Problem 1 (10 points): A 1-m-inner-diameter spherical liquid-oxygen storage tank keeps the liquid oxygen at 90 K. The tank consists of a 0.5-cm-thick aluminum \((k = 170 \text{ W/m-K})\) shell whose exterior is covered with a 10-cm-thick layer of insulation \((k = 0.02 \text{ W/m-K})\). The insulation is exposed to the ambient air at 20°C and the heat transfer coefficient on the exterior side of the insulation is 5 W/m\(^2\)-K. Find the rate at which the liquid oxygen gains heat. Neglect the effects of thermal radiation heat transfer. Surface area of a sphere: \(A = 4\pi r^2\), volume of a sphere: \(V = \left(\frac{4\pi}{3}\right)r^3\).

Problem 2 (10 points): Consider a 7.6-cm-diameter cylindrical lamb meat chunk \((\rho = 1030 \text{ kg/m}^3, C_p = 3.49 \text{ kJ/kg-K}, k = 0.456 \text{ W/m-K}, \alpha = 1.3 \times 10^{-7} \text{ m}^2/\text{s})\). The meat chunk is initially at 2°C, and is dropped into boiling water at 95°C with a heat transfer coefficient of 1200 W/m\(^2\)-K. Find the time it takes for the center temperature of the meat chunk to rise to 75°C. Neglect multi-dimensional effects.

Problem 3 (10 points): A residential water heater uses natural convection to transfer heat from a cylindrical electrical resistance heating element that is 1 cm in diameter by 0.65 m long. The voltage applied to the heating element is 110 V. During operation, the surface temperature of the heating element is 120°C while the temperature of the water is 30°C. The Nusselt number (based on diameter) is 6. Considering only the side surface of the heating element (and thus \(A = \pi DL\)), determine the current passing through the electrical heating element.
PROB. 1 (3.23H)

\[ D_1 = 1 \text{ m}, \quad T_1 = 90 \text{ K} \]

\[ \ell_{AL} = 0.5 \text{ cm} = 0.005 \text{ m} \]

\[ K_{AL} = 170 \text{ W/m-K} \]

\[ \ell_{ins} = 10 \text{ cm} = 0.1 \text{ m} \]

\[ K_{ins} = 0.02 \text{ W/m-K} \]

\[ T_{oo} = 20^\circ \text{C} = 293 \text{ K}, \quad h = 5 \text{ W/m}^2\text{-K}, \text{ FIND } Q \]

\[ Q = \frac{T_{oo} - T_1}{R_T} \]

\[ R_T = R_1 + R_2 + R_3 \]

\[ R_1 = \frac{\Gamma_2 - \Gamma_1}{4\pi \Gamma_1 \Gamma_2 K_{AL}} \]

\[ \Gamma_1 = \frac{1}{2} (1 \text{ m}) = 0.5 \text{ m}, \quad \Gamma_2 = 0.5 + 0.005 = 0.505 \text{ m} \]

\[ \Gamma_3 = 0.505 + 0.1 = 0.605 \text{ m} \]

\[ R_1 = \frac{0.505 - 0.5}{4\pi (0.5 \text{ m})(0.505 \text{ m})(170 \text{ W/m-K})} = 9.27 \times 10^{-6} \text{ K/W} \]

\[ R_2 = \frac{\Gamma_3 - \Gamma_2}{4\pi \Gamma_2 \Gamma_3 K_{ins}} \]
PROB. 1  CONT.

\[ R_2 = \frac{(0.605 \text{ m}) - (0.505 \text{ m})}{4\pi (0.505 \text{ m}) (0.605 \text{ m}) (0.02 \frac{\text{ W}}{\text{ m}^2 \cdot \text{ K}})} = 1.302 \ \frac{\text{ K}}{\text{ W}} \]

\[ R_3 = \frac{1}{hA} \]

\[ A = 4\pi r^2 = 4\pi (0.605 \text{ m})^2 = 4.60 \text{ m}^2 \]

\[ R_3 = \frac{1}{(5 \frac{\text{ W}}{\text{ m}^2 \cdot \text{ K}}) (4.60 \text{ m}^2)} = 0.04348 \ \frac{\text{ K}}{\text{ W}} \]

\[ R_T = (9.27 \times 10^{-6} + 1.302 + 0.04348 \ \frac{\text{ K}}{\text{ W}}) = 1.345 \ \frac{\text{ K}}{\text{ W}} \]

\[ \dot{Q} = \frac{(20 + 273 \text{ K}) - (90 \text{ K})}{(1.345 \ \frac{\text{ K}}{\text{ W}})} \]

\[ \dot{Q} = 150.9 \ \text{ W} \]
PROB. 2  4-165 p. 292

D = 7.6 cm,  \( b = 10.30 \ \frac{kg}{m^3} \),  \( \rho = 3.49 \ \frac{kg}{L} \),  

