ME-313 **STRENGTH OF MATERIALS** **October 11, 2000**

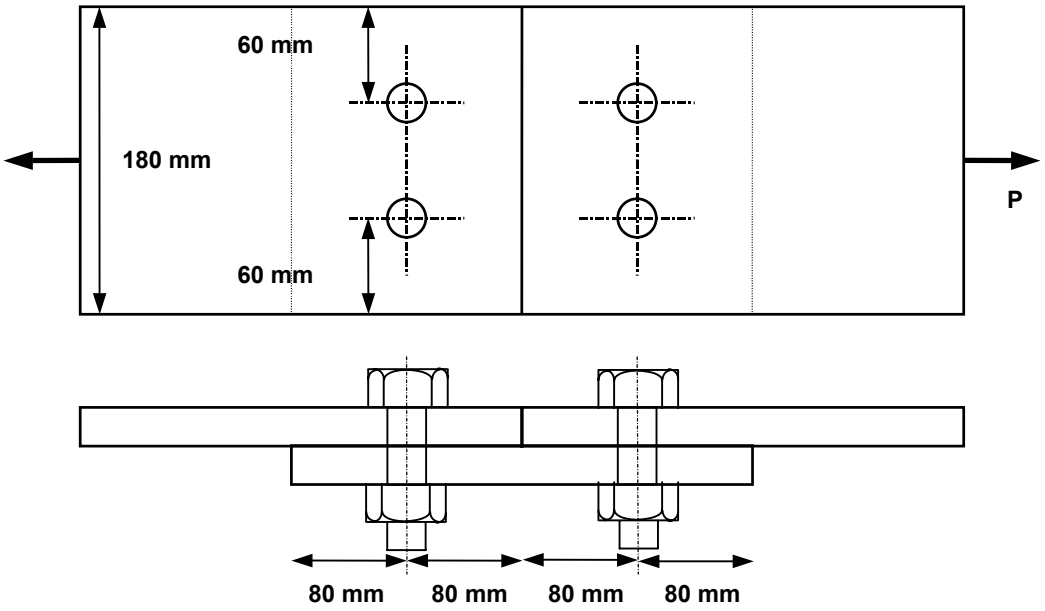
TEST I

OPEN TEXT BOOK

Name:_____

1. Three plates of steel are bolted together as shown in the figure below. Design the bolted connection by choosing both plate thickness and bolt diameter. The connection must carry a load of P = 175 kN, while the allowable stresses are
   - σ_a = 140 MPa allowable tensile stress on the plates
   - σ_b = 260 MPa allowable bearing stress on the plates
   - τ_a = 100 MPa allowable shear stress on the bolts

(25 pts.)



Problem 1

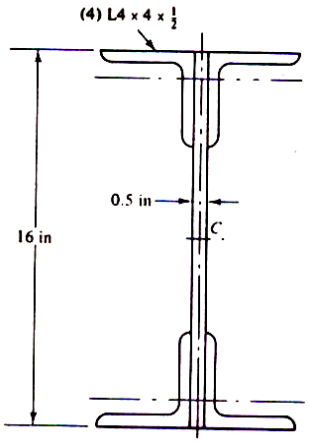
2. A meter for measuring torque uses the angle of twist of a shaft to indicate torque. The shaft is to be 150 mm long and made of 6061-T6 aluminum alloy. Determine the required diameter of the shaft if it is desired to have a twist of 10.0° when a torque of 5.0 N.m is applied to the meter. For the shaft thus designed, compute the torsional shearing stress and the factor of safety with respect to the yield stress in shear. Is this satisfactory? If not, what would you do?

(25 pts.)

