PROB. 3.8

\[ P_2 = P_1 + \rho_0 g h_1 \]
\[ P_3 = P_2 + \rho_0 g h_2 \]
\[ P_4 = P_3 + \rho_w g h_3 \]
\[ P_4 = (P_2 + \rho_0 g h_2) + \rho_w g h_3 \]
\[ P_4 - P_2 = g \left( \rho_0 h_2 + \rho_w h_3 \right) \]

\[ \frac{\rho_{3AE \text{ oil}}}{\rho_{3AE \text{ water}}} = 0.92 = \frac{\rho_0}{\rho_w} \quad \rho_0 = 5 \rho \rho_w \]

\[ P_4 - P_2 = g \rho_w \left( \rho_{3AE \text{ oil}} h_2 + h_3 \right) \]

\[ P_4 - P_2 = (9.81 \, \text{m/s}^2) \times 1000 \left( \frac{\text{kg}}{\text{m}^3} \right) \left[ (0.92 \times 10 \, \text{mm}) + (90 \, \text{mm}) \right] \left( \frac{\text{m}}{1000 \, \text{mm}} \right) \left( \frac{\text{N}}{\text{kg} \cdot \text{m} / \text{s}^2} \right) \]

\[ P_4 - P_2 = 973 \, \text{N/m}^2 \]

\[ \text{FBD} \]

\[ F_B = F_T + mg \]
PROB. 3.8 CONT.

\[ S = \frac{m}{V}, \quad m = gV \]

\[ F_B = F_T + gVg \]

\[ S = \frac{F_B - F_T}{Vg} \]

\[ \rho_B = \frac{F_B}{A}, \quad F_B = \rho_B \cdot A \]

\[ S = \frac{A}{Vg} (\rho_B - \rho_T) \]

\[ \frac{A}{V} = \frac{LW}{LWH} = \frac{1}{H} \]

\[ S = \frac{1}{Hg} (\rho_B - \rho_T) \]

\[ S = \left( \frac{100}{\chi} \times \frac{9.81 \text{ m/s}^2}{973} \right) \left( \frac{\frac{Kg - m^2}{s^2}}{N} \right) \left( \frac{1000 \text{ mm}}{m} \right) \]

\[ S = 992 \frac{Kg}{m^3} \]
PROB. 3.29

\( D = 3^{\text{in}}, \; d = 0.25^{\text{in}}, \; \text{MERIAM RED OIL} \), FIND \( \Theta \) FOR

\( L = 5^{\text{in}}, \; P = 1 \text{ in H}_2\text{O (GAGE)} \), FIND \( S \),

\[ \Delta p = g_w g L \left[ \sin \Theta + \frac{(d - D)}{D} \right] \]

\( g_{oil} = S_{oil} \cdot g_w \)

\[ \Delta p = g_w g h = (62.4 \frac{\text{LBF}}{\text{ft}^3}) \left( 1 \text{ in} \right) \left( \frac{1 \text{ in}}{12 \text{ in}} \right) = \frac{5.21 \text{ LBF}}{\text{ft}^2} \]

\[ \Theta = \sin^{-1} \left[ \frac{\Delta p}{g_{oil} g_w g L} - \left( \frac{d - D}{D} \right)^2 \right] \]

\[ \Theta = \sin^{-1} \left\{ \frac{(5.21 \frac{\text{LBF}}{\text{ft}^2})}{(0.827)(62.4 \frac{\text{LBF}}{\text{ft}^3})(5 \text{ in})(\frac{1 \text{ ft}}{12 \text{ in}})} - \left( \frac{0.25^{\text{in}}}{3^{\text{in}}} \right)^2 \right\} \]

\[ \Theta = 13.6^\circ \]

\[ S = \frac{L}{h_e} = \frac{5^{\text{in}}}{1^{\text{in}}} = 5 \]
FBD

\[ \sum M_B = 0 \Rightarrow \]
\[ R F_A - (H - y') F_R = 0 \]
\[ F_A = \frac{(H - y')}{R} F_R \]  \hspace{1cm} (1)

\[ F_R = P c A = 5 g \frac{1}{2} y_c A \]  \hspace{1cm} (2)

\[ y_c = H - y \]  \hspace{1cm} (3)

