Problem 1 (10 points): Consider an infinitely long pipe in cross flow, where the oncoming freestream velocity and temperature are \( V_\infty \) and \( T_\infty \), respectively. Heat is generated uniformly within the wall of the pipe at a rate of \( \dot{e}_{\text{gen}} \) by applying an electrical current through the wall of the pipe. Assume that the properties of the pipe are constant and the system is at steady state. The interior surface of the pipe is insulated at \( r_i \). Starting with the general conservation of energy equation given below, list the assumptions used to cancel the appropriate terms to result in a reduced form of the energy equation that could be solved to determine the temperature distribution within the pipe wall. Provide a list of boundary conditions needed to solve the energy equation for the temperature distribution within the pipe wall. Do not solve for the temperature distribution.

\[
\begin{align*}
\text{CONSERVATION OF ENERGY EQUATION:} \\
\text{Rectangular Coordinate System:} \\
\frac{\partial}{\partial x}(k \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k \frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(k \frac{\partial T}{\partial z}) + \dot{\epsilon}_{\text{gen}} &= \rho c_p \frac{\partial T}{\partial t} \\
\text{Cylindrical Coordinate System:} \\
\frac{1}{r} \frac{\partial}{\partial r}(kr \frac{\partial T}{\partial r}) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}(k \frac{\partial T}{\partial \phi}) + \frac{\partial}{\partial z}(k \frac{\partial T}{\partial z}) + \dot{\epsilon}_{\text{gen}} &= \rho c_p \frac{\partial T}{\partial t} \\
\text{Spherical Coordinate System:} \\
\frac{1}{r^2} \frac{\partial}{\partial r}(kr^2 \frac{\partial T}{\partial r}) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \phi}(k \frac{\partial T}{\partial \phi}) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta}(k \sin \theta \frac{\partial T}{\partial \theta}) + \dot{\epsilon}_{\text{gen}} &= \rho c_p \frac{\partial T}{\partial t}
\end{align*}
\]
Problem 2 (10 points): A 5-mm-diameter smooth sphere of copper is cooled from an average temperature of 200°C to 50°C by dropping it into a tall column filled with air at 25°C and 101.3 kPa. It can be assumed that the terminal velocity $V_t$ of the sphere is reached quickly such that the entire fall of the sphere occurs at this constant velocity, which is calculated from:

$$V_t = \left[\frac{2(\rho_s - \rho_{air})V_sg}{C_D\rho_{air}A_p}\right]^{1/2}$$

where $V_s =$ volume of the sphere, $g =$ 9.81 m/s$^2$, $\rho_s =$ density of the sphere, $\rho_{air} =$ density of air, $C_D =$ 0.4 = drag coefficient and $A_p =$ projected area of the sphere.

a) Calculate the heat transfer coefficient for the sphere at its mean temperature.

b) Calculate the time required for the sphere to reach the final temperature.

Problem 3 (10 points): A round opaque flat disk of diameter $D =$ 3 m is well insulated on the edges and the lower surface. The disk experiences uniform irradiation at a rate of 2500 W on its top surface. The plate absorbs 2000 W of the irradiation, and the surface is losing heat at a rate of 250 W by convection. If the plate maintains a uniform temperature of 57°C, determine the absorptivity, reflectivity, and emissivity of the plate. Kirchhoff’s law does not apply in this case.

EXTRA CREDIT PROBLEM (10 points): A furnace cavity, which is in the form of a cylinder of 75-mm diameter and 150-mm length, is open at one end to the surroundings that are at 27°C. The sides and bottom may be approximated as blackbodies, are heated electrically, are well insulated, and are maintained at temperatures of $T_1 =$ 1350 and $T_2 =$ 1650°C, respectively. How much power is required to maintain the furnace conditions?
**PROB. 1**

**ENERGY:**

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{e}_{\text{gen}} = \dot{q} \cdot \frac{\partial T}{\partial t} + \dot{e}_{\text{gen}}
\]

**ASSUMPTIONS:**

- **INFINITELY LONG PIPE:** \( \frac{\partial}{\partial z} = 0 \)
- **STEADY STATE:** \( \frac{\partial}{\partial t} = 0 \)

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \dot{e}_{\text{gen}} = 0
\]

**CONSTANT PROPERTIES:**

\[
\frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( \frac{\partial T}{\partial \phi} \right) + \frac{e_{\text{gen}}}{k} = 0
\]

**BOUNDARY CONDITIONS:**

**INSULATED INTERIOR:**

\[
\frac{\partial T}{\partial r} \bigg|_{r=r_2} = 0
\]
CONVECTION + RADIATION EXTERIOR:

\[ h(T_s - T_\infty) + \varepsilon \sigma (T_s^4 - T_{\text{sur}}^4) = -K \frac{\partial T}{\partial r} \bigg|_{r = r_0} \]

