Control Flow Analysis

Adapted from Lectures by
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Outline

- Control-Flow Analysis
- Dominators
- Graph Traversal
- Reducible Graphs
- Interval Analysis

Constant Propagation

```
int sumcalc(int a, int b, int N)
{
    int i;
    int x, y;
    x = 0;
    y = 0;
    for(i = 0; i <= N; i++) {
       x = x + (4*a/b)*i + (i+1)*(i+1);
       x = x + b*y;
    }
    return x;
}
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       x = x + b*0;
    }
    return x;
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```

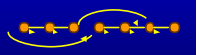
Implementing constant propagation

- Find an RHS expression which is a constant
- Replace the use of the LHS variable with the RHS constant given that:
  - All paths to the use of LHS variable passes the assignment of the LHS variable with the constant
  - There are no intervening definition of the LHS variable
- Need to know the "control-flow" of the program

Representing the control flow of the program

- Most instructions
  - execute the next instruction
    - straight line control-flow

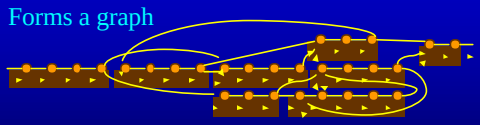
- Jump instructions
  - execute from different location
    - jump in control-flow



- Branch instructions
  - execute either the next instruction or from a different location
    - fork in the control-flow

Representing the control flow of the program

- Forms a graph



- Observation
  - a very large graph
  - lot of straight-line connections
  - simplify the graph by grouping some instructions

Basic Blocks

- A basic block is a maximal sequence of instructions such that
  - Only the first instruction can be reached from outside the basic block
  - All the instructions are executed consequetively if the first instruction is executed
    - No branch or jump instructions in the basic block, except the last instruction
    - No labels within the basic block, except before the first instruction

Control Flow Graph (CFG)

- Control-Flow Graph G = <N, E>
- Nodes(N): Basic Blocks
- Edges(E): (x,y) ∈ E iff first instruction in the basic block y follows the last instruction in the basic block x
  - First instruction in y is the target of branch or jump instruction (last instruction) in the basic block x
  - first instruction of y is next after the last instruction of x in memory and the last instruction of x is not a jump instruction

Control Flow Graph (CFG)

- Block with the first instruction of the procedure is the entry node (block with the procedure label)
- The blocks with the return instruction (jsr) are the exit nodes.
  - Can make a single exit node by adding a special node

Why do we perform Control-flow Analysis

- Loops are important to optimize
  - Programs spend a lot of times in loops and recursive cycles
  - Many special optimizations can be done on loops
- Programmers organize code using structured control-flow (if-then-else, for-loops etc.)
  - Optimizer can exploit this, once they are discovered

Challenges in Control-Flow Analysis

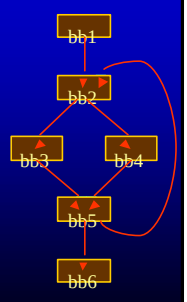
- Unstructured Control Flow
  - Use of goto's by the programmer
  - Only way to build certain control structures

```
L1: x = 0
    if (y > 0) goto L3
L2: if (y < 0) goto L1
L3: y = y + z
    goto L2
```

- Obscured control flow
  - method invocations
  - procedure variables
  - higher-order functions
  - jump tables

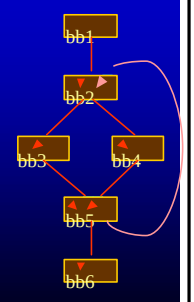
```
Myobject->run()
```


Identifying Recursive Structures Loops



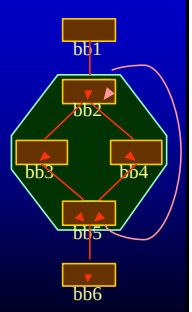
Identifying Recursive Structures Loops

- Identify back edges



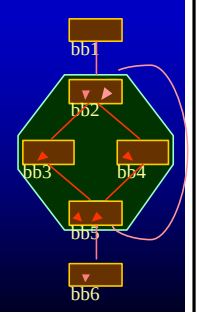
Identifying Recursive Structures Loops

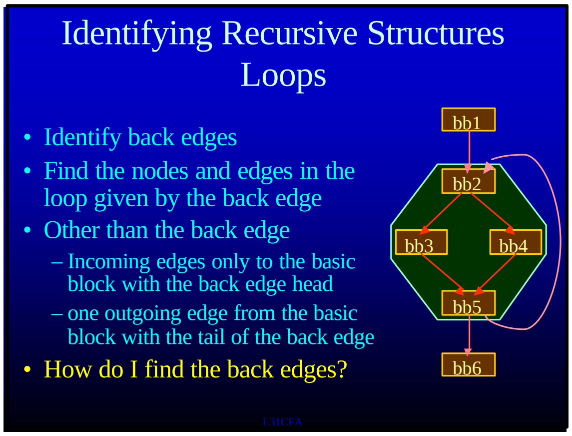
- Identify back edges
- Find the nodes and edges in the loop given by the back edge



Identifying Recursive Structures Loops

- Identify back edges
- Find the nodes and edges in the loop given by the back edge
- Other than the back edge
  - Incoming edges only to the basic block with the back edge head
  - one outgoing edge from the basic block with the tail of the back edge





Identifying Recursive Structures Loops

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- Find the nodes and edges in the loop given by the back edge
- Other than the back edge
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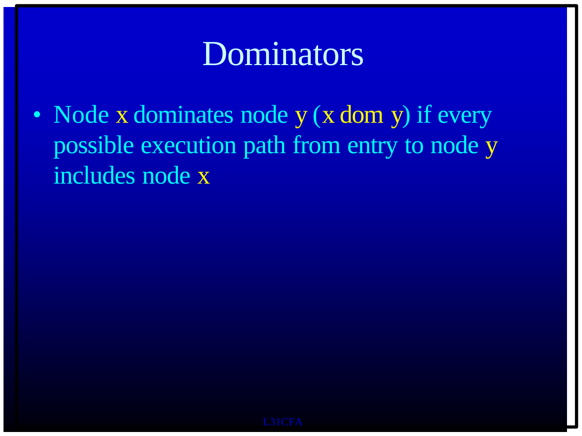
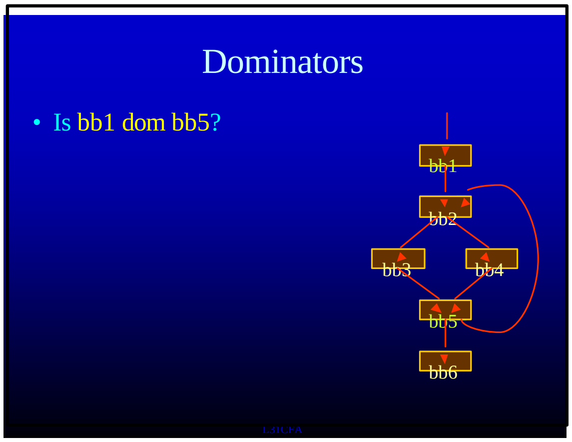
- How do I find the back edges?



