Problem 1 (10 points): A cast iron pipe ($k = 52 \text{ W/m-K}$) is 8-m long, with an outside diameter of $D_o = 10 \text{ cm}$, and an inside diameter of $D_i = 9.2 \text{ cm}$. Steam flows inside the pipe at an average temperature of $200^\circ\text{C}$ with a heat transfer coefficient of $180 \text{ W/m}^2\cdot\text{K}$. The outer surface of the pipe is exposed to ambient air at $12^\circ\text{C}$, with a heat transfer coefficient of $25 \text{ W/m}^2\cdot\text{K}$. Determine the outside surface temperature of the pipe and the heat transfer rate through the pipe wall. Neglect the effects of thermal radiation. (Answers: $T_o = 175.1^\circ\text{C}$, $\dot{Q} = 1.024 \times 10^4 \text{ W}$)

Problem 2 (10 points): Two flanges are to be installed onto the pipe described in Problem 1 as shown below. Each flange is 1 cm thick, and is made of the same material as the pipe. Using the results found in Problem 1, determine the fin efficiency of the flanges and the rate of heat transfer from the flanges.
L = 8 m, K = 52 W/m·K, D₀ = 0.1 m, Dᵢ = 0.092 m,
Tᵢ₀ = 200°C, hᵢ = 180 W/m²·K, Tᵢ₀,₀ = 12°C, h₀ = 25 W/m²·K

Find Tₒ, ᵇ

THERMAL RESISTANCE NETWORK:

\[ Q = \frac{Tᵢ₀ - Tᵢ₀,₀}{R_{\text{TOTAL}}} \]

R_{\text{TOTAL}} = R₁ + R₂ + R₃

R₁ = \frac{1}{hᵢ Aᵢ}

R₂ = \frac{\ln \left( \frac{R₀}{Rᵢ} \right)}{2\pi L K}

R₃ = \frac{1}{h₀ A₀}

Aᵢ = \pi Dᵢ L = \pi (0.092 m)(8 m) = 2.312 m²
\( A_0 = \pi D_0 L = \pi (0.11 \text{ m}) (8 \text{ m}) = 2.31 \text{ m}^2 \)

\[
R_1 = \frac{1}{(180 \frac{\text{ W}}{\text{ m}^2 \cdot \text{ K}}) (2.31 \text{ m}^2)} = 0.002403 \frac{\text{ K}}{\text{ W}}
\]

\[
R_2 = \frac{\ln(10/9.2)}{2\pi (8 \text{ m}) (32 \frac{\text{ W}}{\text{ m} \cdot \text{ K}})} = 3.19 \times 10^{-5} \frac{\text{ K}}{\text{ W}}
\]

\[
R_3 = \frac{1}{(25 \frac{\text{ W}}{\text{ m}^2 \cdot \text{ K}}) (2.31 \text{ m}^2)} = 0.01592 \frac{\text{ K}}{\text{ W}}
\]

\[
R_{\text{TOTAL}} = 0.002403 + 3.19 \times 10^{-5} + 0.01592 = 0.01835 \frac{\text{ K}}{\text{ W}}
\]

\[
\dot{Q} = \frac{(200 - 12^\circ \text{ C})}{(0.01835 \frac{\text{ K}}{\text{ W}})} = 1.024 \times 10^4 \text{ W} \quad \text{HEAT TRANSFER RATE}
\]

\[
\dot{Q} = \frac{T_0 - T_{w,0}}{R_3}
\]

\[
T_0 = T_{w,0} + \dot{Q} \cdot R_3
\]

\[
T_0 = (12^\circ \text{ C}) + (1.024 \times 10^4 \text{ W})(0.01592 \frac{\text{ K}}{\text{ W}}) = 175.1^\circ \text{ C}
\]
PROB. 2

\( D_0 = 10 \text{ cm}, \ D_0, \text{fin} = 18 \text{ cm}, \ \ell = 2 \text{ cm}, \ K = 52 \ \frac{\text{W}}{\text{m} \cdot \text{K}} \)

\( h = 25 \ \frac{\text{W}}{\text{m} \cdot \text{K}}, \ \theta_0 = 12^\circ \text{C}, \ \text{FIND} \ \gamma_{\text{fin}}, \ \dot{Q}_{\text{fin}} \)

\text{FIND} \ \gamma_{\text{fin}} \ : \ \text{FIGURE 3-44, P.172}

\[ \gamma_{2\text{c}} = \gamma_{2} + \ell/2 = (0.09 \text{ m}) + (0.02 \text{ m})/2 = 0.1 \text{ m} \]

\[ L_2 = L + \ell/2 = (0.09 - 0.05 \text{ m}) + (0.02 \text{ m})/2 = 0.05 \text{ m} \]

\[ A_p = L_2 \ell = (0.05 \text{ m}) \times (0.02 \text{ m}) = 0.001 \text{ m}^2 \]

\[ \gamma_{2\text{c}}/\gamma_1 = (0.1 \text{ m})/(0.05 \text{ m}) = 2.0 \]

\[ s = L^{\frac{1}{2}} \cdot \left( \frac{h}{K A_p} \right)^{\frac{1}{2}} \]

\[ s = (0.05 \text{ m})^{\frac{1}{2}} \left[ \frac{(25 \ \frac{\text{W}}{\text{m} \cdot \text{K}})}{(52 \ \frac{\text{W}}{\text{m} \cdot \text{K}} \times 0.001 \text{ m}^2)} \right]^{\frac{1}{2}} = 0.2451 \]

\text{FROM FIGURE 3-44,} \ \sqrt{2} = 0.95 \]

\text{FROM TABLE 3-3,}

\[ A_{\text{fin}} = 2\pi \left( \frac{\ell}{2} \right)^2 \]

\[ A_{\text{fin}} = 2\pi \left[ (0.1 \text{ m})^2 - (0.05 \text{ m})^2 \right] = 0.04712 \text{ m}^2 \]

\[ \dot{Q}_{\text{fin}} = 2_{\text{fin}} \cdot \dot{Q}_{\text{fin}, \text{max}} = 2_{\text{fin}} \cdot \ell A_{\text{fin}} (T_0 - T_\infty) \]

\[ \dot{Q}_{\text{fin}} = (0.95) \times 25 \ \frac{\text{W}}{\text{m} \cdot \text{K}} \times 0.04712 \text{ m}^2 \times 175.1 - 12 \text{C} = 182.5 \text{ W} \]