Type Checking in Cool I

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Outline

- Type concepts in COOL
- Notation for type rules
  - Logical rules of inference
- COOL type rules
- General properties of type systems

Cool Types

- The types are:
  - Class Names
  - SELF_TYPE
- The user declares types for identifiers
- The compiler infers types for expressions
- *Type Checking* is the process of verifying fully typed programs
- *Type Inference* is the process of filling in missing type information
  - The two are different, but are often used interchangeably

Rules of Inference

- We have seen two examples of formal notation specifying parts of a compiler
  - Regular expressions
  - Context-free grammars
- The appropriate formalism for type checking is logical rules of inference

  If E_1 and E_2 have certain types, then E_3 has a certain type

From English to an Inference Rule

- The notation is easy to read with practice
- Start with a simplified system and gradually add features
- Building blocks
  - Symbol $\wedge$ is "and"
  - Symbol $\Rightarrow$ is "if-then"
  - x:T is "x has type T"

From English to an Inference Rule (2)

If e_1 has type Int and e_2 has type Int,
then $e_1 + e_2$ has type Int

(e_1 has type Int $\wedge$ e_2 has type Int) $\Rightarrow$
$e_1 + e_2$ has type Int

(e_1: Int $\wedge$ e_2: Int) $\Rightarrow$ $e_1 + e_2$: Int

From English to an Inference Rule (3)

The statement

$$(e_1\text{: Int} \wedge e_2\text{: Int}) \Rightarrow e_1 + e_2\text{: Int}$$

is a special case of

$$\text{Hypothesis}_1 \wedge \ldots \wedge \text{Hypothesis}_n \Rightarrow \text{Conclusion}$$

This is an inference rule

Notation for Inference Rules

- By tradition inference rules are written

$$\frac{\square\ \text{Hypothesis}_1 \cdots \square\text{Hypothesis}_n}{\square\ \text{Conclusion}}$$

- Cool type rules have hypotheses and conclusions

$$\square\ e\text{:}T$$

- $\square$ means "it is provable that . . ."

Two Rules

$$\frac{i \text{ is an integer}}{\square\ i\text{: int}} \quad [\text{Int}]$$

$$\frac{\square e_1 : \text{ int} \qquad \square e_2 : \text{ int}}{\square\ e_1 + e_2 : \text{ int}} \quad [\text{Add}]$$

Two Rules (Cont.)

- These rules give templates describing how to type integers and + expressions
- By filling in the templates, we can produce complete typings for expressions

$$\frac{\dfrac{1 \text{ is an integer}}{\square\ 1\text{: Int}} \qquad \dfrac{2 \text{ is an integer}}{\square 2\text{: Int}}}{\square\ 1+2\text{: Int}}$$

Soundness

- A type system is *sound* if
  - Whenever $\square$e:T
  - Then e evaluates to a value of type T
- We only want sound rules.

$$\frac{i \text{ is an integer}}{\square i\text{:Object}}$$

Type Checking Proofs

- Type checking proves facts e: T
  - Proof is on the structure of the AST
  - Proof has the shape of the AST
  - One type rule is used for each AST node
- In the type rule used for a node e:
  - Hypotheses are the proofs of types of e's sub-expressions
  - Conclusion is the type of e
- Types are computed in a *bottom-up* pass over the AST

Rules for Constants

$$\frac{}{\square\ \text{false: Bool}} \quad [\text{Bool}]$$

$$\frac{\text{s is a string constant}}{\square\ \text{s: String}} \quad [\text{String}]$$

Rule for New

new T produces an object of type T

- Ignore SELF_TYPE for now . . .

$$\frac{}{\square\ \text{new T: T}} \quad [\text{New}]$$

Two More Rules

$$\frac{\square\ \text{e: Bool}}{\square\ \neg\text{e: Bool}} \quad [\text{Not}]$$

$$\frac{\square\ e_1\text{: Bool} \qquad \square\ e_2\text{: T}}{\square\ \text{while } e_1 \text{ loop } e_2 \text{ pool: Object}} \quad [\text{Loop}]$$

A Problem

- What is the type of a variable reference?

$$\frac{\text{x is an identifier}}{\square\ \text{x: ?}} \quad [\text{Var}]$$

- The local, structural rule does not carry enough information to give x a type.