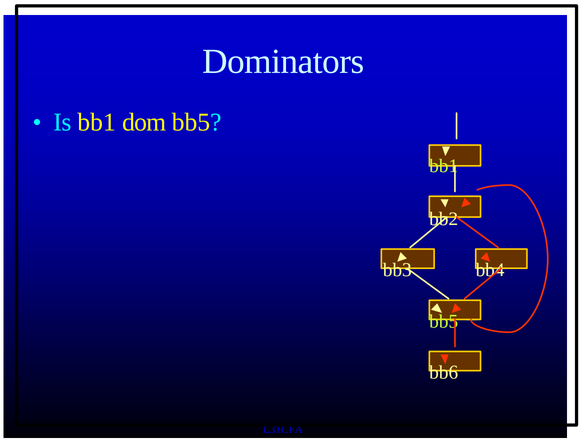
Dominators

- Node x dominates node y (x dom y) if every possible execution path from entry to node y includes node x



Dominators

- Is bb1 dom bb5?



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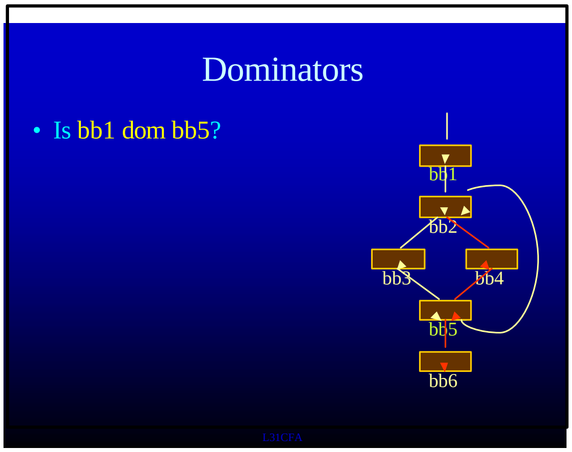

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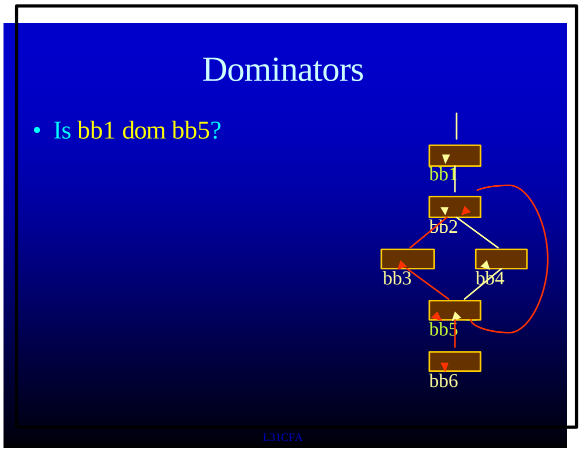

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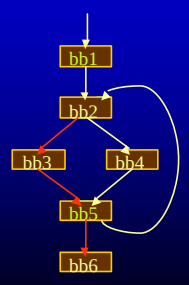
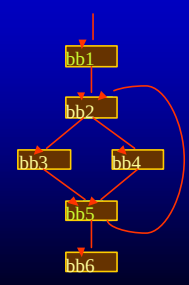


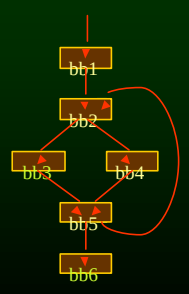
Dominators

- Is bb1 dom bb5? Yes!



QUESTION

- Is bb1 dom bb5? Yes!
- Is bb3 dom bb6?



QUESTION

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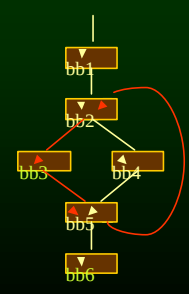
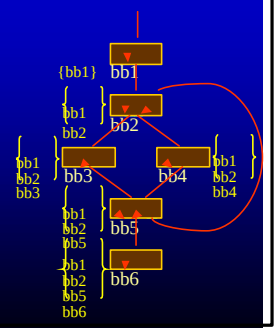
- Is bb3 dom bb6? No!

