Given: Sinusoidal velocity profile for laminar boundary layer 
\[ u = A \sin(By) + c \]

Find: (a) State three applicable boundary conditions.
(b) Evaluate A, B and c.

Solution: For the boundary layer, at

1. \( y = 0, \ u = 0 \) (no slip)
2. \( y = \delta, \ u = 0 \)
3. \( \frac{\partial u}{\partial y} = 0 \) (no shear stress)

Applying these boundary conditions,

1. \( u(0) = A \sin(0) + c = 0 \Rightarrow c = 0 \)
2. \( u(\delta) = A \sin(B\delta) = U \)
   \[ \frac{\partial u}{\partial y} = AB \cos(By) \]
3. \( \frac{\partial u}{\partial y}_{y=\delta} = AB \cos(B\delta) = 0 \Rightarrow BS = \frac{\pi}{2}, \text{ or } B = \frac{\pi}{2\delta} \)

Then from (2), \( A \sin(B\delta) = A \sin(\frac{\pi}{2}) = A = U \), and then

\[ u = U \sin(B \frac{y}{\delta}) \]

\[ A = U, \ B = \frac{\pi}{2\delta}, \ c = 0 \]
Given: Flow in the entrance region of a square duct as shown.

\[ u_0 = 30 \text{ m/s} \]

Fluid is air.

\[ h = 80 \text{ mm} \]

\[ b_2 = 1.0 \text{ mm} \]

Find: Pressure change between sections 1 and 2.

Solution: Apply continuity equation to find \( V_2 \), then use Bernoulli equation to find pressure change.

Basic equations:

\[ \frac{\partial}{\partial t} \int_{CV} \rho \, dv + \int_{CS} \rho \, V \cdot da = 0 \]  

(1)

\[ \frac{p_1}{\rho} + \frac{V_1^2}{2} + \varphi_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + \varphi_2 \]  

(5)

Assumptions:

(1) Steady flow
(2) Incompressible flow
(3) No friction outside boundary layers
(4) Flow along a streamline
(5) \( \varphi_1 = \varphi_2 \)

Then

\[ 0 = \left[ -1 \left( \rho V_c A_1 \right) \right] + \left[ \left( \rho V_c A_2 \right) \right] \] or \( V_1 A_1 = V_2 A_2 \) or \( V_2 = V_1 \frac{A_1}{A_2} \)

and

\[ p_1 - p_2 = \frac{\rho V_c}{2} (V_2 - V_1) = \frac{\rho V_c}{2} \left( \frac{A_1}{A_2, \text{eff}} \right)^2 - 1 \]

At section 1, the area is \( A_1 = h^2 \), but at section 2, the effective flow area is reduced by the wall boundary layers. Using the displacement thickness concept,

\[ A_{2, \text{eff}} = \left( h - 2 \delta^* \right)^2 \] so that \( \left( \frac{A_1}{A_{2, \text{eff}}} \right)^2 = \left[ \frac{h^2}{(h - 2 \delta^*)^2} \right]^2 \)

Thus

\[ p_1 - p_2 = \frac{\rho V_c}{2} \left\{ \left[ \frac{h^2}{(h - 2 \delta^*)^2} \right]^2 - 1 \right\} \]

\[ = \frac{1}{2} \times 1.23 \text{ kg/m}^3 \times (40 \text{ m/s})^2 \times \left[ \left( \frac{80 \text{ mm}}{80 - 2 \delta^*} \right)^2 \right] - 1 \] \times \frac{9.8 \text{ m/s}^2 \times 80^2 \text{ mm}^2}{1 \text{ kg/m}}

\[ = 59.0 \text{ N/m}^2 \] (59.0 Pa)

\[ p_1 - p_2 \]
**Problem 9.32**

**Given:** Thin flat plate in water tunnel.

\[ U = 1.6 \text{ m/s} \quad \rightarrow \quad b = 1 \text{ m} \]

\[ L = 0.3 \text{ m} \]

Velocity profile is \( \frac{u}{U} = 2 \left( \frac{y}{S} \right) - \left( \frac{y}{S} \right)^2 \)

**Find:** Total drag force on plate due to skin friction.

**Solution:** Check \( \text{Re}_L = \frac{UL}{v} = \frac{1.6 \text{ m/s} \times 0.3 \text{ m}}{10^{-6} \text{ m}^2/\text{s}} = 4.8 \times 10^5 \), so laminar.