A Solution

- Put more information in the rules!
- A *type environment* gives types for *free* variables
  - A type environment is a function from ObjectIdentifiers to Types
  - A variable is free in an expression if it is not defined within the expression

Type Environments

Let O be a function from ObjectIdentifiers to Types

The sentence

$$O \square e\text{: T}$$

is read: Under the assumption that variables have the types given by O, it is provable that the expression e has the type T

Modified Rules

The type environment is added to the earlier rules:

$$\frac{i \text{ is an integer}}{O \vdash i : int} \quad [Int]$$

$$\frac{O \vdash e_1 : int \qquad O \vdash e_2 : int}{O \vdash e_1 + e_2 : int} \quad [Add]$$

New Rules

And we can write new rules:

$$\frac{O(x) = T}{O \vdash x : T} \quad [Var]$$

Let

$$\frac{O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x{:}T_0 \text{ in } e_1 : T_1} \quad [Let\text{-}No\text{-}Init]$$

$O[T/y]$ means O modified to return T on argument y

Note that the let rule enforces variable scope

Notes

- The type environment gives types to the free identifiers in the current scope
- The type environment is *passed down* the AST from the root towards the leaves
- Types are *computed up* the AST from the leaves towards the root

Let with Initialization

Now consider let with initialization:

$$\frac{O \vdash e_0 : T_0 \qquad O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x{:}T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \quad [Let\text{-}Init]$$

This rule is weak. Why?

Subtyping

- Define a relation $\leq$ on classes
  - $X \leq X$
  - $X \leq Y$ if X inherits from Y
  - $X \leq Z$ if $X \leq Y$ and $Y \leq Z$
- An improvement

$$\frac{O \vdash e_0 : T \qquad T \leq T_0 \qquad O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x{:}T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \quad [Let\text{-}Init]$$

Assignment

- Both let rules are correct, but more programs typecheck with the second one
- More uses of subtyping:

$$\frac{O(Id) = T_0 \quad O \vdash e_1 : T_1 \quad T_1 \le T_0}{O \vdash Id \leftarrow e_1 : T_1} \quad \text{[Assign]}$$

Initialized Attributes

- Let $O_C(x) = T$ for all attributes $x{:}T$ in class C
- Attribute initialization is similar to let, except for the scope of names

$$\frac{O_C(Id) = T_0 \quad O_C \vdash e_1 : T_1 \quad T_1 \le T_0}{O_C \vdash Id{:}T_0 \leftarrow e_1;} \quad \text{[Attr-Init]}$$

If Then Else

- Consider:
  if e_0 then e_1 else e_2 fi
- The result can be either e_1 or e_2
- The type is either e_1's type or e_2's type
- The best we can do is the smallest supertype larger than the type of e_1 or e_2

Least Upper Bounds

- lub(X,Y), the least upper bound of X and Y, is Z if
  - $X \le Z \wedge Y \le Z$
    Z is an upper bound
  - $X \le Z' \wedge Y \le Z' \Rightarrow Z \le Z'$
    Z is least among upper bounds
- Digression (Examples with ordered sets)
  - Least common multiple, Greatest common divisor
  - Set Union, Set Intersection
  - Maximum, Minimum

If Then Else Revisited

In COOL, the least upper bound of two types is their least common ancestor in the inheritance tree.

$$\frac{O \vdash e_1 : Bool \quad O \vdash e_2 : T_2 \quad O \vdash e_3 : T_3}{O \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \text{ fi} : lub(T_2, T_3)} \quad \text{[If-Then-Else]}$$