SYMMETRY CONDITIONS:

\[ \frac{\partial T}{\partial \phi} \bigg|_{\phi = 0} = 0 \]

\[ \frac{\partial T}{\partial \phi} \bigg|_{\phi = \pi} = 0 \quad \text{or} \quad T(\phi = \pi, r = r_0) = \text{constant} \]
PROB. 2

D = 0.005 m, \( T_i = 200^\circ C, T_f = 50^\circ C, T_o = 25^\circ C, \)
\( p = 101 \text{ kPa}, C_D = 0.4 \)

a) Find terminal velocity, copper properties:

\( \gamma = 8933 \text{ kg m}^{-3}, C_P = 385 \frac{J}{\text{kg} \cdot \text{K}} \quad K = 401 \frac{W}{\text{m} \cdot \text{K}} \)

Air properties: \( T = 25^\circ C \)

\( \gamma = 1.184 \text{ kg m}^{-3}, C_P = 1007 \frac{J}{\text{kg} \cdot \text{K}} \quad K = 0.02551 \frac{W}{\text{m} \cdot \text{K}} \)
\( \nu = 1.849 \times 10^{-5} \frac{\text{m}^2}{\text{s}}, \quad \nu_l = 1.362 \times 10^{-5} \frac{\text{m}^2}{\text{s}}, \quad Pr = 0.7296 \)

\( V_3 = \frac{4}{3} \pi r^3, \quad A_P = \pi r^2 \)

\( \frac{V_5}{A_P} = \frac{\frac{4}{3} \pi r^3}{\pi r^2} = \frac{4}{3} r = \frac{4}{3} (0.0025 \text{ m}) = 0.003 \text{ m} \)

\( V_t = \left[ \frac{2 \left( 8933 - 1.184 \text{ kg m}^{-3} \right) (0.003 \text{ m}) (9.81 \text{ m/s}^2)}{0.4 \times 1184 \text{ kg m}^{-3}} \right]^{1/2} \)

\( V_t = 35.12 \text{ m/s} \)

b) Find h

\( Nu_{4TH} = \frac{hD}{K} = 2 + \left[ 0.4 Re^{1/2} + 0.06 Re^{2/3} Pr^{0.14} \left( \frac{0.05}{m} \right)^{0.4} \right] \)
PROB. 2 CONT.

\[ Re = \frac{VD}{\nu} = \left( \frac{35.12}{3 \times 0.005} \right) \left( \frac{m}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} \right) = 11,242 \]

\[ M^5 = \frac{1}{2} \left( \mu_{200} + \mu_{50^\circ} \right) = \frac{1}{2} \left( 2.577 + 1.963 \right) \times 10^{-5} \]

\[ \mu_s = 2.270 \times 10^{-5} \frac{\text{kg}}{\text{m} \cdot \text{s}} \]

\[ Nu_{ph} = 2 + \left[ 0.4 (11,242)^{1/4} + 0.06 (11,242)^{2/3} \right] (0.7296)^{0.4} \left( \frac{1.849}{2.270} \right)^{1/4} \]

\[ Nu_{ph} = \frac{hD}{K} = 62.73 \]

\[ h = \frac{(62.73 \times 0.02531 \frac{\text{W}}{\text{m} \cdot \text{K}})}{(0.005 \text{ m})} = 320.1 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \]

C) FIND \( t \)

LUMPED CAPACITANCE?

\[ Bi = \frac{hL_c}{K} \]

FOR A SPHERE, \( L_c = \frac{V_0}{3} \)
PROB. 2 CONT.

\[ Bi = \left( \frac{320.1 \frac{W}{m^2 \cdot K} \times \frac{1}{3} \times (0.005 \text{m})}{401 \frac{W}{m \cdot K}} \right) = 6.65 \times 10^{-4} \leq 0.1 \]

**VALID**

\[ \frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \]

\[ t = -\frac{1}{b} \cdot \ln \left( \frac{T_f - T_\infty}{T_i - T_\infty} \right) \]

\[ b = \frac{h \cdot A_s}{gV \cdot C_p} = \frac{h}{gC_p \cdot L_c} = \frac{3h}{gC_p \cdot L_0} \]

\[ b = \frac{3 \left( \frac{320.1}{m^2 \cdot K} \right)}{(8933 \frac{\text{kg}}{m^3}) \cdot 385 \frac{J}{\text{kg} \cdot \text{K}} \cdot (0.0025 \text{m})} = 0.1117 \frac{1}{3} \]

\[ t = -\frac{1}{(0.1117 \frac{1}{3})} \cdot \ln \left( \frac{50 - 25}{200 - 25} \right) \]

\[ t = 17.42^\circ \]
PROB.3

D = 3 m, \( \dot{Q}_i = 2500 \text{ W} \), \( \dot{Q}_a = 2000 \text{ W} \), \( \dot{Q}_c = 250 \text{ W} \)

\( T_s = 857^\circ C \), FIND \( \alpha, \beta, \epsilon \)