Viscous drag for two sides of plate is (Assume \( T = 20^\circ \text{C} \).)

\[
\text{Drag} = 2 \int_0^L \frac{\mu U}{\delta} \frac{d}{dx} \left( \frac{2}{\text{Re}_x} \right) dx.
\]

From the definition, \( \text{Tw} = \mu \frac{dU}{dx} \mid_{y=0} = \frac{\mu U}{\delta} \left( \frac{d(\mu U)}{d(\mu U)} \right) \mid_{y=0} = \frac{\mu U}{\delta} \left( 2 - 2\eta \right) \mid_{\eta=0} = \frac{2\mu U}{\delta} \)

Since \( \frac{S}{x} = \frac{5.48}{\sqrt{\text{Re}_x}} \), \( S = \frac{5.48 S}{\sqrt{\text{Re}_x}} = \frac{8.48}{\sqrt{U/\nu}} \)

\( \mu = \rho \nu \)

Substituting into Eq. 1,

\[
F_D = 2 \int_0^L \frac{2\mu U}{\delta} \frac{d}{dx} \left( \frac{1}{\sqrt{\text{Re}_x}} \right) dx = \frac{4}{8.48} \frac{b \mu U}{\sqrt{U/\nu}} \int_0^L \frac{d}{dx} \left( \frac{1}{\sqrt{2x^7}} \right) dx = \frac{4 \times 1.0 \text{ m/s}}{8.48 \sqrt{U/\nu}} \left[ \frac{1}{7} x \right]_0^L = \frac{8 \times 1.0 \text{ m/s}}{8.48} \sqrt{U/\nu}
\]

\[ \text{Since} \quad \frac{S}{x} = \frac{5.48}{\sqrt{\text{Re}_x}} \]

\[ F_D = 1.62 \text{ N} \]
Given: Turbulent boundary-layer flow of water, $\frac{1}{5}$-power profile.

\[
\frac{U}{U_0} = \left( \frac{y}{\delta} \right)^{1/5} \quad U_0 = 1 \text{ m/sec} \quad \delta \quad \text{Conditions of Example Problem 9.4.}
\]

Find: (a) Expression for wall shear stress, $\tau_w$.
(b) Integrate to obtain expression for drag force, $F_D$.
(c) Evaluate for the conditions shown.

Solution: Apply results from the momentum integral equation.

Computing equation:

\[
C_f = \frac{\tau_w}{\frac{1}{2} \rho U^2} = \frac{0.0594}{(Re_L)^{1/5}}
\]  
(9.27)

Solving for $\tau_w$,

\[
\tau_w = \frac{1}{2} \rho U^2 \left( \frac{0.0594}{(Re_L)^{1/5}} \right) x^{-\frac{1}{15}} - \frac{1}{2} \rho U^2 \left( \frac{0.0594}{(Re_L)^{1/5}} \right) L^{-\frac{1}{15}}
\]

Integrating to obtain $F_D$,

\[
F_D = \int_0^L \tau_w \, dx = \int_0^L \frac{0.0594}{(Re_L)^{1/5}} \left( \frac{1}{2} \rho U^2 \right) x^{-\frac{1}{15}} \, dx = \frac{5}{4} \rho U^2 \left( \frac{0.0594}{(Re_L)^{1/5}} \right) L^{-\frac{1}{15}}
\]

Evaluating, with $L = 1 \text{ m},$

\[
Re_L = \frac{UL}{\nu} = \frac{1 \text{ m} \times 1 \text{ m}}{0.7 \times 10^{-6} \text{ m}^2} = 1.00 \times 10^6 \quad (\nu = 20^\circ C, \text{ Table A.4.6})
\]

\[
F_D = \frac{1}{2} \times \frac{0.0594}{(Re_L)^{1/5}} \times \frac{U^2}{m} \times \frac{1}{2} \rho U^2 \left( \frac{0.0594}{(Re_L)^{1/5}} \right) \frac{N \cdot m}{kg \cdot m} = 2.34 \text{ N}
\]
Given: Air at standard conditions flows at 10 m/s over a flat plate.

Find: \( \delta \) and \( f_w \) at a point 1 m from leading edge for
(a) completely laminar flow (parabolic velocity profile)
(b) completely turbulent flow (\( \kappa \)-power velocity profile)

Solution:

Computing equations:

\[ \delta = \frac{5.48 \times \frac{1}{Re}}{1} \quad \delta = \frac{0.382 \times \frac{1}{Re}}{1} \]

\[ C_f = \frac{0.130 \times \frac{1}{Re}}{1} \quad C_f = \frac{0.0594 \times \frac{1}{Re}}{1} \]

For standard air, \( \rho = 1.23 \text{ kg/m}^3 \), \( \nu = 1.52 \times 10^{-5} \text{ m}^2/\text{s} \) (Table II.10)

The Reynolds number is

\[ Re = \frac{10 \times 1 \times 1.52 \times 10^{-5}}{6.85 \times 10^{-5}} = 6.85 \times 10^5 \]

For laminar flow:

\[ \delta = 5.48 \times \frac{1}{6.85 \times 10^5} = 0.62 \text{ mm} \]

\[ C_f = \frac{0.130 \times \frac{1}{6.85 \times 10^5}}{1} \]

\[ f_w = \frac{1}{2} \left( \frac{0.130 \times \frac{1}{6.85 \times 10^5}}{1} \right) \frac{1}{\sqrt{2}} = 0.054 \frac{N}{m} \frac{N}{m} \]

For turbulent flow:

\[ \delta = \frac{0.382 \times \frac{1}{6.85 \times 10^5}}{1} = 26.0 \text{ mm} \]

\[ C_f = \frac{0.0594 \times \frac{1}{6.85 \times 10^5}}{1} \]

\[ f_w = \frac{1}{2} \left( \frac{0.0594 \times \frac{1}{6.85 \times 10^5}}{1} \right) \frac{1}{\sqrt{2}} = 0.249 \frac{N}{m} \frac{N}{m} \]

Comparing:

\[ \frac{f_w \text{ turb}}{f_w \text{ laminar}} = 4.58 \]

Thus the turbulent boundary layer has a much larger skin friction which causes it to grow more rapidly.
Problem 9.64

Given: Stabilizing fin on Bonneville land speed record auto.

\[ z = 1.340 \text{ m} \]

\[ V = 560 \text{ km/hr} \]

\[ L = 1.65 \text{ m} \]

\[ H = 0.785 \text{ m} \]

Find: (a) Evaluate \( Re \)
(b) Location of \( x_t \)
(c) Power to overcome skin friction drag on fin.