For a semicircle,
\[ \frac{y}{r} = \frac{4R}{3\pi} \rightleftharpoons A = \frac{\pi}{2} R^2 \rightleftharpoons I_x = \frac{1}{8} \pi R^4 \]  \hspace{1cm} (4)

\[ F_R = 5g \frac{(H - \frac{3R}{\pi})\left(\frac{\pi}{2} R^2\right)}{3\pi} \]

For \( P \) the same on both sides,
\[ y' = y_c + \frac{I_x}{2y_c \lambda_y} \]
P.A.T.:

\[ I_x = I_{x'} + A \gamma^2 \]
\[ I_{x'} = I_x - A \gamma^2 \]

\[ I_{x'} = \frac{\pi}{8} R^4 - \left( \frac{\pi}{2} R^2 \right) \left( \frac{4R}{3H} \right) \]

Combine eqns. 1 - 5 gives (4 pages of algebra)

\[ F_A = 9.31 \left( \frac{2}{3} HR^2 - \frac{\pi}{8} R^2 \right) \]

\[ F_A = \left( 1000 - \frac{k_g}{m} \right) \left( 9.81 \frac{N}{kg-m/s^2} \right) \left( \frac{N}{kg-m/s^2} \right) \left[ \frac{2}{3} \left( 8^m \right)^2 - \frac{\pi}{8} \left( 3^m \right)^3 \right] \]

\[ F_A = 3.67 \times 10^5 \text{ N} = 367 \text{ KN} \]
PROB. 3.64

FIND \( F_V \) AND LINE OF ACTION.

\[
\begin{align*}
F_V &= \frac{8}{3}gV \\
V &= \left(\frac{11}{4}R^2\right)W \\
F_V &= \frac{11}{4}R^2W \delta g \\
\bar{X} &= \frac{4R}{3\pi} \\
F_H &= P_cA \\
Y_c &= \frac{1}{3}R \\
P_c &= SwgY_c = \frac{1}{3}sgR \\
A &= RW \\
F_H &= \frac{1}{4}RW^2 \delta g \\
Y' &= Y_c + \frac{I_{xxx}}{AY_c} \\
I_{xxx} &= \frac{1}{12}bh^3 = \frac{1}{12}WR^3
\end{align*}
\]
Problem 3.8

A hollow metal cube with sides 100 mm floats at the interface between a layer of water and a lit of SAE 10W oil such that 10% of the cube is exposed to the oil. What is the pressure difference between the upper and lower horizontal surfaces? What is the average density of the cube?

Given: Properties of a cube floating at an interface

Find: The pressures difference between the upper and lower surfaces; average cube density

Solution

The pressure difference is obtained from two applications of Eq. 3.7

\[ p_U = p_0 + \rho_{SAE10} \cdot g \cdot (H - 0.1 \cdot d) \]

\[ p_L = p_0 + \rho_{SAE10} \cdot g \cdot H + \rho_{H2O} \cdot g \cdot 0.9 \cdot d \]

where \( p_U \) and \( p_L \) are the upper and lower pressures, \( p_0 \) is the oil free surface pressure, \( H \) is the depth of the interface, and \( d \) is the cube size

Hence the pressure difference is

\[ \Delta p = p_L - p_U = \rho_{H2O} \cdot g \cdot 0.9 \cdot d + \rho_{SAE10} \cdot g \cdot 0.1 \cdot d \]

\[ \Delta p = \rho_{H2O} \cdot g \cdot d \cdot (0.9 + \text{SG}_{SAE10} \cdot 0.1) \]

From Table A.2, for SAE 10W oil: \( \text{SG}_{SAE10} = 0.92 \)

\[ \Delta p = \frac{999 \cdot \text{kg}}{\text{m}^3} \times 9.81 \text{~m} \cdot \text{s}^{-2} \times 0.1 \cdot \text{m} \times (0.9 + 0.92 \times 0.1) \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \]
\[ \Delta p = 972 \text{ Pa} \]