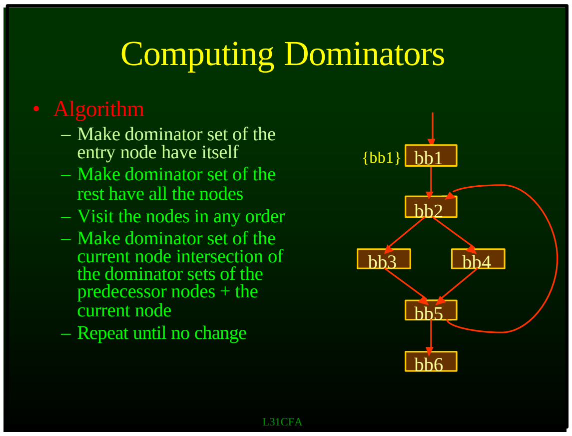


Computing Dominators

- a dom b iff
  - a = b or
  - a is the unique immediate predecessor of b or
  - a is a dominator of all immediate predecessor of b
- Algorithm
  - Make dominator set of the entry node have itself
  - Make dominator set of the rest have all the nodes
  - Visit the nodes in any order
  - Make dominator set of the current node intersection of the dominator sets of the predecessor nodes + the current node
  - Repeat until no change

Dominators (Desired Solution)





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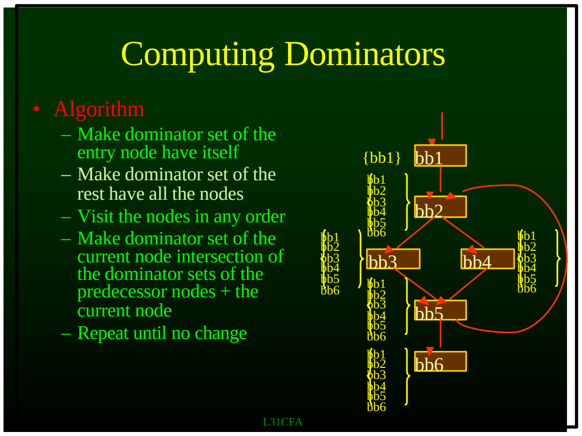

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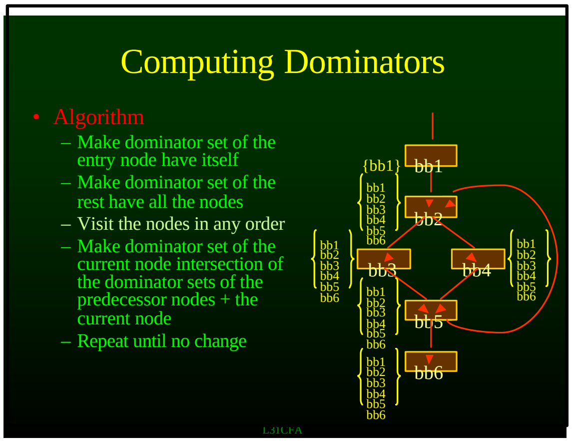

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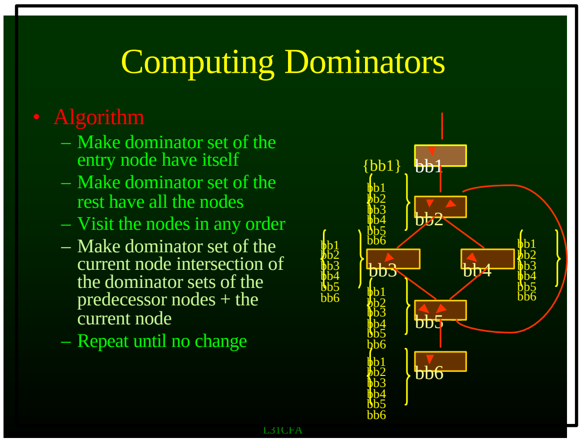

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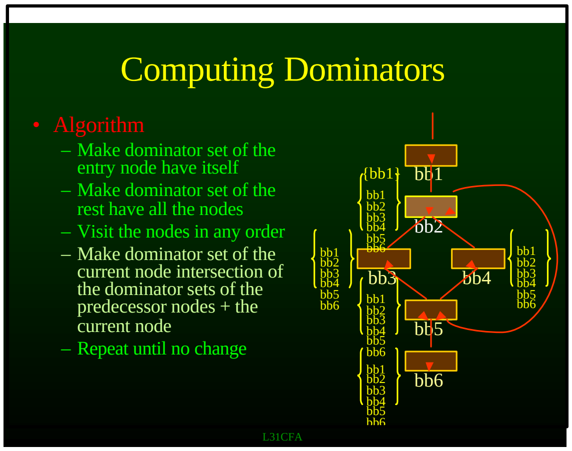