Case

- The rule for case expressions takes a *lub* over all branches

$$\frac{O \vdash e_0 : T_0 \quad O[T_1/x_1] \vdash e_1 : T_1' \quad \vdots \quad O[T_n/x_n] \vdash e_n : T_n'}{O \vdash \text{case } e_0 \text{ of } x_1{:}T_1 \Rightarrow e_1; \cdots; x_n{:}T_n \Rightarrow e_n; \text{ esac} : lub(T_1', \cdots, T_n')} \quad \text{[case]}$$

Method Dispatch

- There is a problem with type checking method calls:

$$\frac{O \vdash e_0 : T_0 \quad O \vdash e_1 : T_1 \quad \vdots \quad O \vdash e_n : T_n}{O \vdash e_0.f(e_1,\cdots,e_n) : ?} \quad \text{[Dispatch]}$$

- We need information about the formal parameters and return type of f

Notes on Dispatch

- In Cool, method and object identifiers live in different name spaces
  - A method foo and an object foo can coexist in the same scope
- In the type rules, this is reflected by a separate mapping M for method signatures

$$M(C,f) = (T_1,\ldots T_n, T_{n+1})$$

means in class C there is a method f

$$f(x_1{:}T_1,\ldots,x_n{:}T_n)\colon T_{n+1}$$

The Dispatch Rule Revisited

$$\frac{O,M \vdash e_0 : T_0 \quad O,M \vdash e_1 : T_1 \quad \vdots \quad O,M \vdash e_n : T_n \quad M(T_0,f) = (T_1',\cdots,T_n',T_{n+1}') \quad T_i \le T_i' \text{ for } 1 \le i \le n}{O,M \vdash e_0.f(e_1,\cdots,e_n) : T_{n+1}'} \quad \text{[Dispatch]}$$

Static Dispatch

- Static dispatch is a variation on normal dispatch
- The method is found in the class explicitly named by the programmer
- The inferred type of the dispatch expression must conform to the specified type

Static Dispatch (Cont.)

$$\frac{O,M \vdash e_0 : T_0 \quad O,M \vdash e_1 : T_1 \quad \vdots \quad O,M \vdash e_n : T_n \quad T_0 \le T \quad M(T,f) = (T_1',\cdots,T_n',T_{n+1}') \quad T_i \le T_i' \text{ for } 1 \le i \le n}{O,M \vdash e_0@T.f(e_1,\cdots,e_n) : T_{n+1}'} \quad \text{[StaticDispatch]}$$

The Method Environment

- The method environment must be added to all rules
- In most cases, M is passed down but not actually used
  - Only the dispatch rules use M

$$\frac{O,M \vdash e_1 : \text{int} \quad O,M \vdash e_2 : \text{int}}{O,M \vdash e_1 + e_2 : \text{int}} \quad \text{[Add]}$$

More Environments

- For some cases involving SELF_TYPE, we need to know the class in which an expression appears
- The full type environment for COOL:
  - A mapping O giving types to object id's
  - A mapping M giving types to methods
  - The current class C

Sentences

The form of a *sentence* in the logic is

$$O,M,C \vdash e : T$$

Example:

$$\frac{O,M,C \vdash e_1 : \text{int} \quad O,M,C \vdash e_2 : \text{int}}{O,M,C \vdash e_1 + e_2 : \text{int}} \quad [\text{Add}]$$

Type Systems

- The rules in this lecture are COOL specific
  - More info on rules for self next time
  - Other languages have very different rules
- General themes
  - Type rules are defined on the structure of expressions
  - Types of variables are modeled by an environment
- Warning: Type rules are very compact!

