Distributed Adaptive Fault-Tolerant Control of Nonlinear Uncertain Second-order Multi-agent Systems

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***Abstract*— This paper presents an adaptive fault-tolerant control (FTC) scheme for a class of nonlinear uncertain second-order multi-agent systems. A local FTC component is designed for each agent in the distributed system by using local measurements and suitable information exchanged between neighboring agents. Each local FTC component consists of a fault diagnosis module and a reconfigurable controller module comprised of a baseline controller and two adaptive fault-tolerant controllers activated after fault detection and after fault isolation, respectively. Under suitable assumptions, the closed-loop stability and leader-follower formation properties of the distributed system are rigorously established under different operating modes of the FTC system, including the time-period before possible fault detection, between fault detection and possible isolation, and after fault isolation.**

I. Introduction

Several modern technical systems can be described by means of distributed multi-agent systems, that is, systems comprised of various distributed and interconnected autonomous agents/subsystems. Examples of such systems include cooperative unmanned vehicles, intelligent power grids, air traffic control system, etc. In recent years, cooperative control using distributed consensus algorithms has received significant attention (see, e.g., [1] and [2]). Adaptive methods for achieving consensus in uncertain systems have also been proposed [3], [4], [5].

Since such distributed multi-agent systems need to operate reliably at all times, despite the possible occurrence of faulty behaviors in some agents, the development of fault diagnosis and accommodation schemes is a crucial step in achieving reliable and safe operations. In the last two decades, significant research activities have been conducted in the design and analysis of fault diagnosis and accommodation schemes (see, for instance, [6]). Most of these methods utilize a centralized architecture, where the diagnostic module is designed based on a global mathematical model of the overall system and is required to have real-time access to all sensor measurements. due to limitations of computational resources and communication overhead, such centralized methods are not suitable for large-scale distributed systems. As a result, in recent years, there has been significant research interest in distributed fault diagnosis and accommodation schemes (see, for instance, [7], [8], [9], [10]).

This paper presents a method for detecting, isolating, and accommodating faults in a class of distributed nonlinear uncertain multi-agent systems. A fault-tolerant control component is designed for each agent in the distributed system by utilizing local measurements and suitable information exchanged between neighboring agents. Each local FTC component consists of two main modules: 1) the online health monitoring (fault diagnosis) module consists of a bank of nonlinear adaptive estimators. One of them is the fault detection estimator, while the rest are fault isolation estimators; and 2) the controller (fault accommodation) module consists of a baseline controller and two adaptive fault-tolerant controllers employed after fault detection and after fault isolation, respectively. Under suitable assumptions, the closed-loop stability and leader-following formation properties are established for the baseline controller and adaptive fault-tolerant controllers, respectively. In previous papers, a centralized FDI and fault-tolerant control scheme is presented in [11], and a distributed FDI and fault-tolerant control scheme for first-order multi-agent systems is presented in [12]. This paper extends the results in these papers by generalizing the fault-tolerant control method to the case of leader-follower formation of distributed second-order multi-agent systems.

II. Graph Theory Notations

A directed graph $\mathscr{G}$ is a pair $(\mathscr{V},\mathscr{E})$, where $\mathscr{V} = \{\upsilon_1,\cdots,\upsilon_m\}$ is a set of nodes, $\mathscr{E} \subseteq \mathscr{V}\times\mathscr{V}$ is a set of edges, and m is the number of nodes. An edge is an ordered pair of distinct nodes (υ_j,υ_i) meaning that ith node can receive information from jth node. For an edge (υ_j,υ_i), node υ_j is called the parent node, node υ_i the child node, and υ_j is a neighbor of υ_i. An undirected graph can be considered as a special case of a directed graph where $(\upsilon_i,\upsilon_j)\in\mathscr{E}$ implies $(\upsilon_j,\upsilon_i)\in\mathscr{E}$ for any $\upsilon_i,\upsilon_j\in\mathscr{V}$.

The set of neighbors of node υ_i is denoted by $N_i = \{j : (\upsilon_j,\upsilon_i)\in\mathscr{E}\}$. The weighted adjacency matrix $\mathscr{A} = [a_{ij}] \in \Re^{m\times m}$ associated with the directed graph $\mathscr{G}$ is defined by $a_{ii} = 0$, $a_{ij} > 0$ if $(\upsilon_j,\upsilon_i)\in\mathscr{E}$, and $a_{ij} = 0$ otherwise. The topology of an interconnection graph $\mathscr{G}$ is said to be fixed, if each node has a fixed neighbor set and a_{ij} is fixed. It is clear that for undirected graphs $a_{ij} = a_{ji}$. The Laplacian matrix $L = [\iota_{ij}] \in \Re^{m\times m}$ is defined as $\iota_{ii} = \sum_{j\in N_i} a_{ij}$ and $\iota_{ij} = -a_{ij}$, $i \neq j$. Both $\mathscr{A}$ and L are symmetric for undirected graphs, and L is positive semidefinite.

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III. Problem Formulation

A. Distributed Multi-Agent System Model

Consider a set of M interconnected agents with the dynamics of the ith agent, $i = 1, \cdots, M$, being described by the following second-order dynamics

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= \phi_i(x_i) + u_i + \eta_i(x_i,t) + \beta_i(t-T_i) f_i(x_i), \end{aligned} \quad (1)$$

where $x_i = \begin{bmatrix} p_i \\ v_i \end{bmatrix} \in \Re^2$, $u_i \in \Re$, are the state vector and input vector of the ith agent, respectively. Additionally, $\phi_i : \Re^2 \mapsto \Re$, $\eta_i : \Re^2 \times \Re^+ \mapsto \Re$, $f_i : \Re^2 \mapsto \Re$ are smooth vector fields. The model given by

$$\dot{x}_i = \begin{bmatrix} 0 \\ \phi_i(x_i) \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i \quad (2)$$

represents the *known nominal* dynamics of the ith agent with ϕ_i being the known nonlinearity, while the healthy system is described by

$$\dot{x}_i = \begin{bmatrix} 0 \\ \phi_i(x_i) \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left(u_i + \eta_i(x_i,t)\right). \quad (3)$$

The difference between the nominal model (2) and the actual (healthy) system dynamics (3) is due to the vector field η_i representing the modeling uncertainty in the state dynamics of the ith agent.

The term $\beta_i(t-T_i) f_i(x_i)$ denotes the changes in the dynamics of ith agent due to the occurrence of a fault. Specifically, $\beta_i(t-T_i)$ represents the time profile of a fault which occurs at some unknown time T_i, and $f_i(x_i)$ is an unknown nonlinear fault function. In this paper, the time profile function $\beta_i(\cdot)$ is assumed to be a step function (i.e., $\beta_i(t-T_i) = 0$ if $t < T_i$, and $\beta_i(t-T_i) = 1$ if $t \geq T_i$) which denotes an abrupt fault. The system model (1) allows the occurrence of fault in multiple agents but it is assumed there is only a single fault in each agent at any time.