INCIDENT RADIATION:

\[ G = \frac{\dot{Q}_i}{A} \quad A = \frac{\pi D^2}{4} = \frac{\pi (3 \text{ m})^2}{4} = 7.068 \text{ m}^2 \]

\[ G = \frac{(2500 \text{ W})}{(7.068 \text{ m}^2)} = 353.7 \frac{\text{W}}{\text{m}^2} \]

ABSORBED IRRADIATION:

\[ G_{\text{abs}} = \frac{\dot{Q}_a}{A} = \frac{(2000 \text{ W})}{(7.068 \text{ m}^2)} = 283.0 \frac{\text{W}}{\text{m}^2} \]

ABSORPTIVITY:

\[ \alpha = \frac{G_{\text{abs}}}{G} = \frac{283}{353.7} = 0.8 \]

REFLECTIVITY:

\[ \alpha + \beta + \epsilon = 1 \]

\( \epsilon = 0 \) FOR OPAQUE SURFACES

\[ \beta = 1 - \alpha = 1 - 0.8 = 0.2 \]
PROB. 3 CONT.

CONVECTIVE LOSS:
\[ \dot{q}_c = \frac{\dot{Q}_c}{A} = \frac{250 \text{ W}}{7.068 \text{ m}^2} = 35.37 \text{ W/m}^2 \]

ENERGY BALANCE:
\[ \dot{G}_{\text{abs}} \cdot A = \dot{E} \cdot A \]
\[ \dot{G}_{\text{abs}} = \dot{E} + \dot{q}_c \]

EMISSIVE POWER:
\[ E = \dot{G}_{\text{abs}} - \dot{q}_c = (283 - 35.37 \text{ W/m}^2) = 247.6 \text{ W/m}^2 \]

EMISSION:
\[ E = \frac{E}{E_0} = \frac{E}{0.75} = \frac{247.6 \text{ W/m}^2}{(3.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(57 + 273 \text{ K})^4} \]
\[ E = 0.3683 \]

ALTERNATIVE REFLECTIVITY:
\[ G_{\text{ref}} = G - \dot{G}_{\text{abs}} = 353.7 - 283 \text{ W/m}^2 = 70.7 \text{ W/m}^2 \]
\[ g = \frac{G_{\text{ref}}}{G} = \frac{70.7}{353.7} = 0.2 \]
EXTRA CREDIT

*D = 0.075 m, L = 0.15 m, T_{sur} = 27^\circ C = T_3, T_2 = 1330^\circ C

T_1 = 1650^\circ C*, FIND \dot{Q}_{sur} = \dot{Q}_3

\dot{Q} = \dot{Q}_{1 \rightarrow 3} + \dot{Q}_{2 \rightarrow 3}

\dot{Q}_{1 \rightarrow 3} = A_1 \frac{F_{1 \rightarrow 3}}{D} \Delta (T_1^4 - T_3^4)

\dot{Q}_{2 \rightarrow 3} = A_2 \frac{F_{2 \rightarrow 3}}{D} \Delta (T_2^4 - T_3^4)

A_1 = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.075 m)^2 = 4.418 \times 10^{-3} m^2

A_2 = \pi DL = \pi (0.075 m)(0.15 m) = 3.534 \times 10^{-2} m^2

FIND VIEW FACTORS:

F_{1 \rightarrow 3} = 1 + \frac{1 - \sqrt{4R^2 + 1}}{2R^2} \quad \text{TABLE 13-1}

R = \frac{R}{L} = \frac{0.5(0.075 m)}{0.15 m} = 0.25

F_{13} = 1 + \frac{1 - \sqrt{4(0.25)^2 + 1}}{2(0.25)^2} = 0.05573

SUMMATION RULE:

F_{11} + F_{12} + F_{13} = 1
extra credit cont.

\[ F_{12} = 1 - F_{13} = 1 - (0.05573) = 0.9443 \]

Reciprocity rule:

\[ A_1 F_{12} = A_2 F_{21} \]

\[ F_{21} = \frac{A_1}{A_2} \cdot F_{12} = \frac{(4.418 \times 10^{-3})}{(3.534 \times 10^{-2})} \cdot (0.9443) = 0.1180 \]

Symmetry rule:

\[ F_{21} = F_{23} = 0.1180 \]

\[ \dot{\dot{Q}}_{1 \rightarrow 3} = \frac{(4.418 \times 10^{-3} \text{ m}^2)(0.05573)(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4})}{\left[(1650 + 273 \text{ K})^4 - (27 + 273 \text{ K})^4\right]} \]

\[ \dot{\dot{Q}}_{1 \rightarrow 3} = 190.8 \text{ W} \]

\[ \dot{\dot{Q}}_{2 \rightarrow 3} = \frac{(3.534 \times 10^{-2} \text{ m}^2)(0.1180)(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4})}{\left[(1350 + 273 \text{ K})^4 - (27 + 273 \text{ K})^4\right]} \]

\[ \dot{\dot{Q}}_{2 \rightarrow 3} = 1639 \text{ W} \]

\[ \dot{\dot{Q}} = 190.8 + 1639 = 1829 \text{ W} \]