Simultaneous measurement of sound velocity and wall thickness of a tube

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Abstract

A method for simultaneously measuring the sound propagation velocity and the thickness of each wall on the opposite sides of a tube is presented. The method uses a pair of ultrasound transducers to produce two reflected pulses from the outer and inner surfaces of the tube wall on the each side, and two transmitted pulses, one with and one without the tube sample between the two transducers. Using the time-domain analysis, sound velocity and wall thickness of the tube are determined from the time delays between the three pairs of ultrasound pulses, whereas using the frequency-domain analysis, phase velocity, group velocity, and wall thickness of the tube are determined from the phase differences between the three pairs of ultrasound pulses. Results of measurements on five tube samples are reported. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Measurement of the wall thickness of tubes is an important procedure for quality assurance and quality control in tube manufacture. The most widely used tool for thickness measurement is micrometers which physically measure the distance between two contact points (calipers) that are placed on the inner and outer walls of a sample. When the inner diameter of the tube is too small for insertion of the caliper, or when the tube material is too soft that the applied pressure by the calipers may cause a significant change in wall thickness, the method of using micrometers may not be applicable. Ultrasound provides an alternative technology for thickness measurement in these situations. In ultrasound measurement, there is no physical contact between the ultrasound transducers and the tube surfaces and the method is capable of measuring wall thickness of a tube with a very small diameter. The ultrasound method is particularly useful for continuous in-line thickness measurement during plastic tube extrusions. Since the current ultrasound method actually measures the time delay between two echoes reflected back from the outer and inner tube surfaces, it uses an assumed sound speed to calculate the wall thickness. If the actual sound speed is different from the assumed speed, measurement error results.

In this paper, we present a method that can simultaneously measure the sound propagation velocity in a tube and the thickness of each wall on the opposite sides of the tube. This new method is essentially an extension of a method, first proposed by Kuo et al. [1] and later expanded by Hsu and Hughes [2], which determines the sound velocity and the thickness of a single plate using times-of-flight data. In their method, the ultrasound pulse passes through a single wall while in the tube measurement, the ultrasound pulse has to pass through two walls which may have different thickness. In addition, the proposed method is capable of measuring the phase velocity as well as the group velocity. When dispersion (sound velocity changes with frequency) is significant, the measurement of phase velocity provides a more accurate description of the material property as well as results in a more accurate thickness measurement [2].

2. Theory

Fig. 1 shows the placement of the two ultrasound transducers and the signal paths in the measurement. It is assumed that the entire setup is immersed in the water,
and the inside of the tube is also filled with water. The method requires to perform two reflection measurements and two transmission measurements. First, as shown in the top drawing in Fig. 1, transducer $T_1$ sends a pulse $P_0(t)$ and the echoes reflected back from the outer and inner surfaces of the front wall (with a thickness $L_1$) are recorded as $R_1(t)$. Then, transducer $T_2$ sends a pulse and the echoes reflected back from the two surfaces of the wall on the opposite side (with a thickness $L_2$) is recorded as $R_2(t)$. Next, transducer $T_1$ sends a pulse which passes through two walls and is recorded by $T_2$ as $T_w(t)$, as shown in the middle drawing in Fig. 1. Finally, the tube sample is removed and a pulse which passes through the water path only is recorded as $T_w(t)$, as shown in the lower drawing in Fig. 1.

Fig. 2 shows four signal traces recorded in an experiment with a polyethylene tube which has a nominal wall thickness of 1.58 mm. From the top two plots, one notices the extra 180° phase difference between the first echo (reflected by the outer surface) and the second echo (reflected by the inner surface). If we define the time delay between these two echoes as $\Delta t_1$ and assume the speed of sound is $c$, the current method calculates the wall thickness $L_1$ based on the following formula:

$$L_1 = \frac{c\Delta t_1}{2}$$  \hspace{1cm} (1)

Similarly, the wall thickness $L_2$ can be calculated from $c$ and $\Delta t_2$. We now present the method for simultaneously determining $c$, $L_1$ and $L_2$ based on either the time delays or the phase differences of the pulses using the frequency-domain analysis.

