Learning Objectives:

Understand the basic principles of passive filters.

Laboratory Equipment:

Agilent Oscilloscope Model 54622A
Agilent Function Generator Model HP33120A

Supplies and Components:

Breadboard
4.7 KΩ Resistor
0.047 μF Capacitor

Pre-Lab Questions

1. Define passive filters. List several different kinds of passive filters. What is a notch filter; what is a band pass filter? Where and why are filters used?
2. Why are capacitors preferred over inductors in filter design?
3. Explain how to build a passive filter using only resistive elements.

Post-Lab Questions

1. What is meant by the term *passive filter*?
2. List at least six practical applications of passive filters
3. Discuss the behavior of the RCL filter below at zero frequency.
4. Discuss the behavior of the RCL filter below at infinite frequency.
Laboratory Two - Passive Filters

Background

Filters are frequency selective circuits which utilize combinations of different circuit elements (such as capacitor C and inductor L) whose impedance varies with frequency in such a way that signals with certain frequencies will be passed or amplified while signals with other frequencies will be blocked or suppressed. The frequency range of the signals that can pass the filter is called the pass-band and the frequency range of the signals that are blocked by the filter is called the stop-band.

A) A simple R-C low pass filter

![Schematic diagram of a low pass filter](image)

Figure 1. A schematic diagram of a low pass filter

Figure 1 shows a simple low pass filter made of a resistor R and a capacitor C. The transfer function and its magnitude are (where $\omega = 2\pi f$):

$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{1}{1 + j\omega RC} \quad (1)$$

and

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (2\pi RC)^2}} \quad (2)$$

The physical meaning of $|H(j\omega)|$ can be understood by connecting a sine wave of frequency f and amplitude $|V_{in}(f)|$ to the input of the circuit, and measuring the amplitude of the output sine wave $|V_{out}(f)|$. The magnitude of the transfer function is just the ratio of the two amplitudes:

$$|H| = \frac{|V_{out}(f)|}{|V_{in}(f)|} \quad (3)$$
The following is a plot of $|H|$ as a function of frequency $f$:

Figure 2. The magnitude of the transfer function of a low pass filter

The corner frequency $f_0$ is defined as a frequency at which the magnitude of the transfer function becomes 0.707, as shown in Figure 2. The corner frequency of the above R-C low pass filter is solely determined by the values of R and C:

$$f_0 = \frac{1}{2\pi RC}$$  \hspace{1cm} (4)

B) A simple R-C high pass filter

Figure 3. A schematic diagram of a high pass filter

Figure 3 shows a simple high pass filter made of a resistor R and a capacitor C. The transfer function of the circuit and its magnitude are:

$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{j\omega RC}{1 + j\omega RC}$$  \hspace{1cm} (5)

and

$$|H(j\omega)| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}} = \frac{2\pi RC}{\sqrt{1 + (2\pi RC)^2}}$$  \hspace{1cm} (6)
The following is a plot for $|H|$ of a high pass filter:

![Graph of $|H|$ for a high pass filter](image)

Figure 4. The magnitude of the transfer function of a high pass filter

Again, the corner frequency is $f_0 = 1/(2\pi RC)$ at which the magnitude of the transfer function becomes 0.707.

**Procedures**

1) Experimenting on the low pass filter

![Circuit diagram for low pass filter](image)

Figure 5. The circuit for experimenting on the low pass filter

a) Build the circuit in Figure 5. The input signal is a sine wave generated by the Function Generator, and the output is connected to CH1 of the oscilloscope. The peak-to-peak amplitude of the input signal is 2V and its frequency changes as described below.

b) Calculate the expected corner frequency of the filter according to Eq. (4).

c) Set the frequency of the input signal to the following values (unit is Hz):

- 10, 50, 100, 200, 400, 600, 650, 700, 750, 800, 850, 900, 1K, 2K, 3K, 5K, 10K

At each frequency, measure the peak-to-peak amplitude of the output signal.

d) While maintaining the p-p amplitude of the input signal to be 2 V, adjust the frequency of the input signal until the p-p amplitude of the output signal becomes $2 \times 0.707 \, \text{V} = 1.414 \, \text{V}$. Write down this frequency which is the measured corner frequency. How close are the measured corner frequency and the one determined in b)?
e) Based on the measurements in d), calculate the magnitude of the transfer function at each frequency according to Eq. (3), where \(|V_{\text{in}}(f)| = 2\, \text{V}.

f) Graph the magnitude of the transfer function as a function of the frequency. Mark the corner frequency on your plot.