Remark 1: The distributed multi-agent system model given by (1) is a nonlinear generalization of the double integrator dynamics considered in literature (for instance, [2]). In this paper, in order to investigate the fault-tolerance and robustness properties, the fault function $\beta_i f_i(x_i)$ and modeling uncertainty η_i are included in the system model.

For isolation purposes, we assume that there are r_i types of possible nonlinear fault functions in the fault class associated with the ith agent; specifically, $f_i(x_i)$ belongs to a finite set of functions given by

$$\mathscr{F}_i \stackrel{\triangle}{=} \{f_i^1(x_i), \cdots, f_i^{r_i}(x_i)\}. \quad (4)$$

Each fault function f_i^s, $s = 1, \cdots, r_i$, is described by

$$f_i^s(x_i) \stackrel{\triangle}{=} \left(\theta_i^s\right)^T g_i^s(x_i), \quad (5)$$

where θ_i^s, for $i = 1, \cdots, M$, is an unknown parameter vector assumed to belong to a known compact set Θ_i^s (i.e., $\theta_i^s \in \Theta_i^s \subseteq \Re^{q_i^s}$), and $g_i^s : \Re^2 \mapsto \Re^{q_i^s}$ is a known smooth vector field. As described in [11], the fault model described by (4) and (5) characterizes a general class of nonlinear faults where the vector field g_i^s represents the functional structure of the sth fault affecting the state equation, while the unknown parameter vector θ_i^s characterizes the fault magnitude.

The objective of this paper is to develop a robust distributed fault diagnosis and fault-tolerant leader-following formation control scheme for a class of distributed multi-agent systems described by (1). The following assumptions are made throughout the paper:

Assumption 1. *Each modeling uncertainty, represented by $\eta_i(x_i,t)$ in (1), has a known upper bound, i.e.,*

$$|\eta_i(x_i,t)| \leq \bar{\eta}_i(x_i,t), \quad \forall x_i \in \Re^2, \quad (6)$$

where the the bounding function $\bar{\eta}_i$ is known and uniformly bounded.

Assumption 2. The communication topology among followers is undirected, and the leader has directed paths to all followers.

Assumption 1 characterizes the class of modeling uncertainty under consideration. The bound on the modeling uncertainty is needed in order to distinguish between the effects of faults and modeling uncertainty during the fault diagnosis process [13]. Assumption 2 is needed to ensure that the information exchange among agents is sufficient for the team to achieve the desired team goal.

B. Fault-Tolerant Control Structure

In this paper, we investigate the FTC problem of leader-following formation. Specifically, the objective is to develop distributed robust FTC algorithms to guarantee that each agent's output converges to a given predefined formation around a time-varying leader even in the presence of modeling uncertainty and faults.

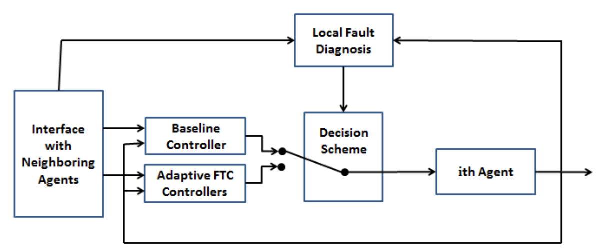

Fig. 1: Distributed FTC architecture for the ith agent

The distributed FTC architecture is shown in Figure 1. First of all, we define three important time–instants: T_i is the fault occurrence time; $T_d > T_i$ is the time–instant when a fault is detected; $T_{\text{isol}} > T_d$ is the time–instant when the monitoring system (possibly) provides a fault isolation decision, that is, which fault in the class $\mathscr{F}_i$ has actually occurred. The structure of the fault-tolerant controller for the ith agent takes on the following general form: [11]

$$\dot{\omega}_i = \begin{cases} b_0(\omega_i, x_i, x_J, x_0, t), & \text{for } t < T_d \\ b_D(\omega_i, x_i, x_J, x_0, t), & \text{for } T_d \leq t < T_{\text{isol}} \\ b_I(\omega_i, x_i, x_J, x_0, t), & \text{for } t \geq T_{\text{isol}} \end{cases}$$

$$u_i = \begin{cases} h_0(\omega_i, x_i, x_J, x_0, t), & \text{for } t < T_d \\ h_D(\omega_i, x_i, x_J, x_0, t), & \text{for } T_d \leq t < T_{\text{isol}} \\ h_I(\omega_i, x_i, x_J, x_0, t), & \text{for } t \geq T_{\text{isol}} \end{cases} \quad (7)$$

where ω_i is the state vector of the distributed controller, x_0 is the time-varying bounded leader states, x_J contains the state variables of neighboring agents that directly communicate with agent i, i.e., $J = \{j : j \in N_i\}$; b_0, b_D, b_I and h_0, h_D, h_I are nonlinear functions to be designed according to the following qualitative objectives:

1) In a fault free mode of operation, a baseline controller guarantees the output of ith agent $x_i(t)$ should track the formation around a leader's time-varying output x_0, even in the possible presence of plant modeling uncertainty.
2) If a fault is detected, the baseline controller is reconfigured to compensate for the effect of the (yet unknown) fault, that is, the fault-tolerant controller is designed in such a way to exploit the information that a fault has occurred, so that the controller may recover some control performances. This new controller should guarantee the boundedness of system signals and some leader-following formation performance, even in the presence of the fault.
3) If the fault is isolated, then the controller is reconfigured again. The second fault-tolerant controller is designed using the information about the type of fault that has been isolated so as to improve the control performances.

IV. Baseline Controller Design

In this section, we design the baseline controller and investigate the closed-loop system stability and performance before fault occurrence. Without loss of generality, let the leader be agent number 0 with a reference output (i.e., $x_0(t) = \begin{bmatrix} p_0(t) & v_0(t) \end{bmatrix}^T$ where $p_0 = \dot{v}_0$). The baseline controller for the ith agent is designed as follows:

$$\begin{aligned} u_i &= -\sum_{j\in N_i} k_{ij}\Big(\alpha(p_i - \bar{p}_i - p_j + \bar{p}_j) + \gamma(v_i - v_j)\Big) \\ &\quad -\phi_i(x_i) - (\bar{\eta}_i + \kappa)\, sgn\Big(\Xi_i\Big), \end{aligned} \tag{8}$$

where $\bar{p}_i$ and $\bar{p}_j$ are the constant desired distance between the leader and agents i and j, respectively, κ is a positive bound on $|\dot{v}_0|$ (i.e., $\kappa \geq |\dot{v}_0|$), $sgn(\cdot)$ is the sign function, $\Xi_i = \sum_{j\in N_i} k_{ij}\big(\epsilon(p_i - \bar{p}_i - p_j + \bar{p}_j) + \rho(v_i - v_j)\big)$, N_i is the set of neighboring agents that directly communicate with the ith agent including the leader as agent number 0 with $\bar{p}_0 = 0$, k_{ij} are positive constants for $j \in N_i$, and α, γ, ρ, and ϵ are positive constants to be defined in Lemma 1. Notice that $k_{il} = 0$, for $l \notin N_i$. First, we need the following Lemma:

Lemma 1: Consider a positive definite square matrix $\Psi \in \Re^{M\times M}$. Define

$$A = \begin{bmatrix} 0_{M\times M} & I_M \\ -\alpha\Psi & -\gamma\Psi \end{bmatrix}, \; P = \begin{bmatrix} \rho\Psi & \epsilon\Psi \\ \epsilon\Psi & \rho\Psi \end{bmatrix}, \tag{9}$$

where I is the identity matrix, $\rho > \epsilon$, and $\rho, \epsilon, \gamma, \alpha > 0$. The matrix $Q = PA + A^T P$ is negative definite if the following conditions are met:

$$\gamma\epsilon = \alpha\rho\,, \quad \frac{\epsilon}{\alpha\epsilon + \rho\gamma} < \mu_{min}\,, \quad \frac{\rho^2}{4\alpha(\rho^2 - \epsilon^2)} < \mu_{min}\,, \tag{10}$$

where μ_{min} is the smallest eigenvalue of Ψ.