2.1. Determine the sound speed and wall thickness using the time-domain analysis

If we denote $\Delta t_1$, $\Delta t_2$ and $\Delta t_3$ as the time delay between the two echoes associated with $L_1$, the time delay between the two echoes associated with $L_2$, and the time delay between $T_w(t)$ and $T_w(t)$, respectively, as shown in Fig. 2, we have the following set of equations:

$$\Delta t_1 = \frac{2L_1}{c}$$  \hspace{1cm} (2)

$$\Delta t_2 = \frac{2L_2}{c}$$  \hspace{1cm} (3)

$$\Delta t_3 = \frac{L_1 + L_2 - L_1 + L_2}{c_w}$$  \hspace{1cm} (4)

where $c_w$ is the speed of sound in water ($c_w$ depends on the temperature only). From the above three equations, $c$ can first be solved:

$$c = c_w \left[ \frac{2\Delta t_3}{\Delta t_1 + \Delta t_2 + 1} \right]$$  \hspace{1cm} (5)
The wall thickness $L_1$ and $L_2$ can then be solved as:

$$L_1 = \frac{c\Delta t_1}{2} \quad \text{and} \quad L_2 = \frac{c\Delta t_2}{2}$$  \hspace{1cm} (6)

2.2. Determine the phase velocity and wall thickness using the frequency-domain analysis

If we use $A(f)e^{-j\phi(f)}$ to represent the Fourier transform of a pulse $P(t)$, use $V_p(f)$ to represent the phase velocity of the tube material, and assume the dispersion of water is negligible [3], the following set of equations can be derived [4]:

$$\frac{1}{V_p(f)} = \frac{\Delta \theta_1(f)}{4\pi f L_1}$$  \hspace{1cm} (7)

$$\frac{1}{V_p(f)} = \frac{\Delta \theta_2(f)}{4\pi f L_2}$$  \hspace{1cm} (8)

$$\frac{1}{V_p(f)} = -\frac{\Delta \theta_1(f)}{2\pi f(L_1 + L_2)} + \frac{1}{c_w}$$  \hspace{1cm} (9)

where $\Delta \theta_1(f)$ and $\Delta \theta_2(f)$ are, respectively, the differences between the phase spectra of the two echoes associated with $L_1$ and the two echoes associated with $L_2$, after the extra 180° phase shift of the second echo is removed, and $\Delta \theta_1(f)$ is the difference between the phase spectra of the two transmitted pulses $T_n(t)$ and $T_c(t)$. From these three equations, we can solve for $V_p(f)$, $L_1$ and $L_2$:

$$V_p(f) = c_w \left[ \frac{2\Delta \theta_1(f)}{\Delta \theta_1(f) + \Delta \theta_2(f)} + 1 \right]$$  \hspace{1cm} (10)

$$L_1 = \frac{\Delta \theta_1(f)}{4\pi f V_p(f)} \quad \text{and} \quad L_2 = \frac{\Delta \theta_2(f)}{4\pi f V_p(f)}$$  \hspace{1cm} (11)

Eq. (10) gives the phase velocity as a function of frequency. From $V_p(f)$, the group velocity $V_g(f)$, can be calculated [5]:

$$V_g(f) = \frac{V_p(f)}{1 - \int \frac{dV_p(f)}{df}}$$  \hspace{1cm} (12)

If the medium is dispersionless ($V_p$ is a constant), then the phase velocity $V_p$, group velocity $V_g$, and the “speed of sound” $c$ defined in the time-domain analysis are all the same. On the other hand, if the dispersion is not negligible, $V_g$ can be significantly different from $V_p$, and $c$ takes a value of $V_g$ at some frequency near the center frequency of the transducer.

3. Experiments and results

Two experiments are performed. In the first experiment, a polyethylene tube sample is used which has a nominal outer diameter of 9.53 mm (3/8 in.) and a nominal inner diameter of 6.35 mm (1/4 in.). Both time-domain analysis and frequency-domain analysis are performed. In the second experiment, four tube samples made of different materials are used. The outer diameters of these tubes range from 4.5 to 21.7 mm. Only time-domain analysis is performed.

3.1. Experiment and results with the polyethylene tube

A pair of identical transducers (Panametrics V309, 5.0 MHz, 13-mm aperture, point focus, 25.4-mm focal distance) are used as the $T_1$ and $T_2$ shown in Fig. 1. The front surface of the tube is placed at near the focal distance of the transducer. The pulser/receiver used in the experiment is Panametrics 5052PR. The amplified pulse is A/D converted by a SONY/TEK 390AD programmable digitizer which has an adjustable digital delay for triggering the sampling window. Each sampling window contains 256 samples and the sampling frequency is 60 MHz. The signals are averaged 30 times and then transferred to a PC and processed using a software package MATLAB (Math Works, MA). The water temperature is 21 °C which gives $c_w = 1485$ m/s [6].

