Problem 2.2

2.2 A polystyrene rod of length 12 in. and diameter 0.5 in. is subjected to an 800-lb tensile load. Knowing that $E = 0.45 \times 10^6$ psi, determine (*a*) the elongation of the rod, (*b*) the normal stress in the rod.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.5)^2 = 0.19635 \text{ in}^2$$

(a) $$\delta = \frac{PL}{AE} = \frac{(800)(12)}{(0.19635)(0.45\times10^6)} = 0.1086$$ $\delta = 0.1086$ in.

(b) $$\sigma = \frac{P}{A} = \frac{800}{0.19635} = 4074 \text{ psi}$$ $\sigma = 4.07$ ksi

Problem 2.9

2.9 A block of 250-mm length and 50 × 40 mm cross section is to support a centric compressive load **P**. The material to be used is a bronze for which $E = 95$ GPa. Determine the largest load which can be applied, knowing that the normal stress must not exceed 80 MPa and that the decrease in length of the block should be at most 0.12% of its original length.

$A = (50)(40) = 2000 \text{ mm}^2$
$= 2\times10^{-3} \text{ m}^2$

$\sigma_a = 80 \text{ MPa} = 80\times10^6 \text{ Pa}$ $\quad E = 95\times10^9 \text{ Pa}$

Considering allowable stress:

$$\sigma = \frac{P}{A} \qquad P = A\sigma = (2\times10^{-3})(80\times10^6) = 160\times10^3 \text{ N}$$

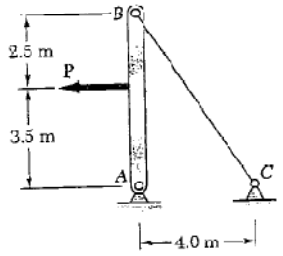
Considering allowable deformation:

$$\delta = \frac{PL}{AE} \qquad P = AE\left(\frac{\delta}{L}\right) = (2\times10^{-3})(95\times10^9)(0.0012) = 228\times10^3 \text{ N}$$

The smaller value governs. $P = 160\times10^3$ N $\quad P = 160.0$ kN

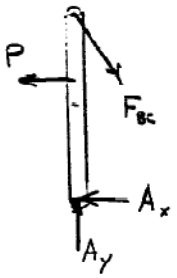
Problem 2.14

2.14 The 4-mm-diameter cable BC is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.



$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111$ m

Use bar AB as a free body.



$+\circlearrowleft \sum M_A = 0 \quad 3.5P - (6)\left(\frac{4}{7.2111} F_{BC}\right) = 0$

$P = 0.9509\ F_{BC}$

Considering allowable stress: $\sigma = 190 \times 10^6$ Pa

$A = \pi d^2 = \pi (0.004)^2 = 12.566 \times 10^{-6}\ m^2$

$\sigma = \frac{F_{BC}}{A} \therefore F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3$ N

Considering allowable elongation: $\delta = 6 \times 10^{-3}$ m

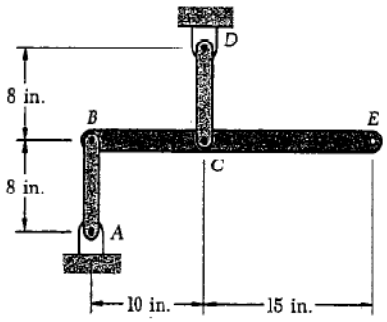
$\delta = \frac{F_{BC} L_{BC}}{AE} \therefore F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3$ N

Smaller value governs. $F_{BC} = 2.091 \times 10^3$ N

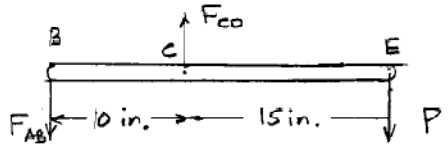
$P = 0.9509\ F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3$ N $\quad P = 1.988$ kN

Problem 2.26

2.26 Each of the links *AB* and *CD* is made of steel ($E = 29 \times 10^6$ psi) and has a uniform rectangular cross section of $\frac{1}{4} \times 1$ in. Determine the largest load which can be suspended from point *E* if the deflection of *E* is not to exceed 0.01 in.



Statics. Free body BCE.



$+\circlearrowleft \Sigma M_C = 0$: $\quad 10\,F_{AB} - 15\,P = 0$

$F_{AB} = 1.5\,P$

$+\circlearrowleft \Sigma M_B = 0$: $\quad 10\,F_{CD} - 25\,P = 0$

$F_{CD} = 2.5\,P$

Area of link:

$A = (\frac{1}{4})(1) = 0.25\ \text{in}^2$

Length:

$L = 8$ in.