Proof: Using (9), the matrix Q can be obtained as

$$Q = \begin{bmatrix} -2\alpha\epsilon\Psi^2 & \rho\Psi - (\epsilon\gamma + \alpha\rho)\Psi^2 \\ \rho\Psi - (\epsilon\gamma + \alpha\rho)\Psi^2 & 2\epsilon\Psi - 2\gamma\rho\Psi^2 \end{bmatrix}. \tag{11}$$

The eigenvalues are found using the following characteristic equation:

$$|sI - Q| = \begin{vmatrix} sI + 2\alpha\epsilon\Psi^2 & -\rho\Psi + (\epsilon\gamma + \alpha\rho)\Psi^2 \\ -\rho\Psi + (\epsilon\gamma + \alpha\rho)\Psi^2 & sI - 2\epsilon\Psi + 2\gamma\rho\Psi^2 \end{vmatrix} = 0\,.$$

Note that $\begin{vmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{vmatrix} = |\hat{A}\hat{D} - \hat{C}\hat{B}|$ if $\hat{A}$ and $\hat{C}$ commute. Also, it can be shown that $|s^2 I - h_1(\Psi)s - h_2(\Psi)| = \prod_{i=1}^{M}\big(s^2 - h_1(\mu_i)s - h_2(\mu_i)\big)$, where $h_1(\cdot)$ and $h_2(\cdot)$ are polynomials, and μ_i is the ith eigenvalue of Ψ. Thus, we have

$$\begin{aligned} |sI - Q| &= \prod_{i=1}^{M}\big[s^2 + 2\big(-\epsilon\mu_i + (\alpha\epsilon + \rho\gamma)\mu_i^2\big)s + 4\alpha\epsilon\mu_i^2 \\ &\quad \cdot(\rho\gamma\mu_i^2 - \epsilon\mu_i) - \big(\rho\mu_i - (\gamma\epsilon + \alpha\rho)\mu_i^2\big)^2\big]\,. \end{aligned}$$

To have all the eigenvalues in the left-half complex plane, the coefficient of s and the constant need to be positive. Since $\mu_i > 0$, the following conditions guarantee that the eigenvalues of Q lie in the left-half complex plane:

$$\begin{cases} -\epsilon + (\alpha\epsilon + \rho\gamma)\mu_i > 0 \\ -(\gamma\epsilon - \alpha\rho)^2\mu_i^2 + (-4\alpha\epsilon^2 + 2\rho\big(\gamma\epsilon + \alpha\rho)\big)\mu_i - \rho^2 > 0\,. \end{cases}$$

The above inequalities are guaranteed by the conditions given in (10). Thus, the proof is completed. □

The following result characterizes the closed-loop stability and leader-following formation performance properties of the overall multi-agent system prior to any fault occurrence.

Theorem 1: In the absence of faults in the ith agent, using the baseline controller described by (8), the leader-follower formation control is achieved asymptotically with a time-varying reference state, i.e. $p_i(t) - p_0(t) \to \bar{p}_i$ and $v_i(t) - v_0(t) \to 0$ as $t \to \infty$.

Proof: Based on (8) and (1), the closed-loop system dynamics, in the absence of a fault (i.e., for $t < T_i$), are given by

$$\begin{aligned} &\dot{p}_i = v_i \\ &\dot{v}_i = -\sum_{j\in N_i} k_{ij}\big(\alpha\tilde{p}_{ij} + \gamma\tilde{v}_{ij}\big) + \eta_i - (\bar{\eta}_i + \kappa)sgn\Big(\Xi_i\Big), \end{aligned} \tag{12}$$

where $\tilde{p}_{ij} \triangleq (p_i - \bar{p}_i) - (p_j - \bar{p}_j)$ and $\tilde{v}_{ij} \triangleq v_i - v_j$. We represent the collective closed-loop dynamics as

$$\dot{\tilde{x}} = A\tilde{x} + \begin{bmatrix} \mathbf{0}_M \\ \zeta - \bar{\zeta} - \mathbf{1}_M\dot{v}_0 \end{bmatrix}, \tag{13}$$

where A is defined in Lemma 1 with the stable matrix $\Psi = \mathscr{L} + \mathrm{diag}\{k_{10}, k_{20}, \cdots, k_{M0}\}$ [14], $\mathscr{L}$ is the communication graph Laplacian matrix, $\tilde{x} = [\tilde{p}^T\; \tilde{v}^T]^T \in \Re^{2M}$ in which $\tilde{p}$ is the column stack vector of $\tilde{p}_i = p_i - \bar{p}_i - p_0$ and $\tilde{v}$ is the column stack vector of $\tilde{v}_i = v_i - v_0$, the terms $\zeta \in \Re^M$ and $\bar{\zeta} \in \Re^M$ are defined as

$$\zeta \triangleq \begin{bmatrix} \eta_1 & \cdots & \eta_M \end{bmatrix}^T \tag{14}$$

$$\bar{\zeta} \triangleq \begin{bmatrix} \bar{\zeta}_1 & \cdots & \bar{\zeta}_M \end{bmatrix}, \tag{15}$$

where $\bar{\zeta}_i = (\bar{\eta}_i + \kappa_i)sgn\big(\Xi_i\big)$, $i = 1, \cdots, M$. We consider the following Lyapunov function candidate:

$$V = \tilde{x}^T P \tilde{x}, \tag{16}$$

where P is a positive definite matrix defined in Lemma 1. Then, the time derivative of the Lyapunov function (16) along the solution of (13) is given by

$$\dot{V} = \tilde{x}^T Q \tilde{x} + 2\tilde{x}^T P \begin{bmatrix} \mathbf{0}_M \\ \zeta - \bar{\zeta} - \mathbf{1}_M \dot{v}_0 \end{bmatrix}, \tag{17}$$

where Q is defined in Lemma 1. Based on (9) and (14), we have

$$\begin{aligned} \tilde{x}^T P \begin{bmatrix} \mathbf{0}_M \\ \zeta \end{bmatrix} &= \epsilon \tilde{p}^T \Psi \zeta + \rho \tilde{v}^T \Psi \zeta \\ &= \sum_{i=1}^{M} \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) \eta_i . \end{aligned} \tag{18}$$