For the cube density, set up a free body force balance for the cube

\[ \Sigma F = 0 = \Delta p \cdot A - W \]

Hence
\[ W = \Delta p \cdot A = \Delta p \cdot d^2 \]

\[ \rho_{\text{cube}} = \frac{m}{d^3} = \frac{W}{d^3 \cdot g} = \frac{\Delta p \cdot d^2}{d^3 \cdot g} = \frac{\Delta p}{d \cdot g} \]

\[ \rho_{\text{cube}} = \frac{972 \cdot \frac{N}{m^2} \times \frac{1}{0.1 \cdot m} \times \frac{s^2}{9.81 \cdot m} \times \frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{s}^2}} \]

\[ \rho_{\text{cube}} = 991 \frac{\text{kg}}{m^3} \]
Given: Inclined manometer as shown filled with oil, \( \sigma_g = 0.897 \)

Find: Angle, \( \theta \), such that applied pressure of 1 in. H\(_2\)O gage gives 5" oil deflection along incline. Also determine sensitivity.

Solution:

Basic equation: \( \frac{\Delta p}{g} = \theta \)

Assumptions:
1. Static fluid
2. Gravity is only body force
3. \( \theta \) axis directed vertically

\[ \Delta p = \Delta \rho g = \rho g \Delta z \]

For constant \( \nu \), \( \Delta p = P_1 - P_2 = \rho g (h_1 - h_2) \)

Under applied pressure \( \Delta p = \sigma_{oil} \left( L \sin \theta + x \right) \)

where \( \Delta p = 1 \text{ in. H}_2\text{O} = \frac{1}{12} \text{ in} \times \frac{1}{12} \text{ ft} = 0.0833 \text{ lb f ft}^2 \)

Since the volume of the oil must remain constant:

\[ x \text{ Area } = \frac{L A_{tube}}{\text{ Area}} \]

\[ x = \frac{L A_{tube}}{\text{ Area}} \]

And:

\[ \Delta p = \sigma_{oil} \left( L \sin \theta + L \frac{A_t}{A_r} \right) = \sigma_{oil} \left[ L \sin \theta + L \left( \frac{\Delta h}{D} \right) \right] \]

Solving for \( \sin \theta \):

\[ \sin \theta = \frac{\Delta p}{\sigma_{oil} L} - \left( \frac{\Delta h}{D} \right)^2 \]

\[ \theta = \arcsin \left( \frac{5.2 \text{ lb f ft}^2}{0.0833 \text{ lb f in} \times \frac{1}{12} \text{ in} \times \frac{1}{12} \text{ ft}^2} \right) \]

\[ \theta = 12.5^\circ \]

The manometer sensitivity, \( s = \frac{\Delta h}{\Delta \rho} = \frac{5 \text{ in}}{\text{ in}} = 5 \)
Given: Semicircular plate gate \( AB \) is hinged along \( B \) and held in place by a horizontal force \( F_h \).

Find: Force \( F_h \) required to hold gate in place

Solution:

Basic equations: \( \text{d}p = \rho g \text{d}y \); \( F_h = \int \rho g h \text{d}A \); \( \Sigma M = 0 \)

Assumptions: (i) static fluid (ii) \( \rho = \text{constant} \) (iii) door is in equilibrium.

Since \( \Sigma M = 0 \) for equilibrium, taking moments about the hinge \( B \), \( F_h = \frac{1}{R} \int_0^R p \rho g \text{d}A \)

Since \( \text{d}p = \rho g \text{d}y \) and \( p = \rho g + \rho g h \),

Because the free surface is at atmospheric pressure, and atmospheric pressure acts on the outside of the gate, \( p = \rho g \),

and \( F_h = \frac{1}{R} \int_0^R \rho g \text{d}A \)

For the circular gate, \( \text{d}A = r \text{d}r \text{d}\theta \), \( y = r \sin \theta \), \( h = H - y \)