Deformations.

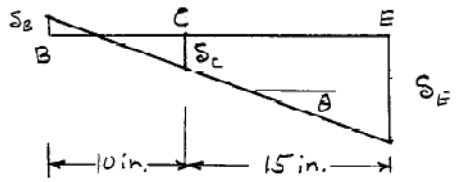
$$\delta_{AB} = \frac{F_{AB} L}{EA} = \frac{(1.5P)(8)}{(29\times10^6)(0.25)} = 1.6552\times10^{-6}\,P$$

$$\delta_{CD} = \frac{F_{CD} L}{EA} = \frac{(2.5P)(8)}{(29\times10^6)(0.25)} = 2.7586\times10^{-6}\,P$$

Deflections. Point B. $\quad \delta_B = \delta_{AB}$ $\quad \delta_B = 1.6552\times10^{-6}\,P \uparrow$

Point C. $\quad \delta_C = \delta_{CD}$ $\quad \delta_C = 2.7586\times10^{-6}\,P \downarrow$

Geometry change.



$$\theta = \frac{\delta_B + \delta_C}{L_{BC}} = \frac{1.6552\times10^{-6}\,P + 2.7586\times10^{-6}\,P}{10} = 0.44138\times10^{-6}\,P$$

$$\delta_E = \delta_F + L_{CE}\,\theta = 2.7586\times10^{-6}\,P + (15)(0.44138\times10^{-6}\,P) = 9.3793\times10^{-6}\,P \downarrow$$

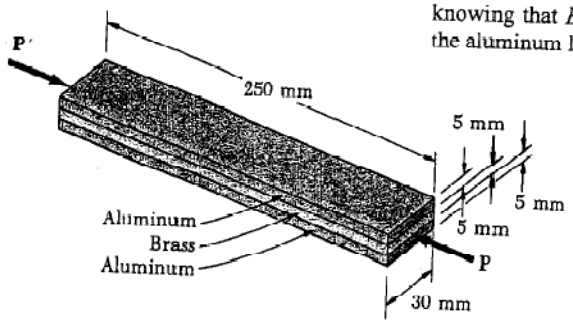
Limiting value of δ_E: $\quad \delta_E = 0.01$ in.

Limiting value of P: $\quad 9.3793\times10^{-6}\,P = 0.01$

$P = 1066$ lb $\qquad P = 1.066$ kips

Problem 2.33

2.33 A 250-mm bar of 15 × 30-mm rectangular cross section consists of two aluminum layers, 5-mm thick, brazed to a center brass layer of the same thickness. If it is subjected to centric forces of magnitude $P = 30$ kN, and knowing that $E_a = 70$ GPa and $E_b = 105$ GPa, determine the normal stress (a) in the aluminum layers, (b) in the brass layer.



For each layer,

$A = (30)(5) = 150\ mm^2 = 150 \times 10^{-6}\ m^2$

Let P_a = load on each aluminum layer,

P_b = load on brass layer.

Deformation. $\delta = \dfrac{P_a L}{E_a A} = \dfrac{P_b L}{E_b A}$ $\qquad P_b = \dfrac{E_b}{E_a} P_a = \dfrac{105}{70} P_a = 1.5 P_a$

Total force. $P = 2P_a + P_b = 3.5\ P_a$

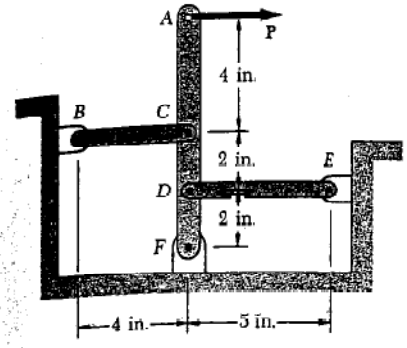
Solving for P_a and P_b, $\qquad P_a = \dfrac{2}{7} P \qquad P_b = \dfrac{3}{7} P$

(a) $\sigma_a = -\dfrac{P_a}{A} = -\dfrac{2}{7}\dfrac{P}{A} = -\dfrac{2}{7}\dfrac{30 \times 10^3}{150 \times 10^{-6}} = -57.1 \times 10^6\ Pa$ $\qquad \sigma_a = -57.1\ MPa$

(b) $\sigma_b = -\dfrac{P_b}{A} = -\dfrac{3}{7}\dfrac{P}{A} = -\dfrac{3}{7}\dfrac{30 \times 10^3}{150 \times 10^{-6}} = -85.7 \times 10^6\ Pa$ $\qquad \sigma_b = -85.7\ MPa$

Problem 2.45

2.45 Links *BC* and *DE* are both made of steel ($E = 29 \times 10^6$ psi) and are $\frac{1}{2}$ in. wide and $\frac{1}{4}$ in. thick. Determine (*a*) the force in each link when a 600-lb force **P** is applied to the rigid member *AF* shown, (*b*) the corresponding deflection of point *A*.