Fig. 2 shows the waveforms of the two reflected signals $R_1(t)$, $R_2(t)$ and the two transmitted signals $T_n(t)$, $T_w(t)$. For the time-domain analysis, the time delays $\Delta t_1$ and $\Delta t_2$ are measured from the zero-crossing right before the largest positive peak of the first echo to the zero-crossing right before the largest negative peak of the second echo to take account for the extra 180° phase shift between the two echoes. The time delay $\Delta t_3$ is measured from the zero-crossing right before the largest positive peak of $T_n(t)$ to the zero-crossing right before the largest positive peak of $T_w(t)$. From the measured time delays, $c$, $L_1$ and $L_2$ are calculated using Eqs. (5) and (6). The results are summarized in Table 1.

To perform the frequency-domain analysis, the two echoes contained in $R_i(t)$ are first separated while keeping their temporal relation. The phase difference $\Delta \theta_1$ between the two echoes is then determined using a procedure described in Ref. [4]. The phase difference $\Delta \theta_2$ between the two echoes contained in $R_3(t)$ is determined in the same way. Finally, the phase difference $\Delta \theta_3$ between $T_n(t)$ and $T_c(t)$ is also determined. The phase velocity $V_p(f)$ is then calculated using Eq. (10). Fig. 3 plots the measured $V_g(f)$ (solid line) in the frequency range of 2-8 MHz. A linear regression line (dashed line) having a slope of 2.156 (m/s)/MHz is also shown in Fig. 3. Using this value of the slope and a phase velocity of 2078 m/s at 5 MHz, the group velocity at 5 MHz is calculated using Eq. (12) as 2089 m/s which is 1% smaller than the value of $c$ (2110 m/s) determined using the time-domain analysis.
Table 1
Results from the polyethylene tube using the time-domain analysis and frequency-domain analysis

<table>
<thead>
<tr>
<th>Time-domain analysis</th>
<th>Frequency-domain analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta t_1 = 1.418$ (µs)</td>
<td>$V_p = 2067 + 2.156f$ (m/s)</td>
</tr>
<tr>
<td>$\Delta t_2 = 1.415$ (µs)</td>
<td>([f = 2-8 \text{ MHz}])</td>
</tr>
<tr>
<td>$\Delta t_3 = 0.597$ (µs)</td>
<td>$V_p = 2089$ (m/s) ([f = 5 \text{ MHz}])</td>
</tr>
<tr>
<td>$c = 2.11 \times 10^3$ (m/s)</td>
<td>$L_1 = 1.483 \pm 0.005$ (mm) ([f = 3-7 \text{ MHz}])</td>
</tr>
<tr>
<td>$L_1 = 1.50$ (mm)</td>
<td>$L_2 = 1.484 \pm 0.004$ (mm) ([f = 3-7 \text{ MHz}])</td>
</tr>
</tbody>
</table>

For the time-domain analysis, $\Delta t_1$, $\Delta t_2$, and $\Delta t_3$ are defined in Fig. 2, and $c$ is the speed of sound. For the frequency-domain analysis, the linear equation of the phase velocity $V_p$ represents the regression line shown in Fig. 3. $V_p$ is the group velocity at 5 MHz.

![Graph showing phase velocity vs. frequency](image)

Fig. 3. Solid line: phase velocity of the polyethylene tube calculated using Eq. (10) and the signals shown in Fig. 2. Dashed line: least-squares line fitted to the phase velocity data.

When Eq. (11) is used to calculate the wall thickness, one actually obtains $L_1$ and $L_2$ as functions of frequency, but the changes of $L_1$ and $L_2$ (noise) within the useful frequency range of the measurement system should be very small. The means and standard deviations of $L_1$ and $L_2$ in the frequency range of 3–7 MHz are listed in Table 1. The difference between the thickness determined by the time-domain analysis and the frequency-domain analysis is about 1%.

3.2. Experiments and results with four tube samples

Four tube samples are provided by a company named Beta LaserMike (Dayton, OH 45424) to test the new methods. The outer diameters of the four samples are 0.45, 8.62, 11.26 and 21.72 mm, respectively. The materials of these tubes are not exactly known. The two transducers (Panametrics M3100, 10.0 MHz, 6.35-mm aperture, line focus, 19-mm focal distance) used in this set of experiment are also provided by the company. The rest of the instruments are the same as the ones used in the experiment with the polyethylene tube.

For each sample, after recording the four signals $R_1(t)$, $R_2(t)$, $T_1(t)$ and $T_2(t)$, the measurement site are carefully marked and the tube is then cut in two pieces at the marked location. The same thickness $L_1$ and $L_2$ are then measured independently by an engineer at Beta LaserMike using a laser device (BenchMile, Model 283-20) manufactured by the company. For the ultrasound measurement, only the time-domain analysis is performed.