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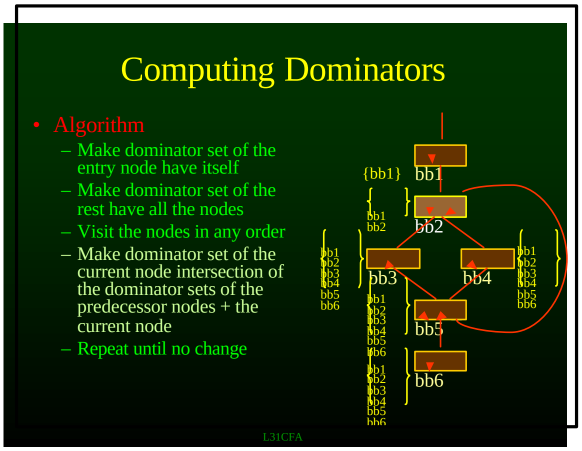

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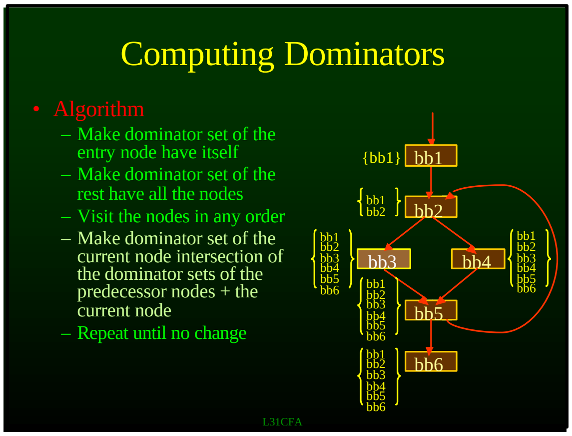

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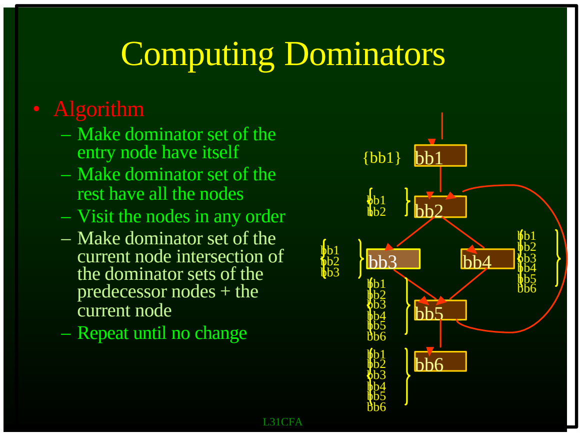

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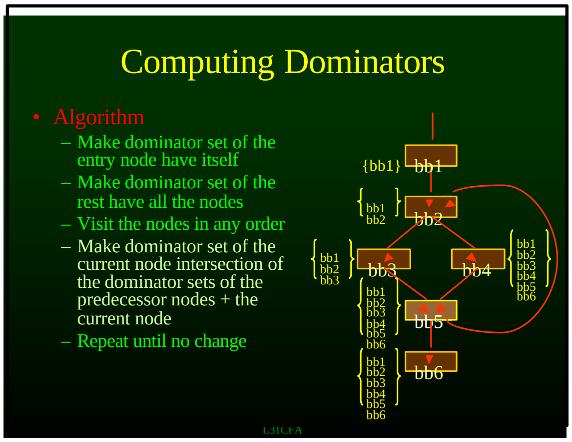

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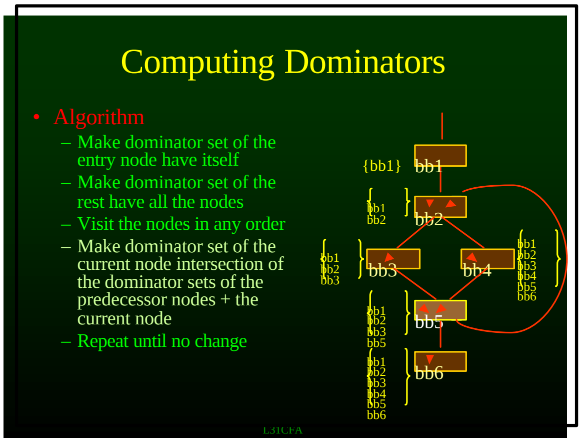

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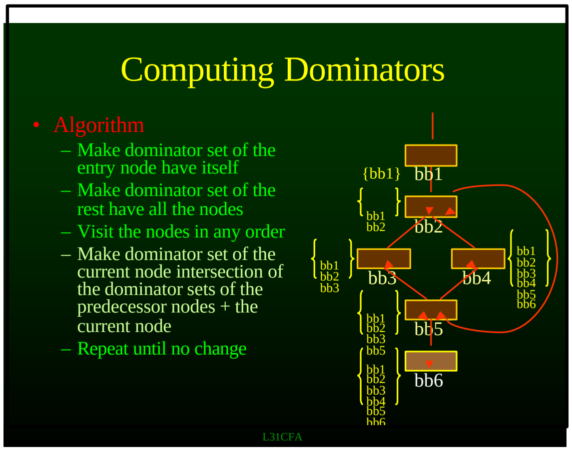

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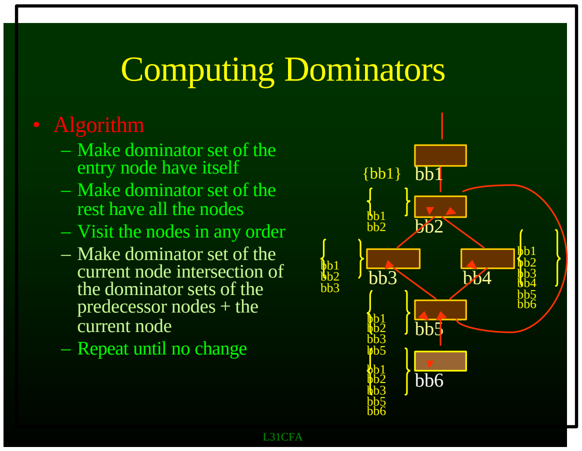

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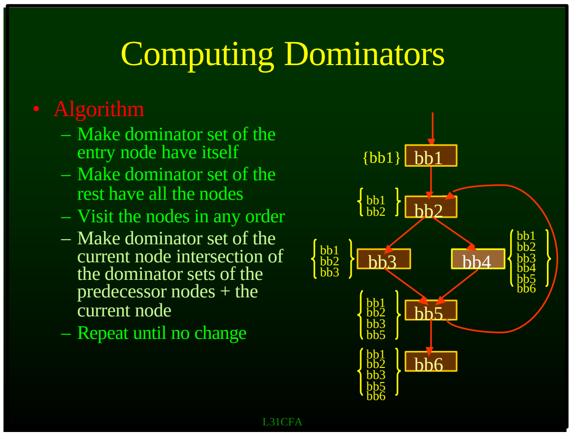

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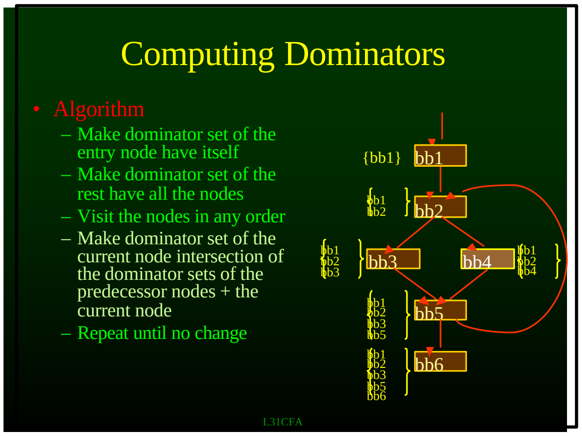

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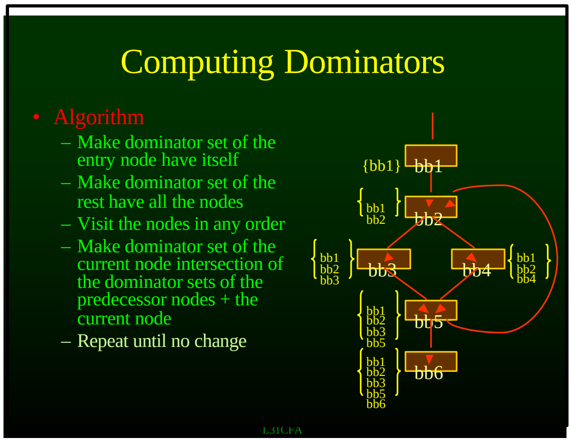

