1. (15 points) The motion of the backhoe bucket shown is controlled by the hydraulic cylinders $AD$, $CD$, and $EF$. As a result of an attempt to dislodge a portion of a slab, a 2-kip force $P$ is exerted on the bucket teeth at $J$. Knowing that $\theta = 45^\circ$, determine the force exerted by cylinder $EF$. 

\[ X = 51.4 \]
2. (15 points) A stadium roof truss is loaded as shown. Determine the force in members $AE$, $EF$, and $FJ$. 

\[ \text{Diagram of the truss with forces and dimensions.} \]
3. (20 points) For the stop bracket shown, locate the $x$ coordinate of the center of gravity. The bracket is made entirely of the same material.
4. (50 points total) Part 1: (15 points) Draw the freebody diagrams for the following situations.

Part 2: (35 points) Solve problem (d). Hint: use $M_{EA} = \vec{r}_{EA} \cdot \vec{M}_E = 0$.

Member $ABC$ is supported by a pin and bracket at $B$ and by an inextensible cord attached at $A$ and $C$ and passing over a frictionless pulley at $D$. The tension may be assumed to be the same in portions $AD$ and $CD$ of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at $B$.

The bent rod $ABEF$ is supported by bearings at $C$ and $D$ and by wire $AH$. Knowing that portion $AB$ of the rod is 250 mm long, determine the tension in the wire $AH$ and the reactions at $C$ and $D$. Assume that the bearing at $D$ does not exert any axial thrust.

A 250 x 400-mm plate of mass 12 kg and a 300-mm-diameter pulley are welded to axle $AC$ which is supported by bearings at $A$ and $B$. For $\beta = 30^\circ$, determine the tension in the cable and the reactions at $A$ and $B$. Assume that the bearing at $B$ does not exert any axial thrust.

The bent rod $ABDE$ is supported by ball-and-socket joints at $A$ and $E$ and by the cable $DF$. If a 60-lb load is applied at $C$ as shown, determine the tension in the cable.
Determine forces exerted by each cylinder.

**FBD of Bucket:**

\[ \vec{F}_{cc} \]

\[ \vec{P} = (2 \cos 45^\circ) \hat{c} + (-2 \sin 45^\circ) \hat{v} \text{ kip} \]

\[ \vec{P} = (1.41) \hat{c} + (-1.41) \hat{v} \text{ kip} \]

\[ \theta = \tan^{-1} \left( \frac{60}{45} \right) = 53.1^\circ \]

\[ \vec{F}_{cc} = (F_{cc} \cos 53.1^\circ) \hat{c} + (F_{cc} \sin 53.1^\circ) \hat{v} \text{ kip} \]

\[ \vec{F}_{cc} = (0.6 F_{cc}) \hat{c} + (-0.8 F_{cc}) \hat{v} \]

\[ \Sigma M_H = 0 \iff \]

\[ (8\text{ in})(1.41 \text{ kip}) + (16\text{ in})(1.41) - (10\text{ in})(0.8 F_{cc}) - (10\text{ in})(0.6 F_{cc}) = 0 \]
FBD of ARM IDB, ARM ABH and Bucket:

\[ \vec{p} = (1.41)^2 + (-1.41)^2 \]

\[ \theta = \tan^{-1} \left( \frac{16}{40} \right) = 21.8^\circ \]

\[ \vec{F}_{EF} = (-\vec{F}_{EF} \cos 21.8^\circ) \hat{i} + (-\vec{F}_{EF} \sin 21.8^\circ) \hat{j} \text{ kip} \]

\[ \vec{F}_{EF} = (-0.928 \vec{F}_{EF}) \hat{i} + (-0.371 \vec{F}_{EF}) \hat{j} \text{ kip} \]

\[ \sum M_x = 0 \]

\[ (34'' \times 0.928 \vec{F}_{EF}) - (40'' \times 0.371 \vec{F}_{EF}) + (28'' \times 1.41) - (120'' \times 1.41) = 0 \]

\[ \vec{F}_{EF} = 7.76 \text{ kip tension} \]
EXAMPLE PROB. 6.56

\[ \sum M_E = 0 + 9 \]
\[ -(2.4m \times 10.5\, \text{kN}) + (2.4)(2) + (1.6)(4) + (0.8)(4) - (0.4)F_{EG} = 0 \]

\[ F_{EG} = -27\, \text{kN} \quad \text{(compression)} \]
\[ \theta = \tan^{-1} \left( \frac{9}{8} \right) = 48.4^\circ \]

\[ \vec{F}_{AE} = (-\cos 48.4^\circ F_{AE}) i + (-\sin 48.4^\circ F_{AE}) j \text{ kips} \]

\[ \vec{F}_{AE} = (-0.664 F_{AE}) i + (-0.748 F_{AE}) j \]

\[ \Sigma F_x = 0 : \]

\[ -F_{FE} - 0.664 F_{AE} = 0 \]

\[ F_{FE} = -0.664 F_{AE} \quad \text{(eqn. 1)} \]

\[ F_{AE} = -1.51 F_{FE} \quad \text{(eqn. 1)} \]

\[ \Sigma M_A = 0 \quad \text{for} \]

\[ -(9 \text{ ft}) F_{FE} - (12)(1.8) - (26)(1.8) - (40)(0.9) = 0 \]
\[ F_{FE} = -11.6 \text{ kips (compression)} \]
\[ F_{AE} = -1.5 \times (-11.6) = 17.5 \text{ kips (tension)} \]
\[ \sum M_E = 0 \]
\[-(8 \times 14 \times F_{FT}) - (8)(0.9) - (20)(1.8) - (34)(1.8) - (48)(0.9) = 0 \]
\[ \Rightarrow F_{FT} = -18.4 \text{ kips (compression)} \]