One Pass Type Checking

- COOL type checking can be implemented in a single traversal over the AST
- Type environment is passed down the tree
  - From parent to child
- Types are passed up the tree
  - From child to parent

Implementing Type Systems

$$\frac{O,M,C \vdash e_1 : \text{int} \quad O,M,C \vdash e_2 : \text{int}}{O,M,C \vdash e_1 + e_2 : \text{int}} \quad [\text{Add}]$$

```
TypeCheck(Environment, e1 + e2) = {
  T1 = TypeCheck(Environment, e1);
  T2 = TypeCheck(Environment,e2);
  Check T1 == T2 == Int;
  return Int; }
```


Soundness : Java Digression

Kinds of Conversion

- Identity Conversions
- String Conversions
- Widening Primitive Conversions (19)
  - *byte ⇒ short ⇒ int ⇒ long ⇒ float ⇒ double*
  - *char ⇒ int ⇒ long ⇒ float ⇒ double*
    - ***int ⇒ float*** may lose precision
- Narrowing Primitive Conversions (19+4)
  - *char ⇒ short, char ⇒ byte,*
  - *byte ⇒ char, short ⇒ char*
    - may lose sign and magnitude information

- Widening Reference Conversions
  - *null* type ⇒ any reference type
  - any reference type ⇒ *class* Object
  - *class* S ⇒ *class* T, if S *extends* T
  - *class* S ⇒ *interface* K, if S *implements* K
  - *interface* J ⇒ *interface* K, if J *extends* K
  - **array SC [] ⇒ array TC [], if SC ⇒ TC, and both SC and TC are reference types.**
  - array T ⇒ *interface* Cloneable
  - array T ⇒ *interface* Serializable

- Narrowing Reference Conversions
  - Permitted among reference types that can *potentially share common instances*.
  - These conversions require *run-time test* to determine if the actual reference is a legitimate value of the new type.
    - Observe the interpretation of *final class*.
    - Observe that every *non-final class* can potentially be *extended* to *implement* an interface.

Narrowing Reference Conversions

- *class* Object ⇒ any reference type
- *class* S ⇒ *class* T, if T *extends* S
- *class* S ⇒ *interface* K, if S is not *final* and S does not *implement* K
- *interface* K ⇒ *class* T, if T is not *final*
- *interface* K ⇒ *class* T, if T is *final* and T *implements* K
- *interface* J ⇒ *interface* K, if K does not *extend* J and there is no common method with same signature but different return types.
- **array SC [] ⇒ array TC [], if SC ⇒ TC, and both SC and TC are reference types.**

Motivation for *Primitive Array Type Constraints*

```
int i = 10;
long l = i;
byte b  = (byte) i;
int[]   iA  =  new int[5];
long[] lA  = (long[]) iA;  //error
 lA[2] = l;                  // reason
byte[] bA = (byte[]) iA;  // error
          b = bA[2];        // reason
```


Motivation for *Reference Array Type Constraints*

```
      Point  i   =    new Point();
     Object l   =    i;
 ColoredPoint b  =   (ColoredPoint) i;
        // throws exception
     Point[]   iA   = new Point[5];
     Object[] lA  = (Object[]) iA;
            lA[2]  =  l;          // no exception
             b  =  lA[2] ;        // *error*
             l  = lA[2]
ColoredPoint[] bA   = (ColoredPoint[]) iA;
        // throws exception
```

Another Example: Primitive Array Type

```
void f(int[] a) {
    int i = a[0];
    a[0] = i;
}
```

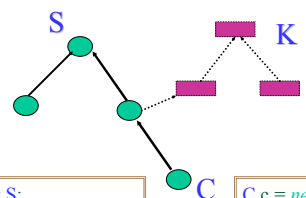
```
long[]  la = new long[8];
short[] sa = new short[8];
f(la);  // banned : compile-time error
f(sa);  // banned : compile-time error
```


Another Example: Reference Array Type

```
void f(Point[] pa) {
   pa[0] = new Point();
}
```

```
Object[] oa = new Object[8];
f(oa);        // compile-time error
f( (Point []) oa);  // throws ClassCastException
ColoredPoint[] cpa =
                  new ColoredPoint[8];
f(cpa);     // array store exception in f
```


class S ⇒ *interface* K



class S;
interface K;
class C *extends*+ S
implements+ K;

C c = *new* C();
S s = c;
K k = (C) s;
s = (C) k;