

# Recitation 2

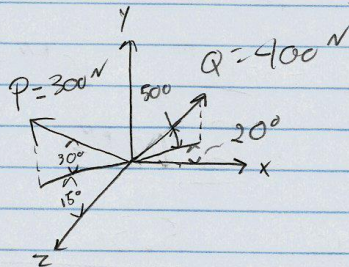
2-80  $\vec{M} = ?$   $\theta_{x,y,z} = ?$   
 $\vec{F} = 240\hat{i} - 270\hat{j} + 680\hat{k}$  N  
 $M = |\vec{F}| = \sqrt{240^2 + (-270)^2 + 680^2}$  N =  $770$  N

$$\cos \theta_x = \frac{R_x}{R} \Rightarrow \theta_x = \arccos \frac{R_x}{R} = \arccos \frac{240}{770} = 71.8^\circ$$

$$\theta_y = \cos^{-1} \frac{-270}{770} = 110.5^\circ$$

$$\theta_z = \cos^{-1} \frac{680}{770} = 28.0^\circ$$

2-91



Q:  $\theta_y = 90 - 50 = 40^\circ$   
 $\phi = -20^\circ$

$$Q_x = Q \sin \theta_y \cos \phi = 241.6 \text{ N}$$

$$Q_y = Q \cos \theta_y = 306.4 \text{ N}$$

$$Q_z = Q \sin \theta_y \sin \phi = -87.9 \text{ N}$$

P:  $\theta_y = 90 - 30 = 60^\circ$   $\phi = 90 + 15 = 105^\circ$

$$P_x = P \sin \theta_y \cos \phi = -67.2 \text{ N}$$

$$P_y = P \cos \theta_y = 150 \text{ N}$$

$$P_z = P \sin \theta_y \sin \phi = 251 \text{ N}$$

$$\vec{R} = \vec{P} + \vec{Q} = (241.6 + (-67.2))\hat{i} + (306.4 + 150)\hat{j} + (-87.9 + 251)\hat{k}$$

$$= \langle 174.4, 456.4, 163.6 \rangle \text{ N}$$

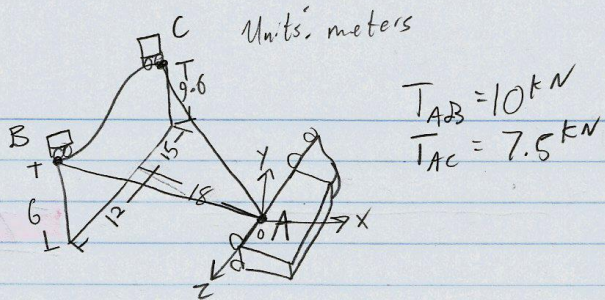
$$|\vec{R}| = 515.2 \text{ N}$$

$$\theta_x = \cos^{-1} \frac{174.4}{515.2} = 70.2^\circ$$

$$\theta_y = \cos^{-1} \frac{456.4}{515.2} = 27.6^\circ$$

$$\theta_z = \cos^{-1} \frac{163.6}{515.2} = 71.5^\circ$$

2-135



$$T_{AB}: d_{AB} = d_{AB} = d_{AB} = d_{AB}$$

$$d_x = -18 - 0 = -18 \text{ m}$$

$$d_y = 6 - 0 = 6 \text{ m}$$

$$d_z = 12 - 0 = 12 \text{ m}$$

$$|d| = \sqrt{d_x^2 + d_y^2 + d_z^2} = 22.4 \text{ m}$$

$$T_{ABx} = \frac{d_x}{d} T_{AB} = \frac{-18 \text{ m}}{22.4 \text{ m}} \cdot 10 \text{ kN} = -8.04 \text{ kN} \hat{i}$$

$$T_{AB_y} = \frac{d_y}{d} T_{AB} = \frac{6 \text{ m}}{22.4 \text{ m}} \cdot 10 \text{ kN} = 2.68 \text{ kN} \hat{j}$$

$$T_{AB_z} = \frac{d_z}{d} T_{AB} = \frac{12 \text{ m}}{22.4 \text{ m}} \cdot 10 \text{ kN} = 5.36 \text{ kN} \hat{k}$$