Let the rigid member ACDF rotate through small angle θ clockwise about point F.

Then $\delta_C = \delta_{BC} = 4\theta$ in. →

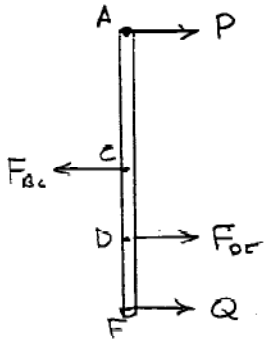
$\delta_D = -\delta_{DE} = 2\theta$ in. →

$$\delta = \frac{FL}{EA} \quad \text{or} \quad F = \frac{EA\delta}{L}$$

For links: $A = (\frac{1}{2})(\frac{1}{4}) = 0.125\ \text{in}^2$, $L_{BC} = 4$ in., $L_{DE} = 5$ in.

$$F_{BC} = \frac{EA\,\delta_{BC}}{L_{AC}} = \frac{(29\times10^6)(0.125)(4\theta)}{4} = 3.625\times10^6\,\theta$$

$$F_{DE} = \frac{EA\,\delta_{DE}}{L_{DE}} = \frac{(29\times10^6)(0.125)(-2\theta)}{5} = -1.45\times10^6\,\theta$$



Use member ACDF as a free body.

$$(+\Sigma M_F = 0: \quad 8P - 4F_{BC} + 2F_{DE} = 0$$

$$P = \tfrac{1}{2}F_{BC} - \tfrac{1}{4}F_{DE}$$

$$600 = \tfrac{1}{2}(3.625\times10^6)\theta - \tfrac{1}{4}(-1.45\times10^6)\theta = 2.175\times10^6\,\theta$$

$$\theta = 0.27586\times10^{-3}\ \text{rad}$$

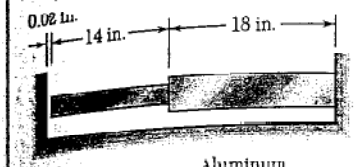
(a) $F_{BC} = (3.625\times10^6)(0.27586\times10^{-3})$ $\quad F_{BC} = 1000$ lb ◄

$F_{DE} = -(1.45\times10^6)(0.27586\times10^{-3})$ $\quad F_{DE} = -400$ lb ◄

(b) Deflection at point A.

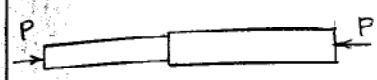
$\delta_A = 8\theta = (8)(0.27586\times10^{-3})$ $\quad \delta_A = 2.21\times10^{-3}$ in. → ◄

Problem 2.58



Bronze
$A = 2.4\ \text{in.}^2$
$E = 15 \times 10^6\ \text{psi}$
$\alpha = 12 \times 10^{-6}/°\text{F}$

Aluminum
$A = 2.8\ \text{in.}^2$
$E = 10.6 \times 10^6\ \text{psi}$
$\alpha = 12.9 \times 10^{-6}/°\text{F}$



2.58 Knowing that a 0.02-in. gap exists when the temperature is 75 °F, determine (*a*) the temperature at which the normal stress in the aluminum bar will be equal to -11 ksi, (*b*) the corresponding exact length of the aluminum bar.

$\sigma_a = -11\ \text{ksi} = -11 \times 10^3\ \text{psi}$

$P = -\sigma_a A_a = (11 \times 10^3)(2.8) = 30.8 \times 10^3\ \text{lb}$

Shortening due to P.

$$\delta_P = \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a}$$

$$= \frac{(30.8 \times 10^3)(14)}{(15 \times 10^6)(2.4)} + \frac{(30.8 \times 10^3)(18)}{(10.6 \times 10^6)(2.8)}$$

$$= 30.657 \times 10^{-3}\ \text{in.}$$

Available elongation for thermal expansion.

