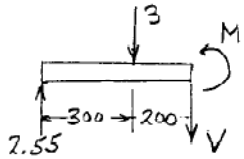
$\circlearrowleft \Sigma M_C = 0$

$-(300)(2.55) + M = 0$

$M = 765$ N·m

C to E. $V = -0.45$ kN

At D⁻.

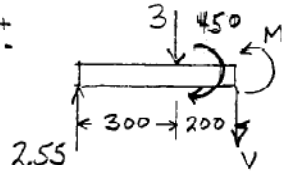


$\circlearrowleft \Sigma M_D = 0$:

$-(500)(2.55) + (200)(3) + M = 0$

$M = 675$ N·m

At D⁺.



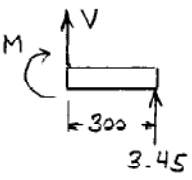
$\circlearrowleft \Sigma M_D = 0$:

$-(500)(2.55) + (200)(3) - 450 + M = 0$

$M = 1125$ N·m

E to B. $V = -3.45$ kN

At E.



$\circlearrowleft \Sigma M_E = 0$:

$-M + (300)(3.45) = 0$

$M = 1035$ N·m

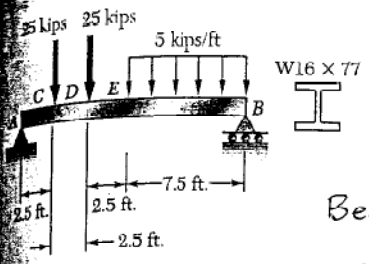
At B. $V = 3.45$ kN, $M = 0$

(a) Maximum $|V| = 3.45$ kN

(b) Maximum $|M| = 1125$ N·m

Problem 5.17

5.17 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.



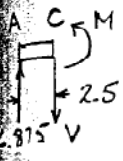
Reaction at A. $+\circlearrowleft \Sigma M_B = 0$:

$$-15.0 R_A + (12.5)(25) + (10.0)(25) + (31.75)(7.5)(5) = 0 \qquad R_A = 46.875 \text{ kips}$$

Bending moment at C. $+\circlearrowleft \Sigma M_C = 0$:

$$-(2.5)(46.875) + M = 0$$

$$M = 117.1875 \text{ kip·ft} = 1.40625 \times 10^3 \text{ kip·in}$$



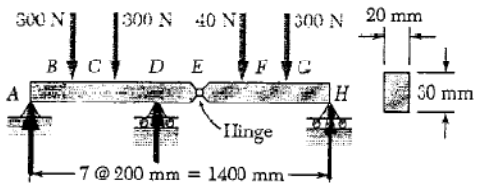
For W16×77 rolled steel section, $S = 134 \text{ in}^3$

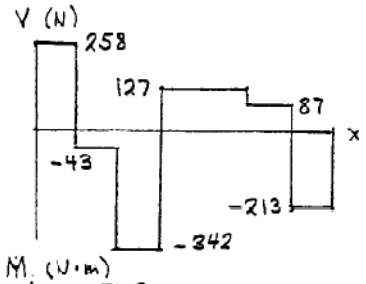
Normal stress at C.

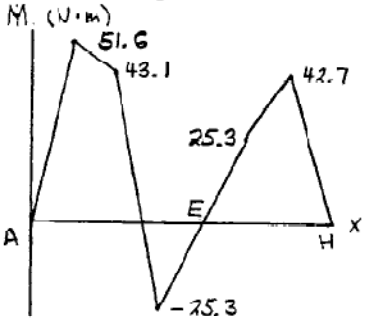
$$\sigma = \frac{M}{S} = \frac{1.40625 \times 10^3}{134} = \qquad \sigma = 10.49 \text{ ksi}$$

Problem 5.24

5.24 and 5.25 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

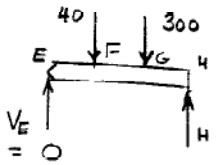






Free body EFGH.

Note that $M_E = 0$ due to hinge.



$\circlearrowleft \Sigma M_E = 0$:

$0.6\,H - (0.2)(40) - (0.4)(300) = 0$

$H = 213.33$ N

$\uparrow \Sigma F_y = 0$: $\quad V_E - 40 - 300 + 213.33 = 0$

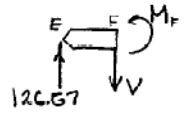
$V_E = 126.67$ N

Shear: E to F. $\quad V = 126.67$ N·m

F to G. $\quad V = 86.67$ N·m

G to H. $\quad V = -213.33$ N·m

Bending moment at F.

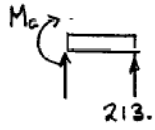


$\circlearrowleft \Sigma M_F = 0$:

$M_F - (0.2)(126.67) = 0$

$M_F = 25.33$ N·m

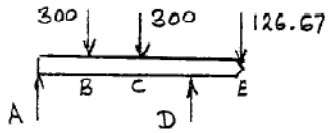
Bending moment at G.



$\circlearrowleft \Sigma M_G = 0$

$-M_G + (0.2)(213.33) = 0$

$M_G = 42.67$ N·m



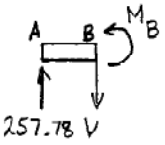
Free body ABCDE.

$+\circlearrowleft \Sigma M_B = 0$: $\quad -0.6\,A + (0.4)(300) + (0.2)(300) - (0.2)(126.67) = 0$

$A = 257.78$ N

$+\circlearrowleft \Sigma M_A = 0$: $\quad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6\,D = 0$

$D = 468.89$ N

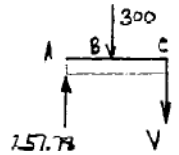


Bending moment at B.

$\circlearrowleft \Sigma M_B = 0$:

$-(0.2)(257.78) + M_B = 0$

$M_B = 51.56$ N·m

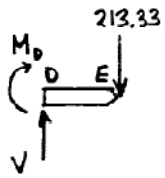


Bending moment at C.

$+\circlearrowleft \Sigma M_C = 0$:

$-(0.4)(257.78) + (0.2)(300) + M_C = 0$

$M_C = 43.11$ N·m



Bending moment at D.

$+\circlearrowleft \Sigma M_D = 0$:

$-M_D - (0.2)(213.33) = 0$

$M_D = -25.33$ N·m

$\max |M| = 51.56$ N·m

$S = \frac{1}{6} b h^2 = \frac{1}{6}(20)(30)^2$

$= 3 \times 10^3$ mm^3 $= 3 \times 10^{-6}$ m^3

Normal stress.

$$\sigma = \frac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6 \text{ Pa}$$

$\sigma = 17.19$ MPa