Solution: Assume standard atmosphere, so \( T = 279 \text{ K} \), \( \rho / \rho_0 = 0.877 \)
(Table A.3); \( \mu = 1.79 \times 10^{-5} \text{ kg/m sec} \) (Table A.17). Then

\[ Re_L = \frac{\rho V L}{\mu} = (0.877) 1.23 \frac{\text{kg}}{\text{m}^3} \times 560 \times 10^3 \frac{\text{m}}{\text{hr}} \times 1.65 \frac{\text{m}}{1.79 \times 10^{-5} \frac{\text{kg}}{\text{m sec}}} \frac{\text{hr}}{3600 \text{s}} \]

\[ Re_L = 1.55 \times 10^7 \]

Assume transition occurs at \( Re_x = 500,000 \). Then

\[ \frac{x_t}{L} = \frac{Re_x}{Re_L} = \frac{500,000}{1.55 \times 10^7} = 0.0323 \]

\[ x_t = 0.0323 \times 1.65 \text{ m} = 0.0532 \text{ m} = 53.2 \text{ mm} \]

Calculate drag force using \( C_D \) from Fig. 9.8: \( F_D = C_D A \frac{1}{2} \rho V^2 \)

\[ C_D = 0.0029 \text{ (Fig. 9.8)} \]

\[ A = 2 \times 1.65 \times 0.785 \text{ m}^2 \]

\[ \frac{1}{2} \rho V^2 = \frac{1}{2} (0.877) 1.23 \frac{\text{kg}}{\text{m}^3} \left( 560 \times 10^3 \frac{\text{m}}{\text{hr}} \times \frac{\text{hr}}{3600 \text{s}} \right) \left( \frac{\text{N} \cdot \text{sec}}{\text{kg} \cdot \text{m}} \right) = 1.31 \times 10^4 \text{ N} \cdot \text{m}^2 \]

\[ F_D = 0.0029 \times 1.31 \times 10^4 \frac{\text{N} \cdot \text{m}^2}{\text{m}^2} = 98.4 \text{ N} \text{ (skin friction drag on fin)} \]

The power required is

\[ P = F_D V = 98.4 \text{ N} \times 560 \times 10^3 \frac{\text{m}}{\text{hr}} \times \frac{\text{hr}}{3600 \text{s}} \times \frac{\text{W} \cdot \text{s}}{\text{N} \cdot \text{m}} = 15.3 \text{ kW} \]

Check using Eq. 9.37b:

\[ C_D = \frac{0.455}{(\log Re)^{2.58} \frac{1610}{Re}} = 0.00270 \]

This is slightly less than from the graph, but reasonable agreement.
Given: Single-vane anemometer made from brass plate, $t$ thick, with $h = 20$ mm and $w = 10$ mm.

Find: (a) Relationship for wind speed as a function of deflection angle, $\theta$.
(b) Plate thickness to give $\theta = 30^\circ$ at $V = 10$ m/s.

Solution: Sum moments about pivot.

$$\Sigma M = F_N \frac{b}{2} - mg \frac{b}{2} \sin\theta = 0$$

$$F_N = \frac{c_D A \frac{1}{2} \rho V^2}{m g \sin\theta}$$

$$c_D A \frac{1}{2} \rho V^2 \cos\theta = m g \sin\theta$$

$$V = \left[ \frac{2 m g \sin\theta}{c_D A \rho \cos\theta} \right]^{\frac{1}{2}}$$

From plate geometry, $m = \rho \frac{w h t}{2}$. From Eq. 1,

$$SG \rho h \omega w h t \sin\theta = c_D A \frac{1}{2} \rho V^2 \cos\theta$$

{From Table A.1, $SG = 8.55$ for brass.}

$$t = \frac{c_D A \rho V^2 \cos\theta}{2 SG \rho h \omega h \sin\theta} = \frac{c_D \rho V^2 \cos\theta}{2 SG \rho h \omega h \sin\theta}$$

Since $A = w h$

From Fig. 9.10, $c_D = 1.2$ at $b/h = 2.0, 50$

$$t = \frac{1.2 \times 1.73 \text{ kg} \times (10)^2 \text{ m}^2 \times \cos(30^\circ) \times m^2}{999 \text{ kg} \times \sin(30^\circ) \times 9.81 \text{ m} \times \frac{1}{1000 \text{ mm}}}$$

$$t = 1.30 \text{ mm}$$