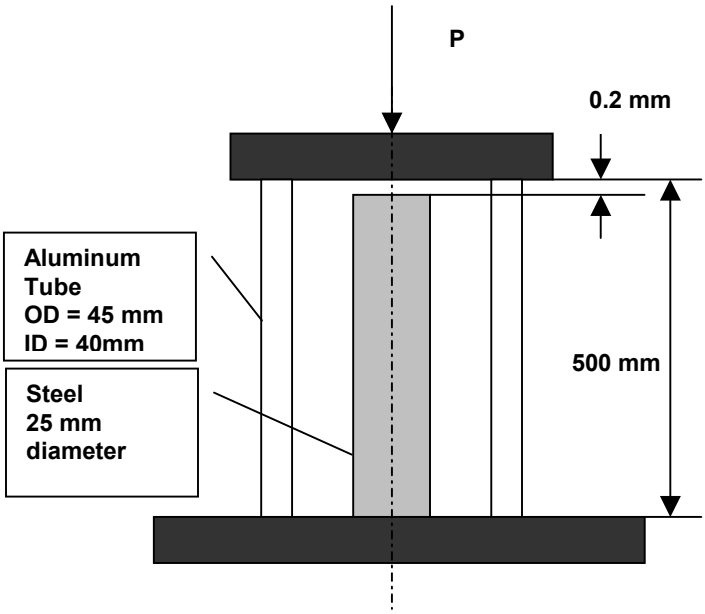
3. A built-up beam is made by fastening four L4 x 4 x $^1/_2$ steel angles to a $^1/_2$ in. x 16 in. steel plate as shown in the figure below. Compute the moment of inertia for the beam.
   (25 pts.)

4. A solid steel rod (25 mm diameter) is surrounded by an aluminum tube (inner diameter = 40 mm, outer diameter = 45 mm) as shown in the figure below. A rigid bearing block is supported on the aluminum tube and a gap of 0.2 mm exists between the block and the top of the steel rod. Determine the permissible load P if the maximum allowable stress is 140 MPa in the steel and 100 MPa in the aluminum.

(25 pts.)



Problem 3



Problem 4

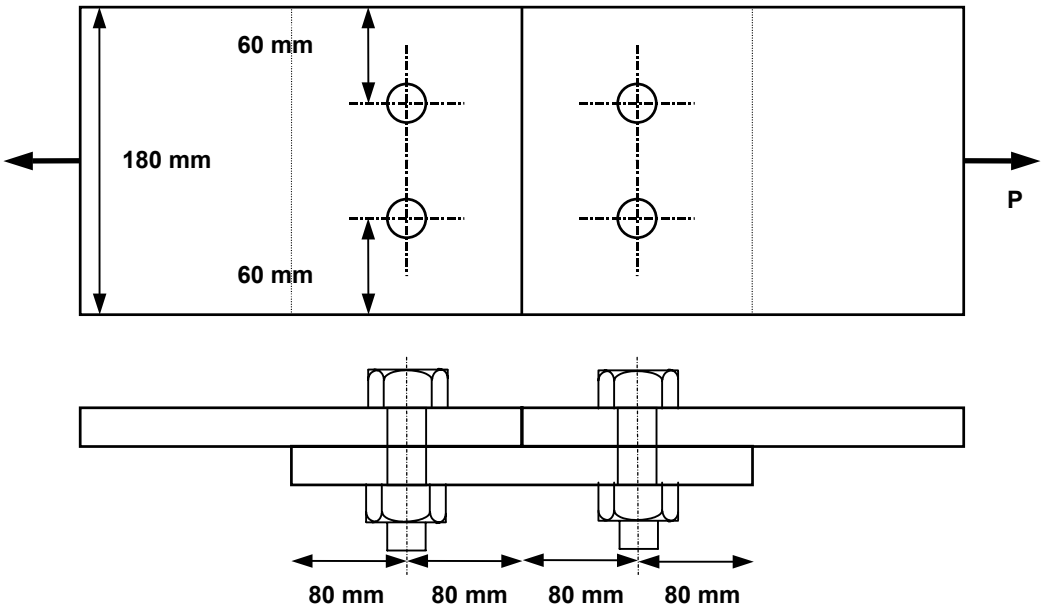
ME-313 **STRENGTH OF MATERIALS** **October 11, 2000**

TEST I

Solutions

1. Three plates of steel are bolted together as shown in the figure below. Design the bolted connection by choosing both plate thickness and bolt diameter. The connection must carry a load of P = 175 kN, while the allowable stresses are
   - σ_a = 140 MPa allowable tensile stress on the plates
   - σ_b = 260 MPa allowable bearing stress on the plates
   - τ_a = 100 MPa allowable shear stress on the bolts

(25 pts.)