**Homework:** #6

**Probs.** 6.15, 6.49, 6.95, 6.145

12-**Class #7 = Prob. 6.1**
PROB. 3

\[ \overline{X} = \frac{\sum X_i W_i}{\sum W_i} = \frac{\sum X_i V_i}{\sum V_i} = \frac{\sum X_i V_i}{\sum V_i} \]

**REGION 1:**

\[ \overline{X}_1 = \frac{1}{2}(100 \ mm) = 50 \ mm \quad V_1 = (100)(12)(88) = 1.056 \times 10^5 \ mm^3 \]

**REGION 2:**

\[ \overline{X}_2 = \frac{1}{2}(100) = 50 \ mm \quad V_2 = (100)(12)(55-12) = 5.16 \times 10^4 \ mm^3 \]
REGION 3:
\[ \bar{x}_3 = \frac{1}{2} (34) = 17 \text{ mm} \]
\[ V_3 = (34)(45)(12) = 1.836 \times 10^4 \text{ mm}^3 \]

REGION 4:
\[ \bar{x} = \frac{1}{3} h = \frac{1}{3} (100 - 34) = 22 \text{ mm} \]
\[ \bar{x}_4 = (22) + (34) = 56 \text{ mm} \]
\[ V_4 = \frac{1}{2} (45)(100 - 34)(12) = 1.782 \times 10^4 \text{ mm}^3 \]

REGION 5:
\[ \bar{x}_5 = 34 + \frac{1}{2} (10) = 39 \text{ mm} \]
\[ V_5 = \frac{1}{2} (51)(62)(10) = 1.581 \times 10^4 \text{ mm}^3 \]

\[ \sum x_i V_i = \left(50 \text{ mm} \right) \left(1.056 \times 10^5 \text{ mm}^3 \right) + \left(50 \right) \left(5.16 \times 10^4 \right) \]
\[ + \left(17 \right) \left(1.836 \times 10^4 \right) + \left(56 \right) \left(1.782 \times 10^4 \right) + \left(39 \right) \left(1.581 \times 10^4 \right) \]
\[ \sum x_i V_i = 9.757 \times 10^6 \text{ mm}^4 \]

\[ \sum V_i = \left(1.056 \times 10^5 \right) + \left(5.16 \times 10^4 \right) + \left(1.836 \times 10^4 \right) + \left(1.782 \times 10^4 \right) \]
\[ + \left(1.581 \times 10^4 \right) \]
\[ \sum V_i = 2.092 \times 10^5 \text{ mm}^3 \]

\[ \bar{x} = \frac{9.757 \times 10^6 \text{ mm}^4}{2.092 \times 10^5 \text{ mm}^3} \]
\[ \bar{x} = 46.8 \text{ mm} \]
SOLUTION OF PROBLEM (d):

Find \( \overrightarrow{TD} \):

Point D: \( X_D = 16'' \), \( Y_D = 0 \), \( Z_D = 24'' \)

Point F: \( X_F = 0 \), \( Y_F = 11'' \), \( Z_F = 24 - 8 = 16'' \)

\[
\begin{align*}
\Delta x &= X_F - X_D = 0 - 16 = -16'' \\
\Delta y &= Y_F - Y_D = 11 - 0 = 11'' \\
\Delta z &= Z_F - Z_D = 16 - 24 = -8'' \\
\end{align*}
\]

\[
\overrightarrow{d} = \sqrt{(-16)^2 + 11^2 + (-8)^2} = 21''
\]

\[
T_x = \left( \frac{-16}{21} \right) = -0.762 T
\]
\[ T_4 = \left( \frac{11}{21} \right) T = 0.524 \ T \]
\[ T_2 = \left( \frac{-8}{21} \right) T = -0.381 \ T \]
\[ \vec{T} = (-0.762 T) \hat{i} + (0.524 T) \hat{j} + (-0.381 T) \hat{k} \]

**UNIT VECTOR ALONG EA:**

**POINT A:** \( x_A = 7, \ y_A = 0, \ z_A = 0 \)

**POINT E:** \( x_E = 0, \ y_E = 0, \ z_E = 24 \)

\[ d x = x_A - x_E = 7 - 0 = 7 \]
\[ d y = y_A - y_E = 0 - 0 = 0 \]
\[ d z = z_A - z_E = 0 - 24 = -24 \]
\[ d = \sqrt{7^2 + 24^2} = 25 \]
\[ \vec{x}_{EA} = \left( \frac{7}{25} \right) \hat{i} + \left( \frac{-24}{25} \right) \hat{k} \]

**POSITION VECTOR FROM E TO D:**
\[ \vec{r}_{ED} = (16) \hat{i} \]

**TAKE MOMENTS ABOUT E:**

**POINT C:** \( x_c = 16, \ y_c = 0, \ z_c = 10 \)
\[ \vec{M}_E = \vec{M}_1 + \vec{M}_2 = (-840) \hat{z} + (6.096T) \hat{j} + (8.384T - 960) \hat{i} \]

\[ M_{EA} = \hat{X}_{EA} \cdot \vec{M}_E = 0 \]

\[ M_{EA} = (0.28)(-840) + (-0.96)(8.384T - 960) = 0 \]

\[-235 - 8.05T + 922 = 0 \]

\[ 8.05T = 687 \]

\[ T = 85.3 \text{ lb} \]