CHAPTER 8: INTERNAL FORCED CONVECTION

OBJECTIVES

- UNDERSTAND INTERNAL FLUID FLOWS
  - HYDRO BOUNDARY LAYER, HYDRO ENTRY LENGTH, FULLY DEVELOPED FLOW
  - LAMINAR VS. TURBULENT FLOW (REYNOLDS #)
  - MEAN VELOCITY DEFINITION
  - HYDRAULIC DIAMETER
  - PRESSURE DROP, FRICTION FACTOR, PUMPING POWER

- UNDERSTAND INTERNAL HEAT TRANSFER
  - THERMAL BOUNDARY LAYER, THERMAL ENTRY LENGTH, THERMALLY FULLY DEVELOPED FLOW
  - DETERMINE EXIT TEMPERATURE FOR CONSTANT HEAT FLUX, CONSTANT SURFACE TEMPERATURE
  - DEVISE NUSSELT NUMBER (DIMENSIONLESS HEAT TRANSFER COEFFICIENT)
  - NUS CORRELATIONS
    - EMPIRICAL
INTERNAL FLUID FLOWS

FIRST RECALL LAMINAR FLOW OVER A FLAT PLATE:

\[ \delta = \delta(x) \]
\[ u = u(x, y) \]

LAMINAR FLOW: SMOOTH, NO FLUCTUATIONS, SELF-DAMPING FLOW, CHARACTERIZED BY REYNOLDS #: \( Re = \frac{\text{INERTIAL FORCES}}{\text{VISCOS FORCE}} \)

LOW VELOCITIES, HIGH VIScosITIES \( \Rightarrow \) LAMINAR FLOW

LAMINAR PIPE FLOW: \( Re = \frac{V_{m}D}{\nu} \leq 2300 \)

HYDRODYNAMIC ENTRY LENGTH:
\[ L_{h, \text{LAM}} \approx 0.05 \text{ Re} D \]
HYDRODYNAMICALLY FULLY-DEVELOPED LAMINAR FLOW: UNCHANGING VELOCITY PROFILE AFTER $L_H, LAM$

$$\frac{dU}{dz} = 0$$

TURBULENT PIPE FLOW: $Re > 10,000$
HIGH VELOCITIES, SPONTANEOUS FLUCTUATIONS (EDDIES)

F-D FLAT VELOCITY PROFILE

HYDRO ENTRY LENGTH:

$L_H, TURB = 10D$

LAMINAR FLOW: LOW $\Delta P$, LOW $h$
TURBULENT FLOW: HIGHER $\Delta P$, HIGHER $h$ SHORTER ENTRY LENGTH

MEAN VELOCITY DEFINITION:

$$\dot{m} = \rho V_m A_c = \int_{A_c} \rho u(r) dA_c$$

$$A_c = \pi r_c^2, \quad dA_c = 2\pi r_c dr$$
FOR CONSTANT PROPERTIES,

\[ V_m = \frac{1}{\pi r_0^2} \int_0^{r_0} u(r) \cdot 2\pi r \, dr \]

\[ V_m = \frac{2}{r_0^2} \int_0^{r_0} u(r) \cdot r \, dr \]

FOR HFD LAMINAR FLOW,

\[ u(r) = -\frac{r_0^2}{4\mu} \frac{dP}{dz} \left(1 - \frac{r^2}{r_0^2}\right) = 2V_m \left(1 - \frac{r^2}{r_0^2}\right) \]

FOR NON-CIRCULAR TUBES, THE REYNOLDS NUMBER IS CAST IN TERMS OF THE HYDRAULIC DIAMETER:

\[ Re = \frac{V_m D_h}{\nu} \]

\[ D_h = \frac{4A}{P}, \quad A = \text{CROSS-SECTIONAL AREA} \]

\[ P = \text{WETTED PERIMETER} \]

FOR SQUARE TUBING,

\[ \frac{1}{W} = \frac{4A}{P} = \frac{4 \cdot W^2}{4W} = W \]

