Problem 1 (10 points): A cool fluid flows over a flat plate heated to a uniform temperature over its entire length $L$. Show that the average heat transfer coefficient can be found by integrating the local heat transfer coefficient over the length of the plate:

$$\bar{h} = \frac{1}{L} \int_{0}^{L} h_x \, dx = 2h_{x=L}$$

where $h_{x=L}$ is the local heat transfer coefficient evaluated at the end of the plate.

Problem 2 (10 points): Air flows over a flat plate at a freestream velocity of 2 m/s and a freestream temperature of 20°C. The plate is heated to a uniform temperature of 60°C over its entire length ($L = 40$ cm). Using the result found in Problem 1, determine the heat transfer rate from the plate if the width of the plate is 1.0 m.

$$\dot{Q} = \bar{h}A(T_s - T_\infty)$$
Local Nusselt Number:

\[ N_u_x = \frac{h_x X}{K} = 0.332 \, Pr^{\frac{1}{3}} \, Re_x^{\frac{1}{2}} \]

\[ h_x = 0.332 \, Pr^{\frac{1}{3}} \, K \cdot \left( \frac{Re_x^{\frac{1}{2}}}{X} \right) \]

\[ h_x = 0.332 \, Pr^{\frac{1}{3}} \, K \cdot \sqrt{\frac{V_0}{2g}} \cdot \frac{X^{\frac{1}{2}}}{X} \]

\[ h_x = 0.332 \, Pr^{\frac{1}{3}} \, K \sqrt{\frac{V_0}{2g}} \cdot X^{\frac{1}{2}} \]

\[ \bar{h} = \frac{1}{L} \int_0^L h_x \, dx \]

\[ \bar{h} = 0.332 \, Pr^{\frac{1}{3}} \, K \sqrt{\frac{V_0}{2g}} \cdot \frac{1}{L} \int_0^L X^{-\frac{1}{2}} \, dx \]

\[ \int x^n \, dx = \frac{1}{n+1} \cdot x^{n+1} \]

\[ \int_0^L x^{-\frac{1}{2}} \, dx = \left[ \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_0^L \]

\[ = \left[ 2 \cdot x^{\frac{1}{2}} \right]_0^L \]

\[ = 2 \, L^{\frac{1}{2}} \]
\[ \bar{h} = 0.332 \Pr^{\frac{1}{3}} K \cdot \sqrt{\frac{V_{00}}{D}} \cdot \frac{1}{L} \cdot (2L) \]

\[ \bar{h} = 2 \left( 0.332 \Pr^{\frac{1}{3}} K \cdot \frac{1}{L} \cdot \sqrt{\frac{V_{00} L}{D}} \right) \]

\[ \bar{h} = 2 \left[ 0.332 \Pr^{\frac{1}{3}} K \cdot \left( \frac{Re_k^{\frac{1}{2}}}{L} \right) \right] \]

\[ \bar{h} = 2 \cdot h_{x=L} \]
PROB. 2

AIR, $V_{in} = 2 \frac{m}{s}$, $T_{in} = 20^\circ C$, $T_s = 60^\circ C$, $L = 0.4 \text{ m}$, $W = 1.0 \text{ m}$

FIND $\dot{Q}$

$\dot{Q} = \overline{h} A (T_s - T_0)$

Local heat transfer coefficient at the end of the plate:

\[ h_L = 0.332 Pr^{1/3} \frac{V_2}{L} \cdot \frac{Re_L}{L} \]

\[ Re_L = \frac{V_0 L}{v} \]

Air properties at $T_{in} = \frac{1}{2} (20 + 60^\circ C) = 40^\circ C$:

\[ k = 0.02662 \left( \frac{W}{m \cdot K} \right), \quad \frac{L}{A} = 1.702 \times 10^{-3} \frac{m^2}{s}, \quad Pr = 0.7255 \]

\[ Re_L = \frac{\left( \frac{2}{3} \times 0.4 \text{ m} \right)}{\left( 1.702 \times 10^{-3} \frac{m^2}{s} \right)} = 47,003 \]

\[ h_L = 0.332 \left( 0.7255 \right)^{1/3} \left( 0.02662 \left( \frac{W}{m \cdot K} \right) \right) \times \sqrt{\frac{47,003}{0.4 \text{ m}}} \]

\[ h_L = 4.304 \frac{W}{m^2 \cdot K} \]

\[ \overline{h} = 2 h_L = 2 \left( 4.304 \frac{W}{m^2 \cdot K} \right) = 8.608 \frac{W}{m^2 \cdot K} \]

\[ \dot{Q} = \left( 8.608 \frac{W}{m^2 \cdot K} \right) (0.4 \text{ m}) (1.0 \text{ m}) (60 - 20^\circ C) = 137.7 \text{ W} \]