Problem 1 (10 points): On a still clear night, the sky appears to be a blackbody with an equivalent temperature of 250 K. The strawberry field cools to 0°C and freezes when the heat transfer coefficient between the plants and air is 8 W/m²-K because of a light breeze. Assume that the plants have an emissivity of 0.85. What is the air temperature when this occurs?

Problem 2 (10 points): Steam at 1 atm pressure ($T_{\text{sat}} = 100{\degree}\text{C}$) is exposed to a 30-by-30-cm vertical square plate which is cooled such that 3.78 kg of liquid water per hour is condensed. Assuming that the plate temperature on the face where condensation occurs is 100°C, calculate the temperature on the cold side of the plate. The plate is 1 cm thick and is made of commercial bronze. Neglect the effects of thermal radiation heat transfer, and assume that the heat conduction is one-dimensional through the plate.

Problem 3 (10 points): You turn on your kitchen sink faucet and hot water falls onto the bottom of the cold sink. Consider the conduction of heat within the material that the sink is made of in the area under the thin film of water before it reaches the hydraulic jump. Write down the complete differential heat conduction equation for this situation. Justify the elimination of all unnecessary terms by stating your assumptions. Rewrite the heat conduction equation after the appropriate terms have been eliminated. Propose boundary and/or initial conditions that might be appropriate for this situation.

CONSERVATION OF ENERGY EQUATION:

Rectangular Coordinate System:

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{e}_{\text{gen}} = \rho C \frac{\partial T}{\partial t}$$

Cylindrical Coordinate System:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{e}_{\text{gen}} = \rho C \frac{\partial T}{\partial t}$$

Spherical Coordinate System:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{e}_{\text{gen}} = \rho C \frac{\partial T}{\partial t}$$
\[ T_{\text{sur}} = 290^\circ K, \quad T_5 = 0^\circ C = 273^\circ K, \quad h = 8 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}, \quad \varepsilon = 0.85 \]

**Find** \( T_{oo} \)

\[ \dot{Q}_{\text{conv}} = \dot{Q}_{\text{rad}} \]

\[ hA(T_{oo} - T_5) = \varepsilon \sigma A (T_5^4 - T_{\text{sur}}^4) \]

\[ T_{oo} = T_5 + \frac{\varepsilon \sigma A}{h} (T_5^4 - T_{\text{sur}}^4) \]

\[ T_{oo} = (273^\circ K) + \frac{(0.85)(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4})}{(8 \frac{\text{W}}{\text{m}^2 \cdot \text{K}})} \cdot [(273^\circ K)^4 - (290^\circ K)^4] \]

\[ T_{oo} = 282.9^\circ K = 99.3^\circ C \]
STEAM, \( p = 1 \text{ atm} \), \( A = 30 \text{ cm} \times 30 \text{ cm} \) \( \rho, 23 \text{ HOLMAN} \)

\( m = 3.78 \frac{\text{kg}}{\text{hr}} \) FIND PLATE TEMP, \( T_L \)

\[ Q = \dot{m} h_{fg}, \quad L = 0.01 \text{m}, \quad \text{COMMERIAL BRONZE} \]

\[ T_L = 100^\circ \text{C} \]

\[ Q_{\text{cool}} \]

Condensation

\[ Q = \dot{m} h_{fg} \quad h_{fg} = 2257 \frac{\text{kJ}}{\text{kg}} \]

\[ Q = \left( 3.78 \frac{\text{kg}}{\text{hr}} \right) \left( 2257 \frac{\text{kJ}}{\text{kg}} \right) \left( \frac{1 \text{ hr}}{3600 \text{ s}} \right) = 2.37 \text{ kW} \]

\[ Q = -KA \frac{dT}{dx} = \dot{m}K \frac{A}{L} \left( T_H - T_L \right) \]

\[ T_L = \bar{T_H} - \frac{QL}{KA} \]

\[ T_L = \left( 100^\circ \text{C} \right) - \left[ \frac{\left( 2.37 \times 10^3 \text{ W} \right) \left( 0.01 \text{ m} \right)}{32 \frac{\text{W}}{\text{m-k}} \left( 0.3 \text{ m} \right) \left( 0.3 \text{ m} \right)} \right] \]

\[ T_L = 94.9^\circ \text{C} \]
Cylindrical Coordinates:
\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \dot{e}_{\text{gen}} = \frac{\dot{q}}{K} \frac{\partial T}{\partial z}
\]

Assumptions:
- Transient \( \left( \frac{\partial T}{\partial z} \neq 0 \right) \)
- Constant Properties
- No Change in Azimuthal Direction \( \left( \frac{\partial}{\partial \phi} = 0 \right) \)
- No Internal Heat Generation \( \left( \dot{e}_{\text{gen}} = 0 \right) \)

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} = \frac{\dot{q}}{K} \frac{\partial T}{\partial z}
\]

Initial Condition: \( T(r, z) = \text{constant} \)

Boundary Conditions:
@ \( z = 0 \): \( T(r) = \text{constant} \) or \( \frac{\partial T}{\partial r} \bigg|_{r=0} = 0 \)
@ \( z = L \): \( -K \frac{\partial T}{\partial r} = h(T_s - T_0) \)
@ \( r = 0 \): \( \frac{\partial T}{\partial r} = 0 \) Symmetry
@ \( r = r_0 \): \( \frac{\partial T}{\partial r} = 0 \) but \( T = T_0 \)