THE PRESSURE DROP CAN BE ESTIMATED FOR LAMINAR OR TURBULENT FLOWS, SMOOTH OR ROUGH SURFACES, CIRCULAR OR NON-CIRCULAR TUBES USING THE DARCY FRICTION FACTOR:
\[ \Delta P = f\left(\frac{L}{D}\right) \frac{1}{2} \rho V_x^2 \]

\[ f = f_{CN}(Re, E, D) \quad E = \text{SURFACE ROUGHNESS} \]

FOR LAMINAR FLOW,
\[ f = \frac{64}{Re} \]

FOR TURBULENT FLOW IN SMOOTH TUBES,
\[ f = (0.790 \cdot \ln Re - 1.64)^{-2} \]

FOR TURBULENT FLOW IN ROUGH SURFACE TUBES,
\[ \frac{1}{\sqrt{f}} = -2.0 \log \left( \frac{E/D}{3.7} + \frac{2.51}{Re^{1/4}} \right) \quad \text{FIGURE A-20} \]

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THE PUMPING POWER REQUIRED TO PUSH THE FLUID DOWN THE LENGTH OF THE PIPE IS:
\[ \dot{W}_{\text{pump}} = \dot{V} \cdot \Delta P \quad \dot{V} = \text{VOLUMETRIC FLOW RATE} \]

INTERNAL HEAT TRANSFER

FIRST RECALL HEAT TRANSFER TO LAMINAR FLOW OVER A FLAT PLATE:
\[ \delta_T = \delta_T(x), \quad T = T(x,y), \quad \dot{Q} = hA(T_s - T_\infty) \]

For \( Pr = 1 \), \( \delta \approx \delta_T \) (water, air)

For \( Pr \gg 1 \), \( \delta \gg \delta_T \) (oils)

For \( Pr \ll 1 \), \( \delta \ll \delta_T \) (liquid metals)

Laminar, HD Pipe Flow:

\[ \frac{q_s}{T_s} = \frac{\dot{Q}}{T_s/A} = \text{constant} \]

Thermal Entry Length:

\[ LT,\text{LAM} = 0.05 \text{ Re Pr} D = \text{Pr} \cdot LT,\text{LAM} \]

\[ LT,\text{TURB} = 10D \quad (\text{same as } LH,\text{TURB}) \]
FOR $\dot{q}_s$ = CONSTANT,

$T_s = T_s(z)$ \text{ \textit{Surface Temperature}}

$T_m = T_m(z)$ \text{ \textit{Mean Temperature}}

FOR HFD FLOW, $\frac{\partial u}{\partial z} = 0$ \textit{Velocity Profile Invariant with } $z$

FOR THERMALLY FULLY-DEVELOPED FLOW,

$\frac{\partial \Theta}{\partial z} = \frac{\partial}{\partial z} \left( \frac{T_s - T}{T_s - T_m} \right) = 0$

$\frac{\partial \Theta}{\partial z} = \frac{L_f}{\partial z}$ \text{ \textit{TFD}}

$A_x$

USE AN ENERGY BALANCE TO DETERMINE EXIT TEMPERATURE FOR $\dot{q}_s$ = CONSTANT:

$Q_s = \dot{q}_s A = mC_p(T_e - T_c)$

$T_e = T_c + \frac{\dot{q}_s A_s}{mC_p}$
FOR CONSTANT SURFACE TEMPERATURE,

\[ T_e = T_s - (T_s - T_c) \cdot \exp \left( - \frac{ln A_s}{m c_p} \right) \]

MEAN FLUID TEMPERATURE:

\[ 8C_p V_m T_m A_c = \int_{A_c} 8C_p u(r) \cdot T(r) \, dA_c \]

FOR CONSTANT PROPERTIES, CIRCULAR TUBE,

\[ A_c = \pi r_0^2, \quad dA_c = 2\pi r_0 \, dr \]

\[ T_m = \frac{1}{\pi r_0^2 V_m} \int_0^{r_0} u(r) \cdot T(r) \cdot r \, dr \]

\[ T_m = \frac{2}{r_0^2 V_m} \int_0^{r_0} u(r) \cdot T(r) \cdot r \, dr \]

FOR CIRCULAR TUBE, LAMINAR, HPD, TPD, \( q_s = C \):

\[ T_m = T_s - \frac{11}{24} \cdot \frac{q_s r_o}{k} \]