Table 2 lists the sound velocity and wall thickness of each tube measured by the ultrasound method, as well as the wall thickness measured by the laser device. As one can see, the sound velocity of the four samples ranges from $1.99 \times 10^3$ to $2.33 \times 10^3$ m/s. The thickness measured by the ultrasound method tends to be smaller.

![Graph showing sound velocity vs. frequency](image)

Table 2
Comparison of the thickness measured using the ultrasound method ($L_4$) and the laser device ($L_3$) for the four tube samples

<table>
<thead>
<tr>
<th></th>
<th>Ultrasound method</th>
<th>Laser measurement</th>
<th>Percent difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sample #1</td>
<td>Sample #2</td>
<td>Sample #3</td>
</tr>
<tr>
<td>$c$ (m/s)</td>
<td>$2.08 \times 10^3$</td>
<td>$2.11 \times 10^3$</td>
<td>$1.99 \times 10^3$</td>
</tr>
<tr>
<td>$L_1$ (mm)</td>
<td>0.495 (mm)</td>
<td>1.02 (mm)</td>
<td>1.38 (mm)</td>
</tr>
<tr>
<td>$L_2$ (mm)</td>
<td>0.554 (mm)</td>
<td>1.00 (mm)</td>
<td>1.38 (mm)</td>
</tr>
<tr>
<td>$L_3$ (mm)</td>
<td>0.522 (mm)</td>
<td>1.034 (mm)</td>
<td>1.452 (mm)</td>
</tr>
<tr>
<td>$L_4$ (mm)</td>
<td>0.545 (mm)</td>
<td>1.064 (mm)</td>
<td>1.401 (mm)</td>
</tr>
<tr>
<td>Percent difference</td>
<td>5.31</td>
<td>1.36</td>
<td>5.08</td>
</tr>
</tbody>
</table>

The percent difference is defined as $100 \times (L_4 - L_3)/(L_4 + L_3)$. The speed of sound ($c$) of each sample measured by the ultrasound method using the time-domain analysis is also listed.
than that measured by the laser device, but the absolute difference is less than 5% in most cases.

4. Summary and discussion

A method for simultaneously measuring sound velocity and the wall thickness of tubes is presented. The method requires that both the outside and inside of the tube are filled with water so that two reflected and two transmitted signals can be obtained. The main advantage of this new method is its ability to determine the wall thickness based on the actually measured sound velocity. The method is particular useful for real-time measurement of wall thickness when the sound velocity of the specimen may change with time, e.g. due to the change of temperature.

In theory, the frequency-domain analysis can determine the phase velocity and group velocity at different frequencies while the time-domain method can only provide a single value of the speed of sound with a certain degree of uncertainty. This uncertainty comes from at least two sources. First of all, if both phase velocity and group velocity change with frequency, the speed of sound determined by the time-domain method will depend on (the frequency characteristics of) the transducers used in the measurement. Second, there is no precise definition of the “time delay” between two pulses (i.e. $\Delta t_1$, $\Delta t_2$ and $\Delta t_3$ in Fig. 2). For example, instead of defining $\Delta t_3$ in Fig. 2 based on the zero-crossings, one may define $\Delta t_3$ based on the peaks of $T_s$ and $T_w$, and the “time delay” determined by the two choices of the “markers” (zero-crossing vs. peak) may not be exactly the same. In a recent paper, Keith Wear [7] lists eight different choices for designating markers for the calculation of the time delay between two pulses, resulting in various measures of “time delay”. It should be pointed out that the degree of uncertainty in determining the speed of sound using the time-domain method depends on the total amount of dispersion in the frequency range of the measurement system. In the experiment with the polyethylene tube, the change in $V_p$ is less than 1% over the frequency range of 2–8 MHz. As a result, the differences between the velocity and thickness measurements using the frequency-domain analysis and the time-domain analysis are found to be small (1%).

When the curve of $V_p$ in Fig. 3 is compared with the earlier results (smoother curves shown in Figs. 5–7 in Ref. [4]) obtained from flat samples, one notices that the phase velocity obtained from the tube sample is significantly noisier than that obtained from flat samples. A possible explanation for this is the phase distortion produced by the curved tube walls. This distortion should decrease when the ratio of the beam width to the diameter of the tube decreases. To minimize the phase distortion, focused transducers should be used and the front surface of the tube should be placed at the focal distance of the transducer. In the experiment with the polyethylene tube, the $-6$ dB beam width is about 0.6 mm which is 10 times smaller than the inner diameter of the tube. Based on the results of this study, a general guideline is that the beam width at the measurement site should be at least 3–5 times smaller than the inner diameter of the tube.

Acknowledgements

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References