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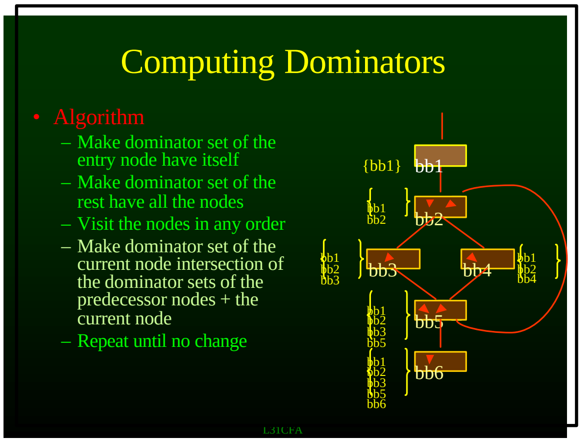

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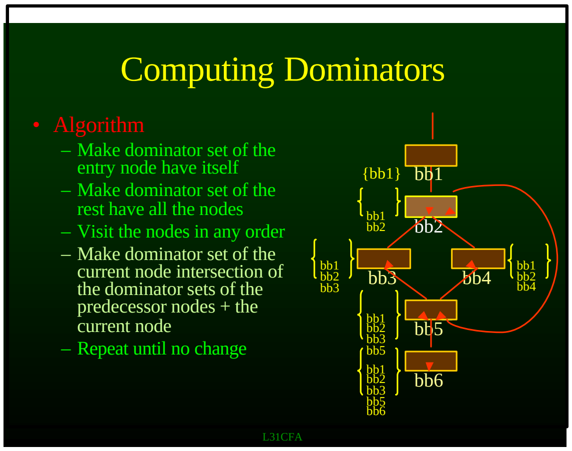

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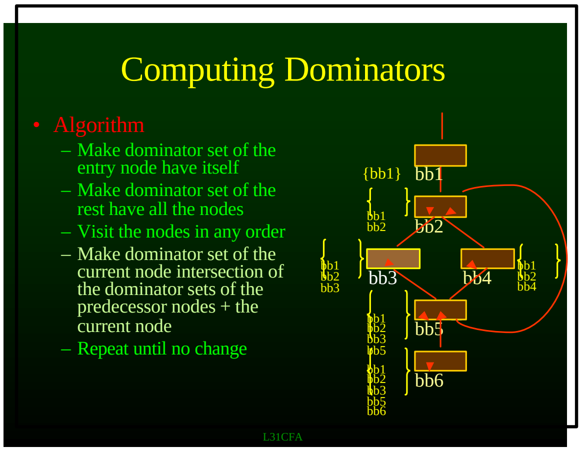

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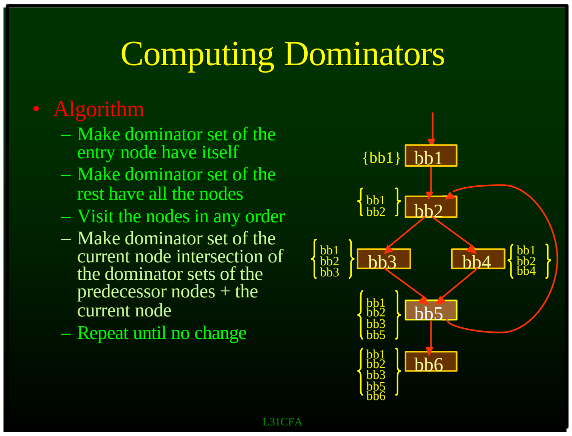

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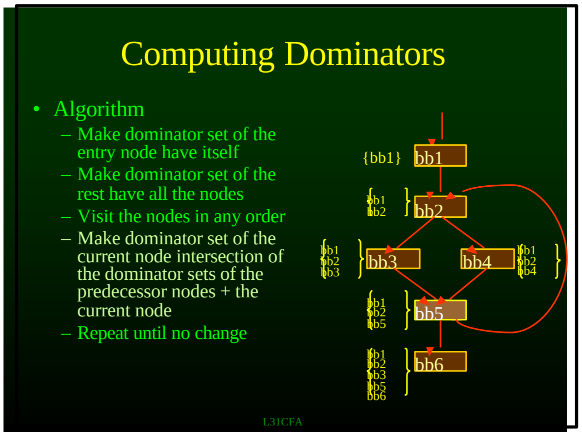

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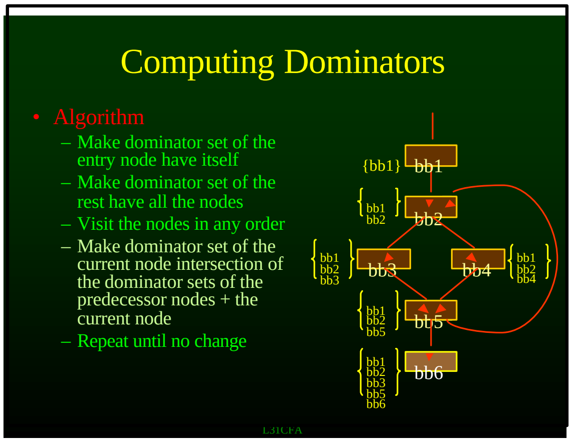

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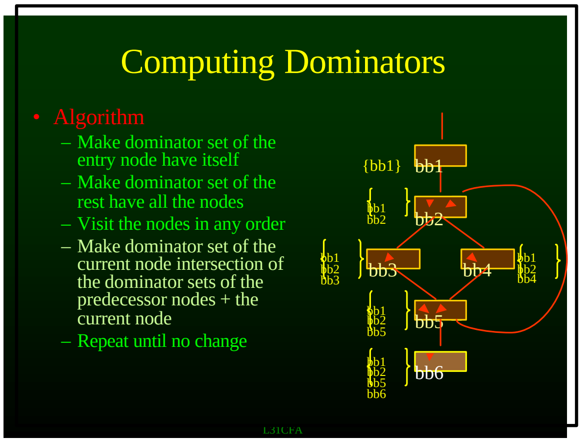

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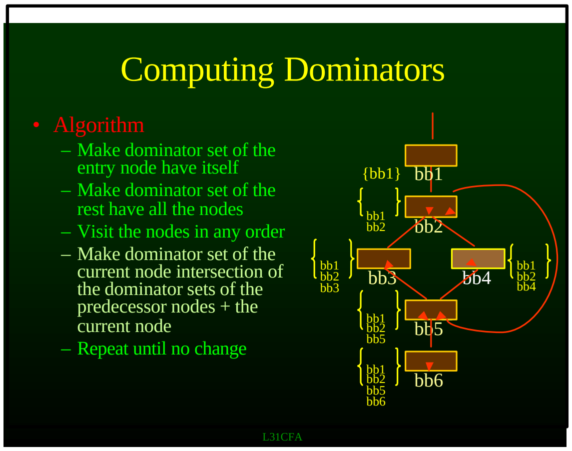

Computing Dominators

- Algorithm
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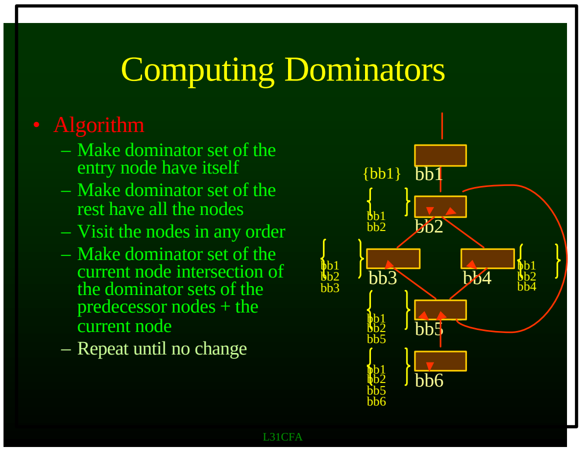

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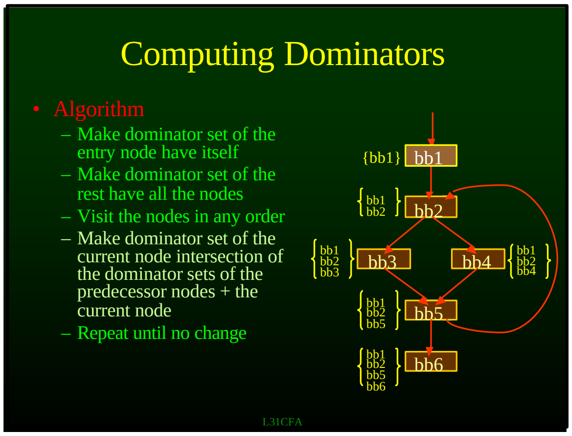