CONSTANT SURFACE TEMPERATURE:

\[ d\dot{Q} \]

\[ T_m \rightarrow \dot{Q}_{\text{conv}} \rightarrow T_m + dT_m \]

FIRST LAW: \( \dot{Q} - \dot{W} = \dot{m}(h_e - h_i) \)
\( \dot{W} = 0, \ h_e - h_i = C_p(T_e - T_i) \)
\( \dot{Q}_{\text{conv}} = \dot{m}C_p(T_e - T_i) \)
\( h(T_s - T_m) \cdot P \, dx = \dot{m}C_p \left[ (T_m + dT_m) - (T_m) \right] \)

\[ \frac{dT_m}{(T_s - T_m)} = \frac{hP}{\dot{m}C_p} \cdot dx \]
INTEGRATE BOTH SIDES:

\[ \int_{T_i}^{T_e} \frac{dT_m}{(T_s - T_m)} = \frac{hP}{\dot{m}C_p} \int_0^L \, dx = \frac{hPL}{\dot{m}C_p} = \frac{hA_s}{\dot{m}C_p} \]

LEFT-HAND SIDE:

LET \( \dot{W} = T_s - T_m, \ \, d\dot{W} = -dT_m, \ \, dT_m = -dW \)

LIMITS: @ \( T_m = T_i, \ \, W = T_s - T_i \) (INLET)
@ \( T_m = T_e, \ \, W = T_s - T_e \) (EXIT)
\[
\begin{align*}
&\int_{T_i}^{T_e} \frac{dT_m}{T_i(T_s-T_m)} = \int_{T_3-T_2}^{T_s-T_e} \left( -\frac{dW}{W} \right) \\
&= \left[ -\ln(W) \right]_{T_s-T_2}^{T_s-T_e} \\
&= -\ln\left( \frac{T_s-T_e}{T_3-T_2} \right) \\
-\ln\left( \frac{T_s-T_e}{T_3-T_2} \right) &= \frac{h A_s}{\dot{m} C_p} \\
\frac{T_s-T_e}{T_3-T_2} &= \exp\left( -\frac{h A_s}{\dot{m} C_p} \right) \\
T_s-T_e &= (T_3-T_2) \cdot \exp\left( -\frac{h A_s}{\dot{m} C_p} \right) \\
T_e-T_3 &= -(T_3-T_2) \cdot \exp\left( -\frac{h A_s}{\dot{m} C_p} \right) \\
T_e &= T_3 - (T_3-T_2) \cdot \exp\left( -\frac{h A_s}{\dot{m} C_p} \right)
\end{align*}
\]
HEAT TRANSFER COEFFICIENT FOR INTERNAL FLOW:
\[ \dot{Q} = h A (T_s - T_m) \]
\[ \frac{\dot{Q}}{A} = h (T_s - T_m) \]

\[ h = \frac{24}{11} \cdot \frac{K}{\text{ft}} = \frac{48}{11} \cdot \frac{K}{D} = 4.36 \frac{K}{D} \]

**DEFINE NUSSELT NUMBER (DIMENSIONLESS HEAT TRANSFER COEFFICIENT):**

\[ Nu = \frac{h D}{K} = 4.36 \quad (\dot{Q}_s = \text{CONSTANT}) \quad \text{HFD, TFD} \]

**FOR CONSTANT SURFACE TEMPERATURE (T_s = C):**

\[ Nu = \frac{h D}{K} = 3.66 \quad (\text{SOLVED NUMERICALLY}) \quad \text{HFD, TFD} \]

**EMPIRICAL Nu CORRELATIONS:**

LAMINAR, HFD, THERMALLY DEVELOPING, T_s = C:

\[ Nu = 3.66 + \frac{0.065 (D/L) \text{Re Pr}}{1 + 0.04 [(D/L) \text{Re Pr}]^{2/3}} \]

TURBULENT, HFD, TFD; T_s = C or \( \dot{Q}_s = C \)

\[ Nu = \left( \frac{f/8}{(\text{Re} - 1000)} \right) \text{Pr} \]

\[ \frac{1 + 12.7 (f/8)^{1/2} (\text{Pr}^{2/3} - 1)}{1} \]

CAN BE USED IN LAM/TURB TRANSITION REGION