By using the same reasoning logic for the other terms in (17) and substituting them into (17), we have

$$\begin{aligned} \dot{V} &= \tilde{x}^T Q \tilde{x} + 2 \sum_{i=1}^{M} \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) (\eta_i - \dot{v}_0) \\ & \quad -2 \sum_{i=1}^{M} \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) (\bar{\eta}_i + \kappa) sgn\big(\Xi_i\big) . \end{aligned} \tag{19}$$

Based on Assumption 1, we have

$$\begin{aligned} & (\eta_i - \dot{v}_0) \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) \\ & \quad -(\bar{\eta}_i + \kappa) \sum_{j \in N_i} k_{ij} (\epsilon \tilde{p}_{ij} + \rho \tilde{v}_{ij}) sgn\big(\Xi_i\big) \le 0 . \end{aligned} \tag{20}$$

Therefore, by applying the above inequality to (19), we obtain $\dot{V} \le \tilde{x}^T Q \tilde{x}$. Using Lemma 1, we know that $\dot{V}$ is negative definite, and $\tilde{p}_i$ and $\tilde{v}_i$ converge to zero as $t \to \infty$. Therefore, the leader-following formation control is reached asymptotically, i.e., $p_i(t) - p_0(t) \to \bar{p}_i$ and $v_i(t) \to v_0(t)$ as $t \to \infty$. □

Remark 2: The baseline controller guarantees the convergence of the leader-following consensus algorithm in the absence of faults. The analysis is an extension of the consensus algorithm given in [14] by considering the presence of modeling uncertainty η_i and by using more control parameters (e.g., ρ and α) to allow certain flexibility in controller design.

V. Distributed Fault Detection and Isolation

The distributed fault detection and isolation (FDI) architecture is comprised of M local FDI components, with one FDI component designed for each of the M agents. The objective of each local FDI component is to detect and isolate faults in the corresponding agent. Specifically, each local FDI component consists of a fault detection estimator (FDE) and a bank of r_i nonlinear adaptive fault isolation estimators (FIEs), where r_i is the number of different nonlinear fault types in the fault set $\mathscr{F}_i$ (4) associated with the corresponding agent. Under normal conditions, each local FDE monitors the corresponding local agent to detect the occurrence of any fault. If a fault is detected in a particular agent i, then the corresponding r_i local FIEs are activated for the purpose of determining the particular type of fault that has occurred in the agent. The FDI design for each agent follows the generalized observer scheme architectural framework [6].

A. Distributed Fault Detection

Based on the agent model described by (1), the FDE for each agent is chosen as:

$$\dot{\hat{x}}_i = \Lambda_i^0 (x_i - \hat{x}_i) + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \big(\phi_i(x_i) + u_i\big), \tag{21}$$

where $\hat{x}_i \in \Re^2$ denotes the estimated local state, $\Lambda_i^0 = \begin{bmatrix} \lambda_{p_i}^0 & 0 \\ 0 & \lambda_{v_i}^0 \end{bmatrix} \in \Re^{2\times 2}$ is a positive definite estimator gain matrix.

For each local FDE, let $\varepsilon_i \stackrel{\triangle}{=} x_i - \hat{x}_i = [\varepsilon_{p_i}\, \varepsilon_{v_i}]^T$ denote the state estimation error of the ith agent. Then, before fault occurrence (i.e., for $0 \le t < T_i$), by using (1) and (21), the estimation error dynamics are given by

$$\dot{\varepsilon}_i = -\Lambda_i^0 \varepsilon_i + \begin{bmatrix} 0 \\ \eta_i(x_i, t) \end{bmatrix} . \tag{22}$$

Therefore, using (22), we have $|\varepsilon_{v_i}| \le \nu_i$, where

$$\nu_i(t) \stackrel{\triangle}{=} \int_0^t e^{-\lambda_{v_i}^0 (t-\tau)} \bar{\eta}_i(x_i, \tau) d\tau + \bar{x}_i e^{-\lambda_{v_i}^0 t} , \tag{23}$$

$\bar{x}_i$ is a conservative bound on the initial state estimation error (i.e., $|\varepsilon_{v_i}(0)| \le \bar{x}_i$). Note that the integral term in the above thresholds can be easily implemented as the output of a linear filter with the input given by $\bar{\eta}_i(x_i, t)$.

Thus, we have the following:

Fault Detection Decision Scheme: The decision on the occurrence of a fault (detection) in the ith agent is made when the modulus of the estimation error (i.e., ε_{v_i}) generated by the local FDE exceeds its corresponding threshold (i.e., $|\varepsilon_{v_i}| > \nu_i$).

B. Distributed Fault Isolation

Now, assume that a fault is detected in the ith agent at some time T_d; accordingly, at $t = T_d$ the FIEs in the local FDI component designed for the ith agent are activated. Each local FIE is designed based on the functional structure of a particular fault type in the agent (see (5)). Specifically, the following r_i nonlinear adaptive estimators are designed as isolation estimators for the ith agent: for $s = 1, \cdots, r_i$,

$$\begin{aligned} \dot{\hat{x}}_i^s &= \Lambda_i^s (x_i - \hat{x}_i^s) + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i \\ & \quad + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \big(\phi_i(x_i) + u_i + \big(\hat{\theta}_i^s\big)^T g_i^s(x_i)\big) , \end{aligned} \tag{24}$$

where $\hat{\theta}_i^s$, for $i = 1, \cdots, M$, and $s = 1, \cdots, r_i$, is the estimate of the fault parameter vector in the ith agent, and $\Lambda_i^s = \begin{bmatrix} \lambda_{p_i}^s & 0 \\ 0 & \lambda_{v_i}^s \end{bmatrix}$ is a diagonal positive definite matrix.

The adaptation in the isolation estimators is due to the unknown fault parameter vector θ_i^s. The adaptive law for updating each $\hat{\theta}_i^s$ is derived by using the Lyapunov synthesis approach [15], with the projection operator $\mathscr{P}$ restricting $\hat{\theta}_i^s$ to the corresponding known set Θ_i^s. Specifically, if we let $\varepsilon_i^s(t) = x_i - \hat{x}_i^s = [\varepsilon_{p_i}^s\, \varepsilon_{v_i}^s]^T$ be the estimation error generated by the sth FIE associated with the ith agent, then the following adaptive algorithm is chosen:

$$\dot{\hat{\theta}}_i^s = \mathscr{P}_{\Theta_i^s}\{\gamma_i^s g_i^s(x_i)\varepsilon_{v_i}^s\},$$

where $\gamma_i^s > 0$ is a constant learning rate.