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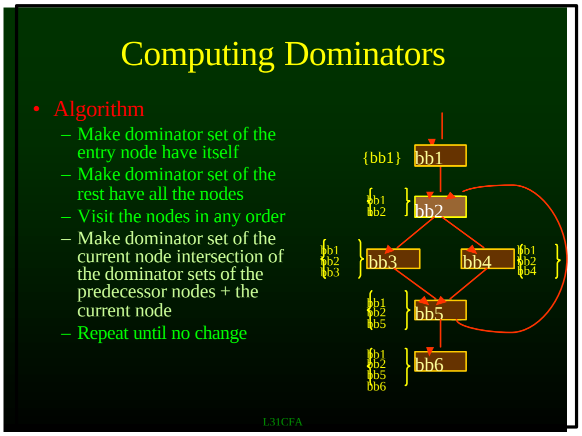

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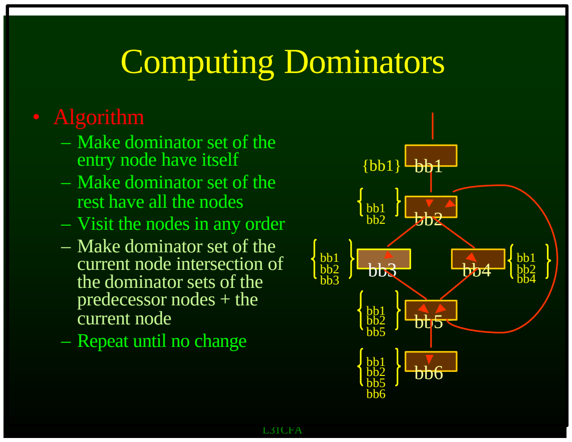

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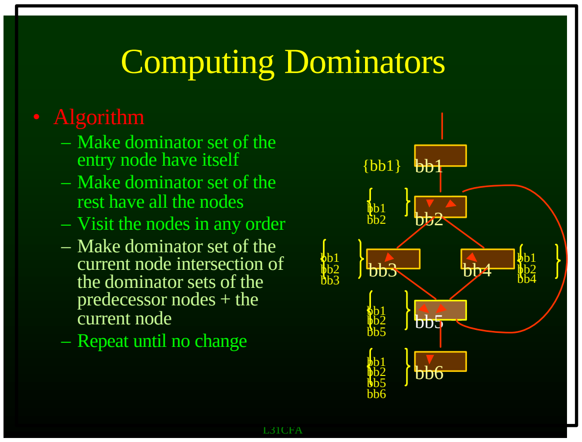

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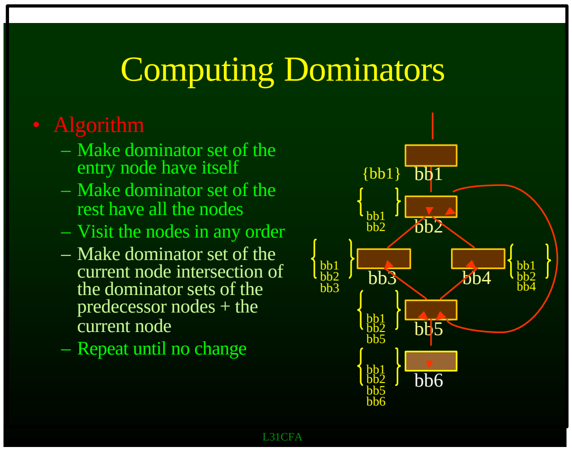

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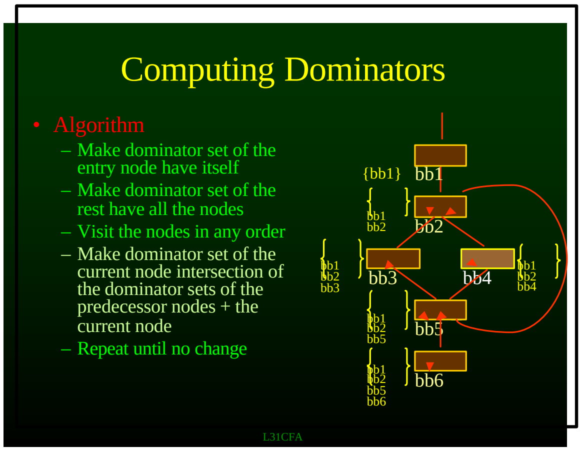

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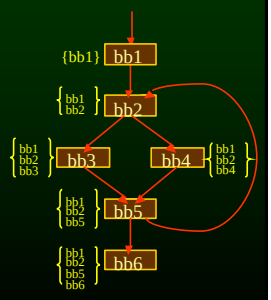
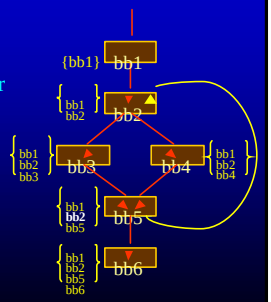


Computing Dominators

- What we just witnessed was an iterative data-flow analysis algorithm in action
  - Initialize all the nodes to a given value
  - Visit nodes in some order
  - Calculate the node's value
  - Repeat until no value changes

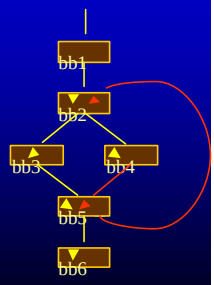
What is a back edge?