$$\vec{T}_{AB} = \langle -8.04, 2.68, 5.36 \rangle \text{ kN}$$

$$T_{AC}: d_x = d_{Cx} - d_{Ax} = -18 - 0 = -18 \text{ m}$$

$$d_y = 9.6 - 0 = 9.6 \text{ m}$$

$$d_z = -15 - 0 = -15 \text{ m}$$

$$|d_{AC}| = 25.3 \text{ m}$$

$$T_{AC_x} = \frac{d_{Ax}}{d_{AC}} \cdot T_{AC} = \frac{-18 \text{ m}}{25.3 \text{ m}} \cdot 7.5 \text{ kN} = -5.34 \text{ kN}$$

$$T_{AC_y} = \frac{d_{Ay}}{d_{AC}} \cdot T_{AC} = \frac{9.6 \text{ m}}{25.3 \text{ m}} \cdot 7.5 \text{ kN} = 2.85 \text{ kN}$$

$$T_{AC_z} = \frac{d_{Az}}{d_{AC}} \cdot T_{AC} = \frac{-15 \text{ m}}{25.3 \text{ m}} \cdot 7.5 \text{ kN} = -4.45 \text{ kN}$$

$$\vec{T}_{AC} = \langle -5.34, 2.85, -4.45 \rangle \text{ kN}$$

2-135

$$\vec{R} = \vec{T}_{AB} + \vec{T}_{AC} = (-8.04 + 5.34)\hat{i} + (2.68 + 2.85)\hat{j} + (5.36 + -4.45)\hat{k} \text{ kN}$$

$$\vec{R} = \langle -13.38, 5.53, .91 \rangle \text{ kN}$$

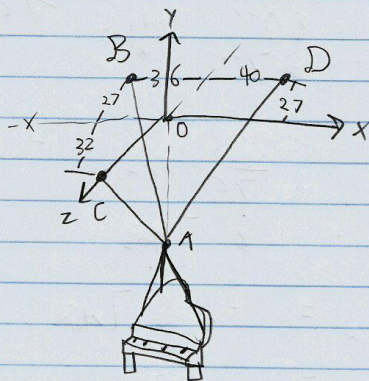
$$|\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = 14.5 \text{ kN}$$

$$\cos \theta_x = \frac{R_x}{R} \quad \theta_x = \arccos\left(\frac{-13.38 \text{ kN}}{14.5 \text{ kN}}\right) = 157.33^\circ$$

$$\theta_y = \cos^{-1}\left(\frac{5.53 \text{ kN}}{14.5 \text{ kN}}\right) = 67.6^\circ$$

$$\theta_z = \cos^{-1}\left(\frac{.91 \text{ kN}}{14.5 \text{ kN}}\right) = 86.4^\circ$$

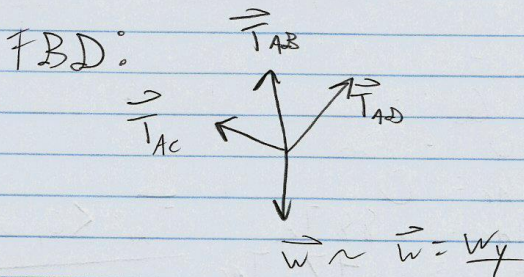
2-104



$$W = ?$$

$$T_{AD} = 616 \text{ lbf}$$

$$\left. \begin{aligned} \sum \vec{F} &= 0 \\ \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0 \end{aligned} \right\} \Rightarrow \underline{\underline{\vec{R} = 0}}$$



2-104

$T_{AD} = 616$

$$T_{AD} \cdot d_i: d_{Di} = d_{Ai}$$

$$dx = d_{Dx} - d_{Ax} = 40 - 0 = 40 \quad d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

$$dy = 0 - -60 = 60 \quad = 77''$$

$$dz = -27 - 0 = -27''$$

$$T_{ADx} = \frac{d_x}{|d|} \cdot T_{AD} = \frac{40''}{77''} \cdot 616 \text{ lbf} = 320 \text{ lbf}$$

$$T_{ADy} = \frac{d_y}{|d|} \cdot T_{AD} = \frac{60''}{77''} \cdot 616 \text{ lbf} = 480 \text{ lbf}$$