Based on (1) and (24), the state estimation error dynamics in the presence of fault s is given by

$$\dot{\varepsilon}_i^s = -\Lambda_i^s\varepsilon_i^s + \begin{bmatrix}0\\1\end{bmatrix}\left(\eta_i(x_i,t) - \left(\tilde{\theta}_i^s\right)^T g_i^s(x_i)\right),$$

where ε_i^s is the state estimation error, and $\tilde{\theta}_i^s = \hat{\theta}_i^s - \theta_i^s$ is the parameter estimation error. Therefore, by using the triangle equality, a bound on the state estimation error can be obtained as $|\varepsilon_{v_i}^s| \le \varsigma_i^s(t)$, where

$$|\varsigma_i^s| \le \int_{T_d}^t e^{-\lambda_{v_i}^s(t-\tau)}\left(\bar{\eta}_i + \xi_i^s\left|g_i^s(x_i)\right|\right)d\tau + \bar{x}_i^s e^{-\lambda_{v_i}^s(t-T_d)},$$

where $\bar{x}_i^s$ is a possibly conservative bound on the initial state estimation error (i.e., $|\varepsilon_{v_i}^s(T_d)| \le \bar{x}_i^s$), and ξ_i^s represents the maximum fault parameter vector estimation error, i.e., $|\theta_i^s - \hat{\theta}_i^s(t)| \le \xi_i^s$. The form of ξ_i^s depends on the geometric properties of the compact set Θ_i^s [11]. For instance, assume that the parameter set Θ_i^s is a hypersphere (or the smallest hypersphere containing the set of all possible $\hat{\theta}_i^s(t)$ with center O_i^s and radius R_i^s); then we have $\xi_i^s = R_i^s + |\hat{\theta}_i^s(t) - O_i^s|$.

The fault isolation decision scheme is based on the following intuitive principle: if fault s occurs at some time T_i and is detected at time T_d, then a set of threshold functions $\varsigma_i^s(t)$ can be designed such that the estimation error generated by the sth estimator satisfies $|\varepsilon_{v_i}^s(t)| \le \varsigma_i^s(t)$ for all $t \ge T_d$. In the fault isolation procedure, if for a particular fault isolation estimator b, the estimation error satisfies $|\varepsilon_{v_i}^b(t)| > \varsigma_i^b(t)$ for some finite time $t > T_d$, then the possibility of the occurrence of corresponding fault type can be excluded. Based on this intuitive idea, the following fault isolation decision scheme is devised.

Distributed fault isolation decision scheme: If for each $b \in \{1,\cdots,r_i\}\backslash\{s\}$, there exist some finite time $t^b > T_d$, such that $|\varepsilon_{v_i}^b(t^b)| > \varsigma_i^b(t^b)$, then the occurrence of fault s in the ith subsystem is concluded.

VI. Fault-Tolerant Controllers

In this section, the design and analysis of the fault-tolerant control schemes are rigorously investigated for two different operating modes of the closed-loop system: 1) during the period after fault detection and before isolation, and 2) after fault isolation.

A. Accommodation before Fault Isolation

After the fault is detected at time $t = T_d$ in agent i, the isolation estimators described in Section *V.B* are activated to determine the particular type of fault that has occurred. Meanwhile, the nominal controller is reconfigured to ensure the system stability and some tracking performances after fault detection. In the following, we describe the design of the fault-tolerant controller using adaptive tracking techniques.

Before the fault is isolated, no information about the fault function is available. Adaptive approximators such as neural-network models can be used to estimate the unknown fault function $\beta_i f_i$. The term "adaptive approximator" [16] is used to represent nonlinear multivariable approximation models with adjustable parameters, such as neural networks, fuzzy logic networks, polynomials, spline functions, etc. Specifically, we consider linearly parametrized network (e.g., radial-basis-function networks with fixed centers and variances) described as follows:

$$\hat{f}_i(x_i,\hat{\vartheta}_i) = \sum_{j=1}^{\varrho} c_j\bar{\varphi}_j(x_i), \tag{25}$$

where $\bar{\varphi}_j(\cdot)$ represents the fixed basis functions, and $\hat{\vartheta}_i \stackrel{\triangle}{=} col(c_j : j = 1,\cdots,\varrho)$ is the adjustable weights of the nonlinear approximator. In the presence of a fault, $\hat{f}_i$ provides the adaptive structure for the online approximation of the unknown fault function $f_i(x_i)$. This is achieved by adapting the weight vector $\hat{\vartheta}_i(t)$.

Remark 3. The objectives of adaptive parameter estimation in the FDI procedure and the fault accommodation procedure are different. The goal of adaptive parameter estimation in the case of FDI is learning, i.e., to approximate the fault function (see for example the fault isolation estimation model given by (24)), while the objective during fault accommodation is to modify the feedback control law via parameter adaptation so as to stabilize the system and guarantee some tracking performance in the presence of a fault. However, the parameters do not necessarily converge to the true parameters unless the condition of persistence of excitation is assumed. In this paper, we do not assume the persistence of excitation condition.

Therefore, the system dynamics described by (1) can be rewritten as

$$\begin{aligned}\dot{x}_i = {} & \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}x_i + \begin{bmatrix}0\\1\end{bmatrix}\big(\phi_i(x_i) + u_i\\ & +\eta_i(x_i,t) + \hat{f}_i(x_i,\vartheta_i) + \delta_i(x_i)\big),\end{aligned} \tag{26}$$

where $\delta_i \stackrel{\triangle}{=} f_i(x_i) - \hat{f}_i(x_i,\vartheta_i)$ is the network approximation error for the ith agent, and ϑ_i is the optimal weight vector given by

$$\vartheta_i \stackrel{\triangle}{=} \arg\inf_{\hat{\vartheta}_i\in\Theta_i}\left\{\sup_{x_i\in\mathscr{X}_i}|f_i(x_i) - \hat{f}_i(x_i,\hat{\vartheta}_i)|\right\},$$

where $\mathscr{X}_i \subseteq \Re^2$ denotes the set to which the variable x_i belongs for all possible modes of the controlled system. To simplify the subsequent analysis, in the following we assume that the bounding conditions on the network approximation error are global, so we set $\mathscr{X}_i = \Re^2$. For each network, we make the following assumption on the network approximation error:

Assumption 3. for each $i = 1,\cdots,M$,

$$|\delta_i| \le \alpha_i\bar{\delta}_i(x_i), \tag{27}$$

where $\bar{\delta}_i$ is a known positive bounding function, and α_i is an unknown constant.

Based on the system model (26), the neural network model (25), and Assumption 3, an adaptive neural controller can be designed using neural-network-based approximation and adaptive bounding control techniques [16]. Specifically, we consider the following controller algorithm:

$$u_i = -\phi_i(x_i) - \sum_{j\in N_i} k_{ij}\big(\alpha\tilde{p}_{ij} + \gamma\tilde{v}_{ij}\big) - \psi_i - \hat{f}_i(x_i,\hat{\vartheta}_i(t)) - (\bar{\eta}_i+\kappa)sgn\Big(\Xi_i\Big) \quad (28)$$

$$\dot{\hat{\vartheta}}_i = \Gamma_i \sum_{j\in N_i} k_{ij}\big(\epsilon\tilde{p}_{ij} + \rho\tilde{v}_{ij}\big)\varphi_i(x_i) \quad (29)$$

$$\psi_i = \hat{\alpha}_i\bar{\delta}_i(x_i)sgn\Big(\sum_{j\in N_i} k_{ij}\big(\epsilon\tilde{p}_{ij} + \rho\tilde{v}_{ij}\big)\Big) \quad (30)$$

$$\dot{\hat{\alpha}}_i = \Upsilon_i\big|\sum_{j\in N_i} k_{ij}\big(\epsilon\tilde{p}_{ij} + \rho\tilde{v}_{ij}\big)\big|\bar{\delta}_i(x_i), \quad (31)$$

where $\hat{\vartheta}_i$ is an estimation of the neural network parameter vector ϑ_i, $\varphi_i \stackrel{\triangle}{=} col(\bar{\varphi}_j : j = 1,\cdots,\varrho)$ is the collective vector of fixed basis functions , $\hat{\alpha}_i$ is an estimation of the unknown constants α_i, and Γ_i and Υ_i are symmetric positive definite learning rate matrices.