Problem 1

Solution

Ignore stress concentration

- *The bolts: Each bolt carries a load of P/2. The diameter of the bolts is determined as below.*

$$\frac{P}{2} = \tau_{all} \times \frac{\pi d^2}{4}$$

$$d = \sqrt{\frac{2P}{\pi \tau_{all}}} = \sqrt{\frac{2 \times 175 \times 10^3 N}{\pi \times 100 \times 10^6 (N/m^2)}} = 3.338 \times 10^{-2} m = 33.38 mm$$

- *The plate thickness is determined by two factors, the allowable bearing stress and the allowable tensile stress.*

- *Bearing stress*

$$\sigma_b = \frac{P/2}{td}$$

$$t = \frac{P}{2\sigma_b d} = \frac{175\times10^3 N}{2\times260\times10^6 (N/m^2)\times3.338\times10^{-2} m} = 1.008\times10^{-2} m = 10.08mm$$

- *Tensile stress*

$$\sigma_a = \frac{P}{Area} = \frac{P}{(0.18m - 2\times.0338m)\times t}$$

$$t = \frac{175\times10^3 N}{0.1132m\times140\times10^6 (N/m^2)} = 1.104\times10^{-2} m = 11.04mm$$

- *Select the larger of the two thickness values, i.e. 11.04 mm*

- *The design therefore requires*
  - *Bolt diameter d=33.38 mm*
  - *Plate thickness t = 11.04 mm*

2. A meter for measuring torque uses the angle of twist of a shaft to indicate torque. The shaft is to be 150 mm long and made of 6061-T6 aluminum alloy. Determine the required diameter of the shaft if it is desired to have a twist of 10.0° when a torque of 5.0 N.m is applied to the meter. For the shaft thus designed, compute the torsional shearing stress and the factor of safety with respect to the yield stress in shear. Is this satisfactory? If not, what would you do?

(25 pts.)

Solution

$$\phi = 10^\circ = 10\frac{\pi}{180} = 0.1745\text{rad}$$

$G = 26\times10^9\,\text{N/m}^2$ (from table at the back of the book)

$$\phi = \frac{TL}{JG}$$

$$\Rightarrow J = \frac{TL}{\phi G} = \frac{5\text{Nm}\times0.15\text{m}}{0.1745\text{rad}\times26\times10^6(\text{N/m}^2)} = 1.653\times10^{-10}\,\text{m}^4$$

$$J = \frac{\pi d^4}{32}$$

$$\Rightarrow d^4 = \frac{32\times1.653\div10^{-10}\,\text{m}^4}{\pi}$$

$$\Rightarrow d = 6.405\times10^{-3}\,\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{Td}{2J} = \frac{5\text{Nm}\times(6.405\times10^{-3}\text{m})}{2\times1.653\times10^{-10}\,\text{m}^4} = 96.88\times10^6(\text{N/m}^2) = 96.88\text{MPa}$$

τ_{yield} for $AA-6061-T6$ is 140MPa from the back of the book.

$$FS = \frac{\tau_{yield}}{\tau_{actual}} = \frac{140\text{MPa}}{96.88\text{MPa}} = 1.445$$

- *This factor of safety appears to be sufficient.*
- *To increase the factor of safety, one could select a different aluminum alloy with a higher yield strength.*
- *Changing the diameter or changing to a material other than aluminum will change the angle of twist for the applied torque, and may not be acceptable.*

3. A built-up beam is made by fastening four L4 x 4 x $^1/_2$ steel angles to a $^1/_2$ in. x 16 in. steel plate as shown in the figure below. Compute the moment of inertia for the beam.
(25 pts.)