- An edge (x, y) $\in$ E is a back edge iff y dom x
  - is node y in the dominator set of node x



Traversing the CFG

- Depth-First Traversal
  - Visit all the descendants of a node before visiting any siblings
- Depth-first spanning tree
  - a set of edges corresponding to a depth-first visitation of CFG

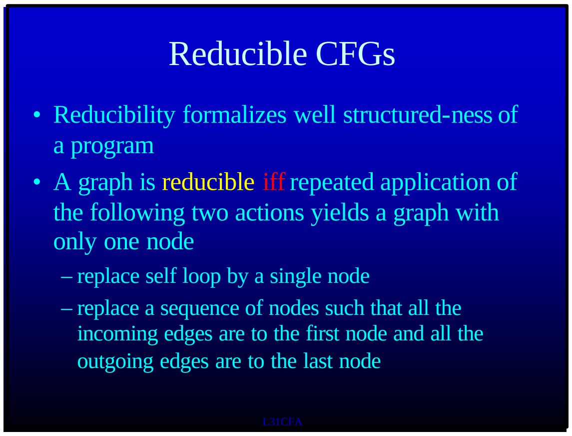


Preorder and Postorder

- In preorder traversal, each node is processed before its descendants in the depth-first tree
- In postorder traversal, each node is processed after its descendants in the depth-first tree

Outline

- Control-Flow Analysis
- Dominators
- Graph Traversal
- Reducible Graphs
- Interval Analysis



Reducible CFGs

- Reducibility formalizes well structured-ness of a program
- A graph is reducible iff repeated application of the following two actions yields a graph with only one node
  - – replace self loop by a single node
  - – replace a sequence of nodes such that all the incoming edges are to the first node and all the outgoing edges are to the last node

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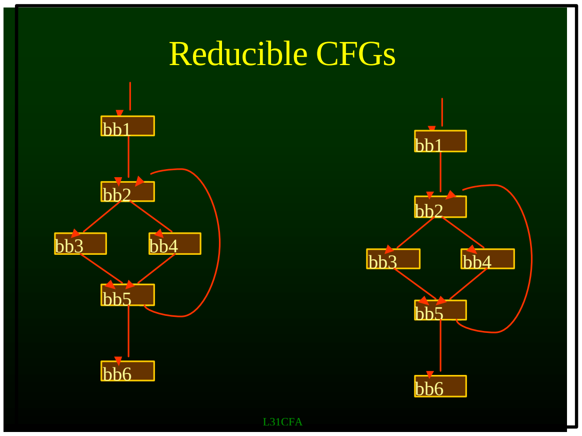

Reducible CFGs

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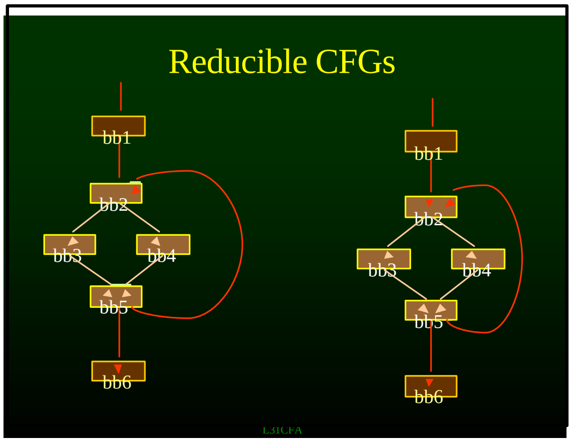

Reducible CFGs

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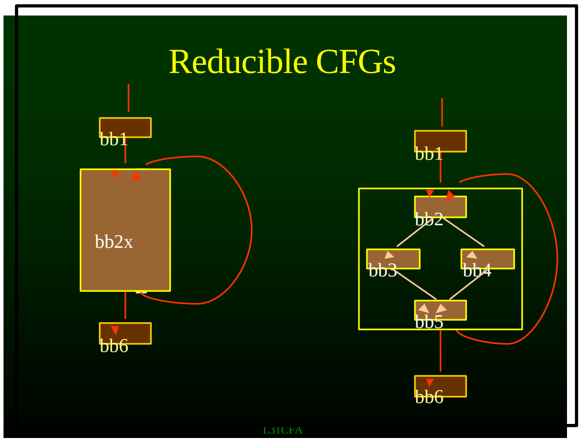

Reducible CFGs

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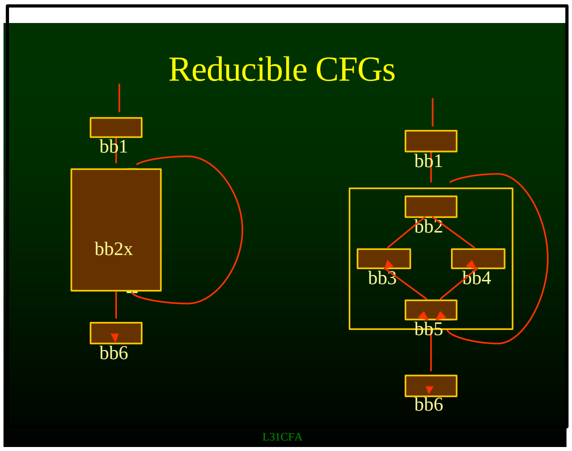

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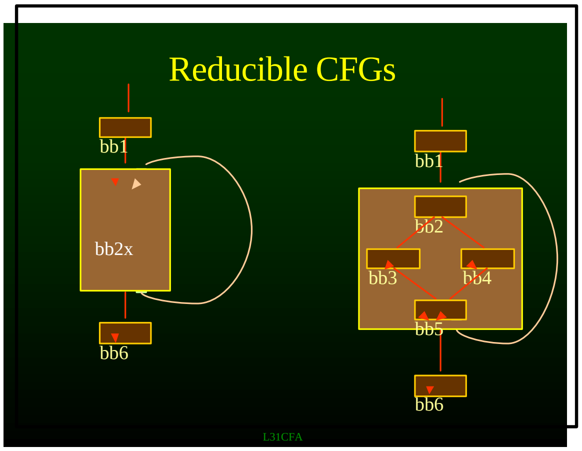

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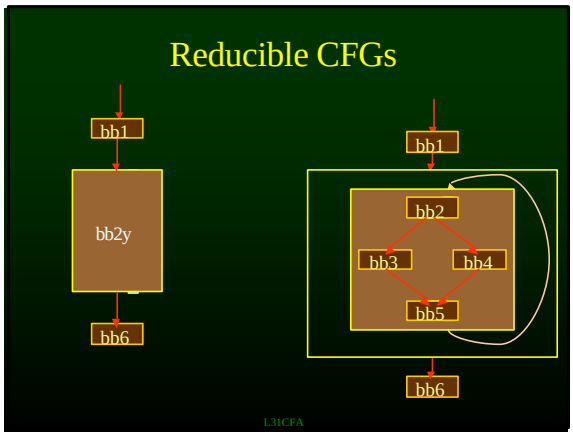

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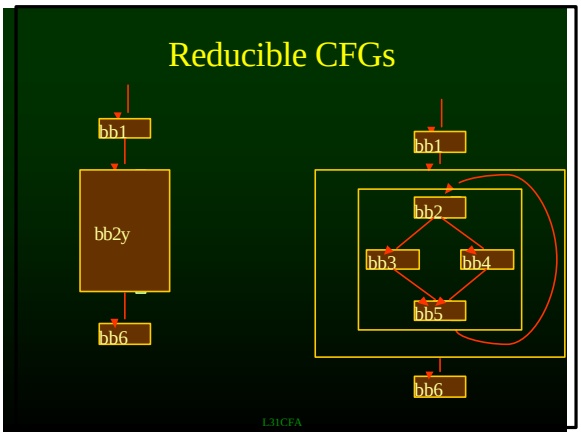