We can represent the collective closed-loop dynamics as

$$\dot{\tilde{x}} = A\tilde{x} + \begin{bmatrix} \mathbf{0}_M \\ \zeta - \bar{\zeta} - \mathbf{1}_M\dot{v}_0 + \tilde{f} \end{bmatrix}, \quad (32)$$

where A is given in Lemma 1, and $\tilde{x} = [\tilde{p}^T\tilde{v}^T]^T$ is defined in a similar way as in (13), the terms $\zeta \in \Re^M$ and $\bar{\zeta} \in \Re^M$ are defined in (14) and (15), the term $\tilde{f} \in \Re^M$ is defined as

$$\tilde{f} \stackrel{\triangle}{=} \begin{bmatrix}\tilde{f}_1 + \delta_1 - \psi_1 & \cdots & \tilde{f}_M + \delta_M - \psi_M\end{bmatrix}^T, \quad (33)$$

where $\tilde{f}_i \stackrel{\triangle}{=} \tilde{\vartheta}_i^T\varphi_i$, and $\tilde{\vartheta}_i = \vartheta_i - \hat{\vartheta}_i$ is the network parameter estimation error associated with the ith agent. To derive the adaptive algorithm and to investigate analytically the stability properties of the closed-loop system, we consider the following Lyapunov function candidate:

$$V = \tilde{x}^T P\tilde{x} + \tilde{\vartheta}^T(\Gamma)^{-1}\tilde{\vartheta} + \tilde{\alpha}^T(\Upsilon)^{-1}\tilde{\alpha}, \quad (34)$$

where P is defined in Lemma 1, $\tilde{\vartheta} = \begin{bmatrix}\tilde{\vartheta}_1^T & \cdots & \tilde{\vartheta}_M^T\end{bmatrix}^T$ is the collective parameter estimation errors, $\tilde{\alpha} = \begin{bmatrix}\tilde{\alpha}_1 & \cdots & \tilde{\alpha}_M\end{bmatrix}^T$ is the collective bounding parameter estimation errors defined as $\tilde{\alpha}_i = \alpha_i - \hat{\alpha}_i$, and $\Gamma = \mathrm{diag}\{\Gamma_1,\cdots,\Gamma_M\}$ and $\Upsilon = \mathrm{diag}\{\Upsilon_1,\cdots,\Upsilon_M\}$ are constant learning rate matrices.

Following the same procedure as given in the proof of Theorem 1, using (33), it can be shown that the time derivative of the Lyapunov function (34) along the solution of (32) satisfies

$$\begin{aligned}\dot{V} = \; & \tilde{x}^T Q\tilde{x} + 2\sum_{i=1}^{M}\Bigg[\sum_{j\in N_i} k_{ij}\big(\epsilon\tilde{p}_{ij} + \rho\tilde{v}_{ij}\big)(\eta_i - \dot{v}_0) \\ & - \sum_{j\in N_i} k_{ij}\big(\epsilon\tilde{p}_{ij} + \rho\tilde{v}_{ij}\big)(\bar{\eta}_i+\kappa)sgn(\Xi_i) \\ & + \tilde{\vartheta}_i^T\Big(\sum_{j\in N_i} k_{ij}\big(\epsilon\tilde{p}_{ij} + \rho\tilde{v}_{ij}\big)\varphi_i - (\Gamma_i)^{-1}\dot{\hat{\vartheta}}_i\Big) \\ & + \sum_{j\in N_i} k_{ij}\big(\epsilon\tilde{p}_{ij} + \rho\tilde{v}_{ij}\big)(\delta_i - \psi_i) - \tilde{\alpha}_i(\Upsilon_i)^{-1}\dot{\hat{\alpha}}_i\Bigg].\end{aligned}$$

Therefore, by using (20) and selecting the adaptive algorithm for $\hat{\vartheta}_i$ as (29), we have

$$\dot{V} \le \tilde{x}^T Q\tilde{x} + 2\sum_{i=1}^{M}\Bigg(\sum_{j\in N_i} k_{ij}\big(\epsilon\tilde{p}_{ij} + \rho\tilde{v}_{ij}\big)(\delta_i - \psi_i) - \tilde{\alpha}_i(\Upsilon_i)^{-1}\dot{\hat{\alpha}}_i\Bigg). \quad (35)$$

By using (30), and based on Assumption 3, we have

$$\begin{aligned}\sum_{j\in N_i} k_{ij}\big(\epsilon\tilde{p}_{ij} + \rho\tilde{v}_{ij}\big)(\delta_i - \psi_i) &= \Xi_i\big(\delta_i - \hat{\alpha}_i\bar{\delta}_i sgn(\Xi_i)\big) \\ &\le |\Xi_i|\tilde{\alpha}_i\bar{\delta}_i, \quad (36)\end{aligned}$$

where Ξ_i is defined in (8). By using (35) and (36), we have

$$\dot{V} \le \tilde{x}^T Q\tilde{x} + 2\sum_{i=1}^{M}\Bigg(\big|\sum_{j\in N_i} k_{ij}\big(\epsilon\tilde{p}_{ij} + \rho\tilde{v}_{ij}\big)\big|\tilde{\alpha}_i\bar{\delta}_i - \tilde{\alpha}_i(\Upsilon_i)^{-1}\dot{\hat{\alpha}}_i\Bigg).$$

Therefore, by using (31), we have

$$\dot{V} \le \tilde{x}^T Q\tilde{x} \le 0, \quad (37)$$

where Q is given in Lemma 1. Thus, we conclude that $\tilde{x}_i$, $\hat{\vartheta}_i$, and $\hat{\alpha}_i$ are uniformly bounded. By integrating both sides of (37), it can be easily shown that $\tilde{x}_i \in L_2$. Since $\tilde{x}_i \in L_\infty \cap L_2$ and $\dot{\tilde{x}}_i \in L_\infty$, based on Barbalat's Lemma [15], we can conclude that the leader-following formation between agents' outputs is reached asymptotically, i.e., $\tilde{x}_i \to 0$ as $t \to \infty$. □

The aforementioned design and analysis procedure is summarized in the following theorem:

Theorem 2: Suppose that the bounding Assumption 3 holds. Then, if a fault is detected, the adaptive fault-tolerant law (28), the weight parameter adaptive law (29), and the bounding parameter adaptive laws (30) and (31) guarantee that all the signals and parameter estimates are uniformly bounded, i.e., x_i, $\hat{\vartheta}_i$, and $\hat{\alpha}_i$ are bounded, and leader-following formation is achieved asymptotically with a time-varying reference state, i.e. $p_i(t) - p_0(t) \to \bar{p}_i$ and $v_i(t) \to v_0(t)$ as $t \to \infty$.