Solution

- *The rectangular portion is being bent about its centroid. Therefore*

$$I_{rect.}\frac{bh^3}{12} = 170.7\text{in}^4$$

- *The moment of inertia I_x of the 4 L shapes about their centroid can be obtained from the tables at the back of the book.*
- *In the built up beam, the centroid of the L shapes is displaced (8 - 1.18)in. from the axis of bending*
- *Therefore, the moment of inertia of the L sections about the axis of bending can be determined using the parallel axis theorem, i.e.*

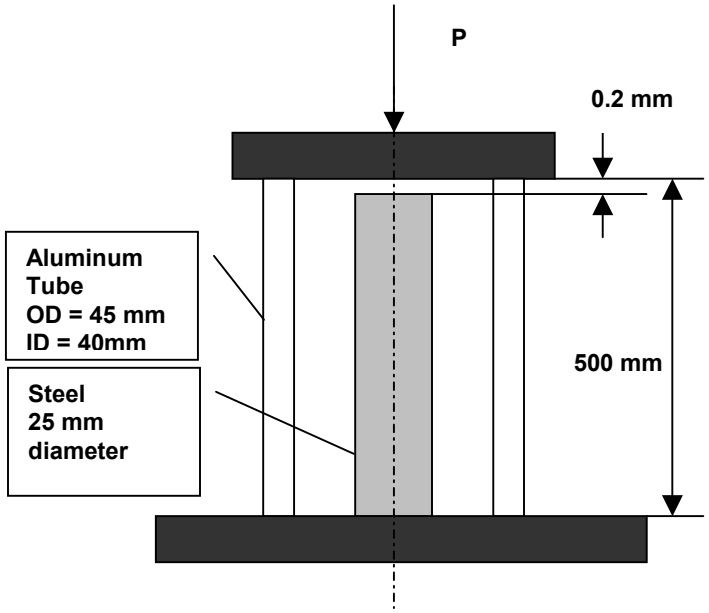
$$I' = I_x + Ay^2 = 5.56\text{in}^4 + 3.75\text{in}^2 \times (6.82\text{in})^2 = 179.98\text{in}^4$$

- *The moment of inertia of the entire cross-section is therefore*

$$I_{total} = 170.7\text{in}^4 + 4 \times 179.98\text{in}^4 = 890.6\text{in}^4$$

4. A solid steel rod (25 mm diameter) is surrounded by an aluminum tube (inner diameter = 40 mm, outer diameter = 45 mm) as shown in the figure below. A rigid bearing block is supported on the aluminum tube and a gap of 0.2 mm exists between the block and the top of the steel rod. Determine the permissible load P if the maximum allowable stress is 140 MPa in the steel and 100 MPa in the aluminum.

(25 pts.)



Solution

- *Assume the steel is loaded to reach its allowable stress value, then*

$$\varepsilon_{steel} = \frac{140MPa}{E_{steel}} = \frac{140MPa}{200GPa} = 7 \times 10^{-4}$$

$$\Delta L_{steel} = \varepsilon_{steel} \times L_{steel} = 7 \times 10^{-4} \times 500mm = 0.35mm$$

- *Since the aluminum tube is 0.2mm longer*

$$\Delta L_{aluminum} = 0.35 + 0.2 = 0.55mm$$

$$\varepsilon_{alumiunum} = \frac{0.55mm}{500mm} = 1.1 \times 10^{-3}$$

$$\sigma_{alumiunum} = \varepsilon_{alumiunum} \times E_{alumiunum} = 1.1 \times 10^{-3} \times 70GPa = 77MPa$$

- *A stress of 77 MPa is less that the allowable stress for aluminum. So this is OK*
- *Total load is determined from the load supported by the steel rod and the aluminum tube.*

$$P_{total} = P_{steel} + P_{aluminum} = \sigma_{steel} \times A_{steel} + \sigma_{aluminum} \times A_{aluminum}$$

$$P_{total} = 140 \times 10^6 (N/m^2) \times \frac{\pi}{4}\left(25 \times 10^{-3} m\right)^2 + 77 \times 10^6 (N/m^2) \times \frac{\pi}{4}\left[\left(45 \times 10^{-3} m\right)^2 - \left(40 \times 10^{-3} m\right)^2\right]$$

$$P_{total} = 68722N + 25702N = 94424N = 94.42kN$$