$$\delta_T = 0.02 + 30.657 \times 10^{-3} = 50.657 \times 10^{-3}\ \text{in.}$$

But, $\delta_T = L_b \alpha_b (\Delta T) + L_a \alpha_a (\Delta T)$

$$= (14)(12 \times 10^{-6})(\Delta T) + (18)(12.9 \times 10^{-6})(\Delta T) = (400.2 \times 10^{-6})\,\Delta T$$

Equating, $(400.2 \times 10^{-6})\Delta T = 50.657 \times 10^{-3}$ $\quad \Delta T = 126.6°\text{F}$

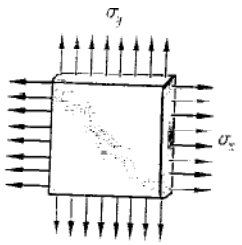
(a) $T_{hot} = T_{cold} + \Delta T = 75 + 126.6 = 201.6°\text{F}$

(b) $\delta_a = L_a \alpha_a (\Delta T) - \dfrac{PL_a}{E_a A_a}$

$$= (18)(12.9 \times 10^{-6})(126.6) - \frac{(30.8 \times 10^3)(18)}{(10.6 \times 10^6)(2.8)} = 10.712 \times 10^{-3}\ \text{in}$$

$L_{exact} = 18 + 10.712 \times 10^{-3} = 18.0107\ \text{in.}$

Problem 2.73



2.73 In many situations it is known that the normal stress in a given direction is zero, for example $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as *plane stress*, show that if the strains ε_x and ε_y have been determined experimentally, we can express σ_x, σ_y and ε_z as follows:

$$\sigma_x = E\frac{\varepsilon_x + \nu\varepsilon_y}{1-\nu^2} \qquad \sigma_y = E\frac{\varepsilon_y + \nu\varepsilon_x}{1-\nu^2} \qquad \varepsilon_z = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

$\sigma_z = 0$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \quad (1) \qquad \varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y) \quad (2)$$

Multiplying (2) by ν and adding to (1)

$$\varepsilon_x + \nu\varepsilon_y = \frac{1-\nu^2}{E}\sigma_x \quad \text{or} \quad \sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

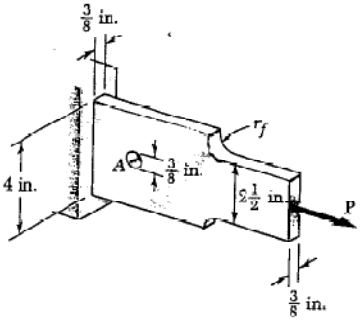
Multiplying (1) by ν and adding to (2)

$$\varepsilon_y + \nu\varepsilon_x = \frac{1-\nu^2}{E}\sigma_y \quad \text{or} \quad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) = -\frac{\nu}{E}\cdot\frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y + \varepsilon_y + \nu\varepsilon_x)$$

$$= -\frac{\nu(1+\nu)}{1-\nu^2}(\varepsilon_x + \varepsilon_y) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

Problem 2.96



2.96 Knowing that the hole has a diameter of $\frac{3}{8}$ in., determine (*a*) the radius r_f of the fillets for which the same maximum stress occurs at the hole *A* and at the fillets, (*b*) the corresponding maximum allowable load **P** if the allowable stress is 15 ksi.

For the circular hole $r = (\frac{1}{2})(\frac{3}{8}) = 0.1875$ in.

$d = 4 - \frac{3}{8} = 3.625$ in $\qquad \frac{r}{d} = \frac{0.1875}{3.625} = 0.0517$

$A_{net} = dt = (3.625)(\frac{3}{8}) = 1.3594$ in^2

From Fig 2.64 a $\qquad K_{hole} = 2.82$

$$\sigma_{max} = \frac{K_{hole}P}{A_{net}}$$

(b) $$P = \frac{A_{net}\sigma_{max}}{K_{hole}} = \frac{(1.3594)(15)}{2.82}$$

(b) $P = 7.23$ kips

(a) For fillet $\qquad D = 4$ in, $\quad d = 2.5$ in $\qquad \frac{D}{d} = \frac{4.0}{2.5} = 1.60$

$A_{min} = dt = (2.5)(\frac{3}{8}) = 0.9375$ in^2

$$\sigma_{max} = \frac{K_{fillet}P}{A_{min}} \quad \therefore \quad K_{fillet} = \frac{A_{min}\sigma_{max}}{P} = \frac{(0.9375)(15)}{7.23} = 1.945$$

From Fig 2.64 b, $\frac{r_f}{d} \approx 0.17$ $\therefore$ $r_f \approx 0.17d = (0.17)(2.5) = 0.425$ in