\( \text{so } F_h = \frac{1}{R} \int_0^R \int_0^\pi \rho g (H - r \sin \theta) r \text{d}r \text{d}\theta \)

\( F_h = \rho g \int_0^R \int_0^\pi \left( H^2 - r^2 \sin^2 \theta \right) r \sin \theta \text{d}r \text{d}\theta = \frac{\rho g R^3}{3} \left[ \frac{4}{3} - \frac{r^4}{4} \sin^2 \theta \right] \text{d}\theta \)

\( F_h = \rho \frac{g R^3}{3} \left[ \frac{4}{3} - \frac{1}{4} \sin^2 \theta \right] \text{d}\theta = \rho \frac{g R^3}{3} \left[ \frac{4}{3} \sin \theta - \frac{1}{3} \sin^3 \theta \right] \text{d}\theta \)

\( F_h = \rho g \left[ \frac{5}{3} - \frac{1}{3} \right] \)

\( = 998 \text{ N/m}^3 \times 9.81 \text{ m/s}^2 \times \left[ \frac{2 \times 8 \text{ m} \times 9.81 \text{ m}^2}{3} - \frac{5 \times 2 \times 8^3 \text{ m}^3}{3} \right] \times \frac{1.5}{\text{m}^2} \times \frac{1}{\text{N}} \times \frac{1}{\text{m}} \)

\( F_h = 366 \text{ kN} \)
Given: Gate formed in the shape of a circular arc has width or w meters. Liquid is water; depth \( h = R \).

Find: (a) magnitude and direction of the net vertical force component due to fluid acting on the gate,
(b) line of action of vertical component of the force.

Solution

Basic equations: \( \vec{F}_x = -\int \rho dF \), \( \frac{dp}{dy} = \rho g \), \( x', F_{x y} = (x dF) \).

Assumptions:
1. Static fluid
2. \( \rho = \) constant
3. \( y \) is measured positive downward from free surface
4. \( F_{x y} = \) \( \int R \sin \theta \) \( w d \theta \)

We can obtain an expression for \( F \) as a function of \( y \):

\[
\frac{dp}{dy} = \rho g \quad dp = \rho g \, dy \quad \text{and} \quad P - P_0 = \int_0^y \rho g \, dy = \rho g y
\]

Since atmospheric pressure acts at the free surface and on the back surface of the gate, then the appropriate expression for \( P \) is \( P = \rho g y \).

Along the surface of the gate, \( y = R \sin \theta \) and hence \( P = \rho g R \sin \theta \).

Thus, \( F_{x y} = \int_0^{\frac{\pi}{2}} \rho g R \sin \theta \, w R \, d \theta = \rho g R^2 \int_0^{\frac{\pi}{2}} \sin \theta \, d \theta = \rho g R^2 \left[ -\cos \theta \right]_0^{\frac{\pi}{2}} = \frac{\rho g R^2 W}{2} \).

\[ F_{x y} = -\rho g w R^2 \left[ \frac{1}{2} \sin^2 \theta \right] \left[ \frac{1}{2} \sin^2 \theta \right]_0^{\frac{\pi}{2}} \]

For any element of surface area \( dA \), the force \( dF \) acts normal to the surface. Thus each \( dF \) has a line of action through the origin. Consequently, the line of action of \( F_{x y} \) must also be through the origin.

We can find the line of action of \( F_{x y} \) by recognizing that the moment of \( F_{x y} \) about an axis through the origin must be equal to the sum of the moments of \( dy \) about the same axis.

\[
x' F_{x y} = \int x \, dF_y = \int (-\rho g R \sin \theta) \sin \theta \, d\theta = -\rho g w R^2 \int_0^{\frac{\pi}{2}} \sin^2 \theta \, d\theta
\]

\[
x' F_{x y} = -\rho g w R^2 \left[ \frac{1}{2} \sin^2 \theta \right]_0^{\frac{\pi}{2}} = -\rho g w R^2 \left[ \frac{1}{2} \sin^2 \theta \right]_0^{\frac{\pi}{2}}
\]

\[
x' = \frac{(1/2) w R}{2}
\]