2) Experimenting on the high pass filter

![Circuit Diagram](image)

Figure 6. The circuit for experimenting on the high pass filter

a) Build the circuit in Figure 6. The input signal is a sine wave generated by the Function Generator, and the output is connected to CH1 of the oscilloscope. The peak-to-peak amplitude of the input signal is 2V and its frequency changes as described below.

b) Calculate the expected corner frequency of the filter according to Eq. (4).

c) Set the frequency of the input signal to the following values (unit is Hz): 10, 50, 100, 200, 400, 600, 650, 700, 750, 800, 850, 900, 1K, 2K, 3K, 5K, 10K

At each frequency, measure the peak-to-peak amplitude of the output signal.

d) While maintaining the p-p amplitude of the input signal to be 2 V, adjust the frequency of the input signal until the p-p amplitude of the output signal becomes \(2 \times 0.707 \, \text{V} = 1.414 \, \text{V}\). Write down this frequency which is the measured corner frequency. How close are the measured corner frequency and the one determined in b) ?

e) Based on the measurements in d), calculate the magnitude of the transfer function at each frequency according to Eq. (3), where \(|V_{\text{in}}(f)| = 2\, \text{V}.

f) Graph the magnitude of the transfer function as a function of the frequency. Mark the corner frequency on your plot.
Grading Rubric: Passive Measurements (Lab 2)

| Points |
| Cover Page - Be sure to use the sample provided! |

I) Circuit Diagrams

| Points |
| 1) Experiment 1 - Low Pass Filter |
| Be sure to label the resistor and capacitor with their measured values, not their labeled values. Also, the AC voltage source (the sine wave from the function generator) should be properly labeled with its peak-to-peak magnitude (2 V<sub>pp</sub>). Note: Since the frequency of the sine was variable, you'll need to mention this either in the circuit diagram itself or in a figure caption. Finally, show where the output voltage was measured. |
| 2) Experiment 2 - High Pass Filter |
| Be sure to label the resistor and capacitor with their measured values, not their labeled values. Also, the AC voltage source (the sine wave from the function generator) should be properly labeled with its peak-to-peak magnitude (2 V<sub>pp</sub>). Note: Since the frequency of the sine was variable, you'll need to mention this either in the circuit diagram itself or in a figure caption. Finally, show where the output voltage was measured. |

II) Data and Results

| Points |
| 1) Experiment 1 - Low Pass Filter |
| a. Measured Values for R and C |
| b. Equation and Calculation for Time Constant, τ |
| c. Equation and Calculation for Corner (Cutoff) Frequency, ω<sub>o</sub> [rad/s] |
| d. Equation and Calculation for Corner (Cutoff) Frequency, f<sub>o</sub> [Hz] |
| e. Data Table Showing: Input Frequency, (f), Input Voltage Magnitude (|V<sub>in(t)</sub>|), Output Voltage Magnitude (|V<sub>out(t)</sub>|), and the calculated Transfer Function Magnitude (|H(jω)|). |
| Make sure the data in your table are labeled with the appropriate unites in the column headers (ex. [V], [Hz], etc.) |
| f. Graph of the Transfer Function Magnitude (|H(jω)|) vs. Frequency (f). |
| Be sure to include a proper title, x-axis label and y-axis label on your graph! Also, be sure to include proper units in the axes labels (where appropriate)! Finally, mark the corner frequency (f<sub>o</sub>) on your graph. |

| Points |
| 2) Experiment 2 - High Pass Filter |
| a. Data Table Showing: Input Frequency, (f), Input Voltage Magnitude (|V<sub>in(t)</sub>|), Output Voltage Magnitude (|V<sub>out(t)</sub>|), and the calculated Transfer Function Magnitude (|H(jω)|). |
| Make sure the data in your table are labeled with the appropriate unites in the column headers (ex. [V], [Hz], etc.) |
| b. Graph of the Transfer Function Magnitude (|H(jω)|) vs. Frequency (f). |
| Be sure to include a proper title, x-axis label and y-axis label on your graph! Also, be sure to include proper units in the axes labels (where appropriate)! Finally, mark the corner frequency (f<sub>o</sub>) on your graph. |
### III) Discussion

#### 1) Curve Fitting - Low Pass Filter

a. **Equation (2) on p. 11 of the lab manual** is the closed-form representation of the transfer function magnitude as a function of frequency (and your measured values for R and C). In order to see how close your experimental results were to what this equation predicts, **plot $|H(jw)|$ using the closed-form equation on a graph with your experimental data (use the same frequency points you used in the lab experiment to calculate $|H(jw)|$**. Comment on how closely your experimental data fits the curve from the closed-form expression. What possible sources of variation (error) could have been present in the experiment to cause the experimental data to vary slightly from the predicted results?  

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#### 2) Curve Fitting - High Pass Filter

a. **Equation (6) on p. 12 of the lab manual** is the closed-form representation of the transfer function magnitude as a function of frequency (and your measured values for R and C). In order to see how close your experimental results were to what this equation predicts, **plot $|H(jw)|$ using the closed-form equation on a graph with your experimental data (use the same frequency points you used in the lab experiment to calculate $|H(jw)|$**. Comment on how closely your experimental data fits the curve from the closed-form expression. What possible sources of variation (error) could have been present in the experiment to cause the experimental data to vary slightly from the predicted results?

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### IV) Post-Lab Questions

### V) References