B. Accommodation after Fault Isolation

In this section, we describe and analyze the adaptive fault-tolerant controller employed after fault isolation. Let us now assume that the isolation procedure described in Section *V.B* provides the information that fault s has been isolated at time T_{isol}. Then, for $t \ge T_{isol}$, using (5) the dynamics of the system takes on the following form:

$$\dot{x}_i = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}x_i + \begin{bmatrix}0\\1\end{bmatrix}\big(\phi_i(x_i) + u_i + \eta_i + \theta_i^{sT}g_i^s(x_i)\big). \quad (38)$$

The control objective is to have the output x_i, $i = 1,\cdots,M$, track the time-varying output of the leader and form a formation around the leader.

After the isolation of the fault type s, i.e., $t \ge T_{isol}$, the following adaptive fault-tolerant controller is adopted:

$$u_i = -\phi_i(x_i) - \sum_{j\in N_i} k_{ij}\big(\alpha\tilde{p}_{ij} + \gamma\tilde{v}_{ij}\big) - \hat{\theta}_i^T g_i^s(x_i) - (\bar{\eta}_i+\kappa)sgn\Big(\Xi_i\Big) \quad (39)$$

$$\dot{\hat{\theta}}_i \;=\; \Gamma_i \sum_{j\in N_i} k_{ij}\big(\epsilon\tilde{p}_{ij}+\rho\tilde{v}_{ij}\big)g_i^s(x_i)\,, \tag{40}$$

where $\hat{\theta}_i$ is an estimation of the unknown fault parameter vector, and Γ_i is a symmetric positive definite learning rate matrix. Then, we have the following:

Theorem 3: Assume that fault s occurs at time T_i and that it is isolated at time T_{isol}. Then, the fault-tolerant controller (39) and fault parameter adaptive law (40) guarantee that all states are bounded, and the leader-following formation is achieved asymptotically with a time-varying reference state, i.e. $p_i(t)-p_0(t)\to\bar{p}_i$ and $v_i(t)\to v_0(t)$ as $t\to\infty$;

Proof: Based on (38) and (39), the closed-loop system dynamics are given by

$$\begin{aligned}\dot{p}_i &= v_i\\ \dot{v}_i &= -\sum_{j\in N_i} k_{ij}\big(\alpha\tilde{p}_{ij}+\gamma\tilde{v}_{ij}\big)+\eta_i-(\bar{\eta}_i+\kappa)sgn\Big(\Xi_i\Big)\\ &\quad +\tilde{\theta}_i^T\; g_i^s(x_i)\,.\end{aligned}$$

We can represent the collective closed-loop dynamics as

$$\dot{\tilde{x}} = -A\tilde{x}+\begin{bmatrix}\mathbf{0}_M\\ \zeta-\bar{\zeta}-\mathbf{1}_M\dot{v}_0+\tilde{f}^s\end{bmatrix} \tag{41}$$

where A is given in Lemma 1, and $\tilde{x}=[\tilde{p}^T\tilde{v}^T]^T$ is defined in a similar way as in (13), the terms ζ and $\bar{\zeta}$ are defined in (14) and (15), and $\tilde{f}^s\in\Re^M$ is defined as

$$\tilde{f}^s \;\triangleq\; \big[\tilde{f}_1^s \quad \cdots \quad \tilde{f}_M^s\big]^T \tag{42}$$

where $\tilde{f}_i^s\triangleq\tilde{\theta}_i^Tg_i^s$, and $\tilde{\theta}_i=\theta_i-\hat{\theta}_i$ is the parameter estimation error corresponding to the ith agent. We consider the following Lyapunov function candidate:

$$V \;=\; \tilde{x}^TP\tilde{x}+\tilde{\theta}^T(\Gamma)^{-1}\tilde{\theta}\,, \tag{43}$$

where P is defined in Lemma 1, $\tilde{\theta}=\big[\tilde{\theta}_1^T \quad \cdots \quad \tilde{\theta}_M^T\big]^T$ is the collective parameter estimation errors, and $\Gamma=\mathrm{diag}\{\Gamma_1,\cdots,\Gamma_M\}$ is a positive definite learning rate matrix. Then, using (18) and a similar reasoning logic for (42), the time derivative of the Lyapunov function (43) along the solution of (41) is given by

$$\begin{aligned}\dot{V} &= \tilde{x}^TQ\tilde{x}+2\sum_{i=1}^{M}\Bigg[\sum_{j\in N_i}k_{ij}\big(\epsilon\tilde{p}_{ij}+\rho\tilde{v}_{ij}\big)(\eta_i-\dot{v}_0)\\ &\quad -\sum_{j\in N_i}k_{ij}\big(\epsilon\tilde{p}_{ij}+\rho\tilde{v}_{ij}\big)(\bar{\eta}_i+\kappa)sgn\big(\Xi_i\big)\\ &\quad +\tilde{\theta}_i^T\Bigg(\sum_{j\in N_i}k_{ij}\big(\epsilon\tilde{p}_{ij}+\rho\tilde{v}_{ij}\big)g_i^s-(\Gamma_i)^{-1}\dot{\hat{\theta}}_i\Bigg)\Bigg]\,,\end{aligned}$$

where Q is defined in Lemma 1. Therefore, using (20) and choosing the adaptive law as (40), we have $\dot{V}\le\tilde{x}^TQ\tilde{x}$. Then, the proof can be concluded by using a similar reasoning logic as reported in the proof of Theorem 2. □

VII. Simulation Results

In this section, a simulation example of a networked multi-agent system consisting of 5 agents is considered to illustrate the effectiveness of the distributed fault-tolerant control method. The dynamics of each agent is given by

$$\dot{x}_i=\begin{bmatrix}0&1\\0&0\end{bmatrix}x_i+\begin{bmatrix}0\\1\end{bmatrix}\big(\phi_i(x_i)+u_i+\eta_i+\beta_i(t-T_i)f_i(x_i)\big)\,,$$

where, for $i=1,\cdots,5$, $x_i=[p_i\,v_i]^T$ is the state of the ith agent, and u_i is the input of ith agent. The nominal term in the dynamics of each agent is $\phi_i(x_i)=v_i^2$.

The unknown modeling uncertainty in the local dynamics of the agents are assumed to be a sinusoidal signal $\eta_i=0.5sin(t)$ which is assumed to be bounded by $\bar{\eta}_i=0.6$. The objective is have each agent follow a virtual leader x_0 given by $\dot{x}_0=\begin{bmatrix}v_0\\0.5\,sin(t)\end{bmatrix}$ with zero initial condition and also keep a formation around the leader with $\bar{p}_1=-4$, $\bar{p}_2=-2$, $\bar{p}_3=0$, $\bar{p}_4=2$, $\bar{p}_5=4$.