\( K = 0.456 \ \frac{W}{m \cdot K} \),  \( \alpha = 1.3 \times 10^{-7} \ \frac{m^2}{s} \),  \( T_c = 2^\circ C \),  

\( T_w = 95^\circ C \),  \( h = 1200 \ \frac{W}{m^2 \cdot K} \),  

\( T = 75^\circ C \).  
NEGLECT MULTI-DIMENSIONAL EFFECTS

CHECK Biot NUMBER:

\( B_i = \frac{hL_c}{K} \)

\( L_c = \frac{r_o}{2} = \frac{1}{2} \left[ \frac{1}{2} \left( \frac{7.6 \text{ cm}}{100 \text{ cm}} \right) \right] = 0.019 \text{ m} \)

\( B_i = \frac{(1200 \ \frac{W}{m^2 \cdot K})(0.019 \text{ m})}{(0.456 \ \frac{W}{m \cdot K})} = 50 \)  CAN'T USE  LUMPED ANALYSIS

ONE TERM APPROXIMATION:

\( \frac{T_o - T_w}{T_c - T_o} = A_1 \ e^{-\frac{x^2}{4}} \)

\( B_i = \frac{hr_o}{K} = \frac{(1200 \ \frac{W}{m^2 \cdot K})(0.038 \text{ m})}{(0.456 \ \frac{W}{m \cdot K})} = 100 \ \ \ \text{TABLE 4-2} \)

\( x_1 = 2.3809, \ A_1 = 1.6015 \)
\[ I = \frac{x_t}{v_0^2} \]

\[ e^{-x_t^2} I = \frac{1}{A_1} \left( \frac{T_0 - T_\infty}{T_c - T_\infty} \right) \]

\[ x_t^2 I = \ln \left[ \frac{1}{A_1} \left( \frac{T_0 - T_\infty}{T_c - T_\infty} \right) \right] \]

\[ I = \frac{x_t}{v_0^2} = -\frac{x_t^2}{A_1} \cdot \ln \left[ \frac{1}{A_1} \left( \frac{T_0 - T_\infty}{T_c - T_\infty} \right) \right] \]

\[ t = - \frac{\rho_0^2}{x_t^2} \cdot \ln \left[ \frac{1}{A_1} \left( \frac{T_0 - T_\infty}{T_c - T_\infty} \right) \right] \]

\[ t = - \frac{(0.038 \text{ m})^2}{(1.3 \times 10^{-7} \text{ m}^2 / \text{s}) (2.3809)^2} \cdot \ln \left[ \frac{1}{(1.6015)} \left( \frac{75 - 95 \text{ ^\circ C}}{2 - 95 \text{ ^\circ C}} \right) \right] \]

\[ t = (3934 \text{ s}) \left( \frac{\text{MIN}}{60 \text{ s}} \right) = 65.57 \text{ MIN} \]

\[ I = \frac{x_t}{v_0^2} = \frac{(1.3 \times 10^{-7} \text{ m}^2 / \text{s}) (3934^2)}{(0.038 \text{ m})^2} = 0.3542 \]

\textit{One-term approximation is valid.}
PROB. 3  (6-84)

\[ D = 1 \text{ cm}, \quad L = 0.65 \text{ m}, \quad V = 110 \text{ V}, \quad T_s = 94120^\circ C \]
\[ T_\infty = 30^\circ C, \quad N_u = \frac{hD}{k} = 6.0, \quad A = \pi DL \]

**Find I**

\[ \dot{Q} = hA(T_s - T_\infty) \]

\[ T_{film} = \frac{1}{2}(T_s + T_\infty) = \frac{1}{2}(120 + 30^\circ C) = 75^\circ C \]

\[ K = 0.667 \frac{\text{W}}{\text{m} \cdot \text{K}} \quad \text{(TABLE A-9)} \]

\[ h = 6 \cdot \frac{k}{D} = 6 \left( \frac{0.667 \frac{\text{W}}{\text{m} \cdot \text{K}}}{0.01 \text{ m}} \right) = 400.2 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \]

\[ \dot{Q} = (400.2 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}) [\pi(0.01 \text{ m})(0.65 \text{ m})] (120 - 30^\circ C) \]

\[ \dot{Q} = 733.5 \text{ W} \]

\[ \dot{Q} = V \cdot I \]

\[ I = \frac{\dot{Q}}{V} = \left( \frac{733.5 \text{ W}}{110 \text{ V}} \right) = 6.686 \text{ A} \]