$$T_{ADz} = \frac{d_z}{|d|} \cdot T_{AD} = \frac{-27''}{77''} \cdot 616 \text{ lbf} = -216 \text{ lbf}$$

$$\vec{T}_{AD} = \langle 320, 480, -216 \rangle \text{ lbf}$$

$$T_{AB} \cdot d_i: d_{Di} = d_{Ai} \quad \underline{d_{Ai} \text{ will be constant}}$$

$$dx = -36 - 0 = -36'' \quad |d| = 75$$

$$dy = 0 - -60 = 60''$$

$$dz = -27 - 0 = -27''$$

$$T_{ABx} = \frac{d_x}{d} \cdot T_{AB} = \frac{-36''}{75''} \cdot T_{AB} = -.48 T_{AB}$$

$$T_{ABy} = \frac{d_y}{d} \cdot T_{AB} = .8 T_{AB}$$

$$T_{ABz} = \frac{d_z}{d} \cdot T_{AB} = -.36 T_{AB}$$

$$\vec{T}_{AB} = \langle -.48 T_{AB}, .8 T_{AB}, -.36 T_{AB} \rangle \text{ lbf}$$

$$T_{AC} \cdot d_x = 0 - 0 = 0''$$

$$dy = 0 - -60 = 60'' \quad |d| = 68''$$

$$dz = 32 - 0 = 32''$$

$$T_{ACx} = \frac{0}{68} \cdot T_{AC} = 0$$

$$T_{ACy} = \frac{60}{68} \cdot T_{AC} = .88 T_{AC}$$

$$T_{ACz} = \frac{32}{68} \cdot T_{AC} = .471 T_{AC}$$

$$\vec{T}_{AC} = \langle 0, .88 T_{AC}, .471 T_{AC} \rangle \text{ lbf}$$

1-104

$$\vec{R}: \vec{R}_x = T_{ADx} + T_{ABx} + T_{ACx} + W_x$$

$$(1) \quad 0 = 320 - 0.48 T_{AB} + 0 + 0 \quad \text{llf}$$

$$\vec{R}_y = T_{ADy} + T_{ABy} + T_{ACy} - W_y$$

$$(2) \quad 0 = 480 + 0.8 T_{AB} + 0.88 T_{AC} - W \quad \text{llf}$$

$$\vec{R}_z = T_{ADz} + T_{ABz} + T_{ACz} + W_z$$

$$(3) \quad 0 = -216 - 0.36 T_{AB} + 0.471 T_{AC} + 0 \quad \text{llf}$$

Method 1: Substitution

$$(1) \quad 0 = 320 - 0.48 T_{AB}$$
$$320 = 0.48 T_{AB}$$
$$T_{AB} = 666.6 \quad \text{llf}$$

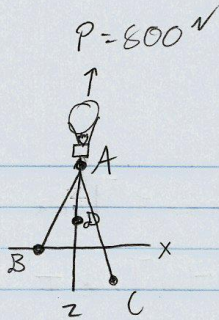
$$(3) \quad 0 = -216 - 0.36 T_{AB} + 0.471 T_{AC}$$
$$456 = 0.471 T_{AC}$$
$$T_{AC} = 968 \quad \text{llf}$$

$$(2) \quad 0 = 480 + 0.8 T_{AB} + 0.88 T_{AC} - W$$
$$W = 1865 \quad \text{llf}$$

Method 2: Matrix: Reduced Row Echelon Form

$$\begin{array}{l} (1) \quad x \\ (2) \quad y \\ (3) \quad z \end{array} \left[ \begin{array}{ccc|c} T_{AB} & T_{AC} & W & R + T_{AB} \\ -0.48 & 0 & 0 & -320 \\ 0.8 & 0.88 & -1 & -480 \\ -0.36 & 0.471 & 0 & 216 \end{array} \right] \begin{array}{l} \text{ref} \\ \Rightarrow \end{array}$$
$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 666.6 \\ 0 & 1 & 0 & 968 \\ 0 & 0 & 1 & 1865 \end{array} \right] \begin{array}{l} = T_{AB} \\ = T_{AC} \\ = W \end{array}$$