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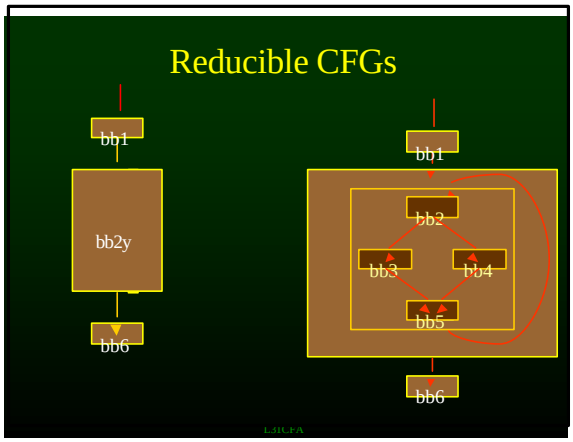

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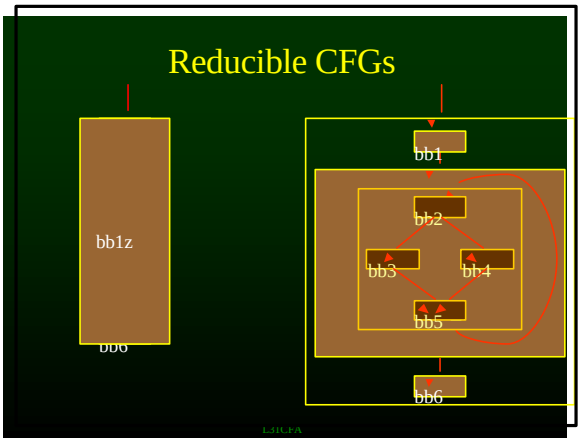

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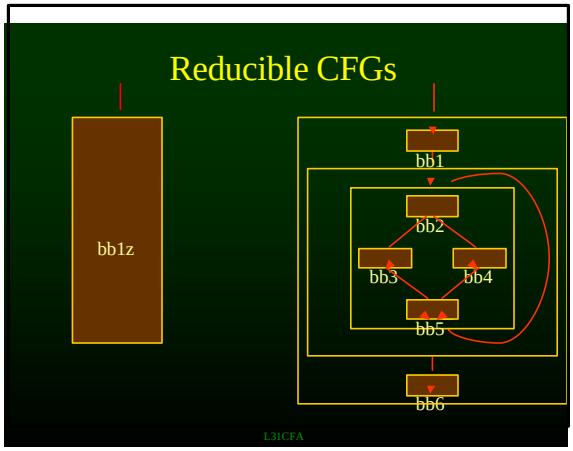

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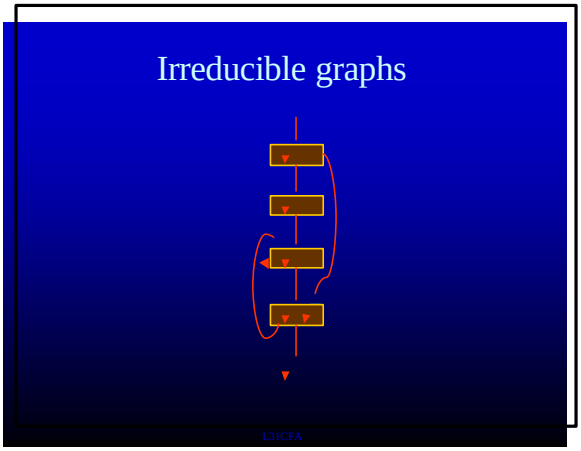
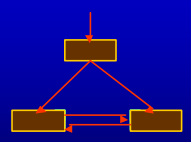


Irreducible graphs

Irreducible graphs



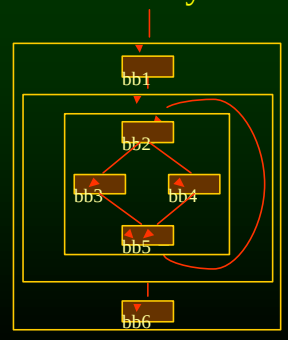
Approaches of Control-Flow Analysis

- Iterative Analysis
  - Use a CFG
  - Propagate values
  - Iterate until no change
- Interval Based Analysis
  - Use a reducible CFG
  - Calculate in hierarchical graphs
  - No iterations (faster)

Interval Based Analysis

- If a node does not include a graph:
  - calculate value
- If a node includes a graph
  - Calculate values of the nodes in that graph
  - Propagate values (no back edges ⇒ no iteration)
  - Use entry (or exit) value as the value of the enclosing node

Interval Analysis



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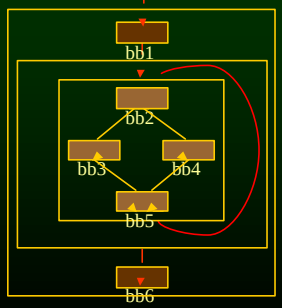

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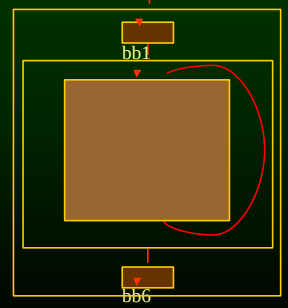

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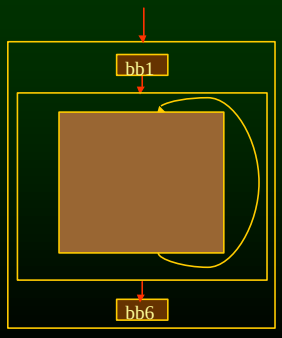

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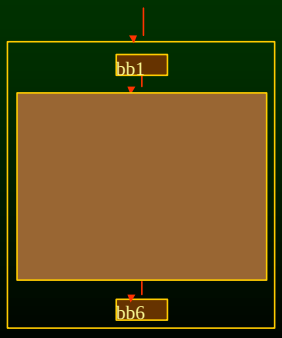

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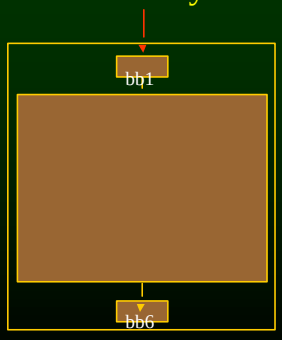

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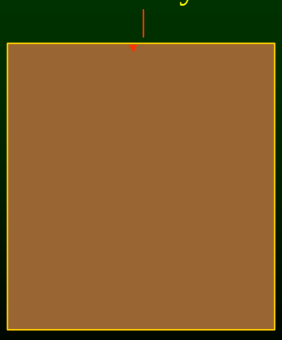

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