The Laplacian matrix of the interconnection graph of agents is given as

$$\mathscr{L}=\begin{bmatrix}2&-1&0&0&-1\\-1&2&0&0&-1\\0&0&2&-1&-1\\0&0&-1&2&-1\\-1&-1&-1&-1&4\end{bmatrix}.$$

The virtual leader only communicates with the second agent (i.e., $k_{20}=1$). The matrix $\Psi=\mathscr{L}+\mathrm{diag}\{0,1,0,0,0\}$ has the minimum eigenvalue of $\mu_{min}=0.13$, and $\alpha=3$, $\gamma=30$, $\epsilon=0.1$, and $\rho=1$ satisfy the conditions given in Lemma 1. The fault class under consideration is defined as

1) A process fault function $f_i^1=\theta_i^1g_i^1$, where $g_i^1=p_i$ is considered as the first fault type, and the magnitude of this fault is considered as $\theta_i^1\in[0\;\;1]$.
2) A process fault function $f_i^2=\theta_i^2g_i^2$, where $g_i^2=p_i\,sin(p_i)$ is considered as the second fault type, and the magnitude of the fault is considered as $\theta_i^2\in[0\;\;1]$.

The estimator gain for the fault detection estimator is chosen as $\lambda_i^0=2$. For fault isolation estimator, $\lambda_i=2$ has been chosen.

A radial basis function (RBF) neural network is used for approximation of the fault after its detection and before its isolation. The RBF network considered in this paper consists of 21 neurons with 21 adjustable gain parameters. The center of radial basis functions are equally distributed on interval $[-10,10]$ with a variance of 0.5. The initial parameter vector of the neural network is set to zero. We set the learning rate as $\Gamma_i=5$ and consider an unknown constant bound on the network approximation error, i.e., $\bar{\delta}_i=1$. The learning rate is chosen as $\Upsilon_i=0.1$.

After fault isolation, the neural-network-based adaptive fault-tolerant controller is reconfigured to accommodate the specific fault that has been isolated. We set the learning rate $\Gamma_i=0.2$ with a zero initial condition (see (40)).

Figure 2 and Figure 3 show the fault detection and isolation results when the first process fault class (i.e., $f_1^1=\theta_1^1g_1^1$) with a magnitude of 0.8 occurs to agent 1 at $T_i=30$ second. As can be seen from Figure 2, the residual corresponding to the output generated by the local FDE designed for agent 1 exceeds its threshold immediately after fault occurrence. Therefore, the process fault in agent 1 is timely detected. It can be seen in Figure 3 that the residual corresponding to the FIE associated with the first fault type always remains below the threshold, while the residual corresponding to the FIE associated with the second fault type exceeds the threshold at approximately $t = 30.6$ second. Thus, based on the fault isolation decision scheme described in Section *V.B* the occurrence of fault type 1 can be concluded.

Regarding the performance of the adaptive fault-tolerant controllers, as can be seen from Figure 4, the leader-following formation is achieved using the proposed adaptive FTCs even after fault occurrence, while the agents cannot achieve the leader-following formation and become unstable without the FTC controllers (see Figure 5).

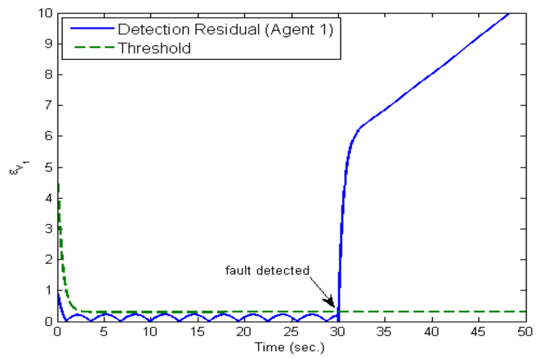

Fig. 2: The case of a process fault in agent 1: fault detection residuals (solid and blue line) and the corresponding threshold (dashed and green line) generated by the local FDE

VIII. Conclusion

In this paper, we investigate the problem of a distributed FDI and FTC for a class of multi-agent uncertain second-order systems. By using on-line diagnostic information, adaptive FTC controllers are developed to achieve the leader-following formation with a time-varying leader in the presence of faults. The closed-loop stability and leader-following formation properties are rigorously established under different modes of the FTC system. The extensions to systems with more general structure is an interesting topic for future research.

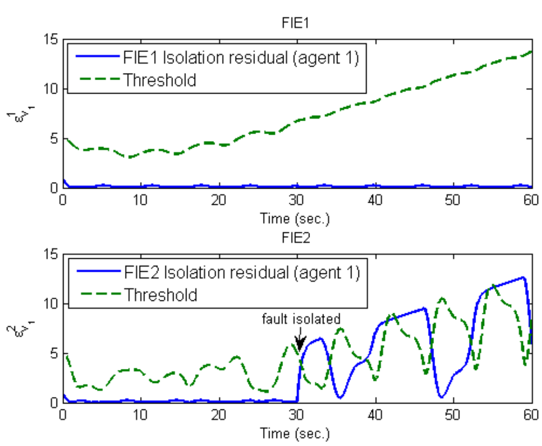

Fig. 3: The case of a process fault in agent 1: the fault isolation residuals (solid and blue line) and the corresponding threshold (dashed and green line) generated by the two FIEs of agent 1

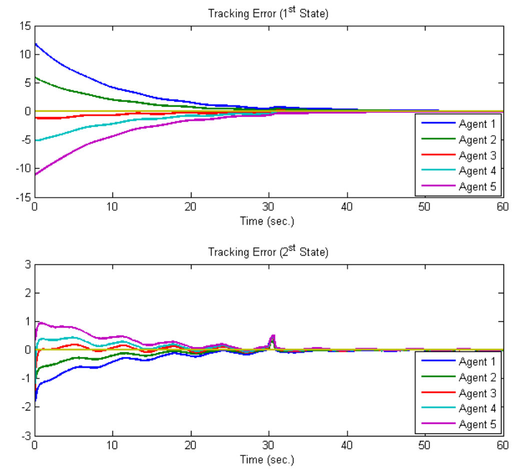

Fig. 4: The tracking errors in the case of a process fault in agent 1: with adaptive fault-tolerant controllers

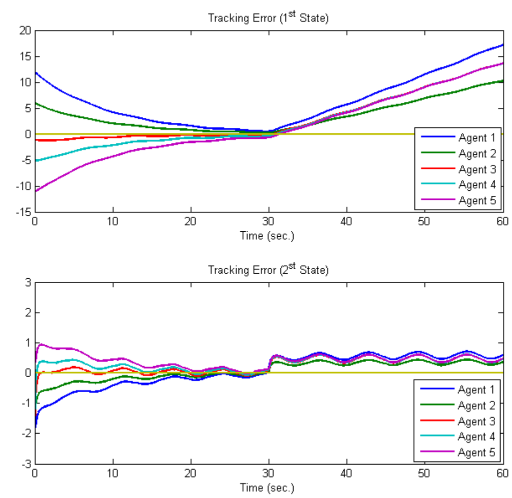

Fig. 5: The tracking errors in the case of a process fault in agent 1: without adaptive fault-tolerant controllers

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