2-102



$$\begin{aligned}
 T_{AC}: \quad & d_{AC}: \quad d_{Ax} = d_{Cx} - d_{Ax} = 2.4^m - 0^m = 2.4 \\
 & d_{Ay} = 0 - 5.6 = -5.6 \\
 & d_{Az} = 4.20 - 0 = 4.2
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} d_{Ax} \\ d_{Ay} \\ d_{Az} \end{aligned}} \right\} m$$

$$|d_{AC}| = \sqrt{d_x^2 + d_y^2 + d_z^2} = 7.4^m$$

$$T_{ACx} = \frac{d_x}{|d|} \cdot T_{AC} = \frac{2.4^m}{7.4^m} \cdot T_{AC} = 0.324 T_{AC}$$

$$T_{ACy} = \frac{-5.6^m}{7.4^m} T_{AC} = -0.757 T_{AC}$$

$$T_{ACz} = \frac{4.2^m}{7.4^m} T_{AC} = 0.568 T_{AC}$$

$$\vec{T}_{AC} = \langle 0.324 T_{AC}, -0.757 T_{AC}, 0.568 T_{AC} \rangle^N$$

$$\begin{aligned}
 T_{AB}: \quad & d_{AB}: \quad d_{Ax} = -4.20 - 0 = -4.2 \\
 & d_{Ay} = 0 - 5.6 = -5.6 \\
 & d_{Az} = 0 - 0 = 0
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} d_{Ax} \\ d_{Ay} \\ d_{Az} \end{aligned}} \right\} m \quad d_{AB} = 7^m$$

$$T_{ABx} = \frac{-4.2}{7} \cdot T_{AB} = -0.6 T_{AB}$$

$$T_{ABy} = \frac{-5.6}{7} \cdot T_{AB} = -0.8 T_{AB}$$

$$T_{ABz} = \frac{0}{7} \cdot T_{AB} = 0$$

$$\vec{T}_{AB} = \langle -0.6 T_{AB}, -0.8 T_{AB}, 0 \rangle^N$$

$$\begin{aligned}
 T_{AD}: \quad & d_{AD}: \quad d_{Ax} = 0 - 0 = 0 \\
 & d_{Ay} = 0 - 5.6 = -5.6 \\
 & d_{Az} = -3.3 - 0 = -3.3
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} d_{Ax} \\ d_{Ay} \\ d_{Az} \end{aligned}} \right\} m \quad |d_{AD}| = 6.5^m$$

$$T_{ADx} = \frac{0^m}{6.5^m} T_{AD} = 0$$

$$T_{ADy} = \frac{5.6^m}{6.5^m} T_{AD} = 0.862 T_{AD}$$

$$T_{ADz} = \frac{-3.3^m}{6.5^m} T_{AD} = -0.508 T_{AD}$$

$$\vec{T}_{AD} = \langle 0, 0.862 T_{AD}, -0.508 T_{AD} \rangle^N$$

2-102

$$\vec{R} = 0 \quad \text{b/c} \quad \sum \vec{F} = 0 \quad \therefore$$

$$\vec{R}_x = 0 = T_{ACx} + T_{ABx} + T_{ADx} + P_x \quad \checkmark$$

$$\textcircled{1} \quad 0 = .324 T_{AC} - .6 T_{AB} + 0 + 0 \quad \checkmark$$

$$\textcircled{2} \quad \vec{R}_y = 0 = -.757 T_{AC} - .8 T_{AB} - .862 T_{AD} + 800 \quad \checkmark$$

$$\textcircled{3} \quad \vec{R}_z = 0 = .568 T_{AC} + 0 - .508 T_{AD} + 0$$

$$\begin{array}{l} \textcircled{1} x \\ \textcircled{2} y \\ \textcircled{3} z \end{array} \left[ \begin{array}{ccc|c} T_{AC} & T_{AB} & T_{AD} & R-P \\ \hline .324 & -.6 & 0 & 0 \\ -.757 & -.8 & -.862 & -800 \\ .568 & 0 & -.508 & 0 \end{array} \right] \xrightarrow{\text{RREF}} \Rightarrow$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 371.6 \\ 0 & 1 & 0 & 200.7 \\ 0 & 0 & 1 & 415.5 \end{array} \right] \checkmark$$

$$\left. \begin{array}{l} T_{AC} = 371.6 \\ T_{AB} = 200.7 \\ T_{AD} = 415.5 \end{array} \right\} \checkmark$$