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Prediction of Spur Gear Mechanical Power Losses Using a Transient Elastohydrodynamic Lubrication Model

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A model to predict load-dependent (mechanical) power losses of spur gear pairs is proposed based on an elastohydrodynamic lubrication (EHL) model. The EHL model includes gear-specific time variations of all key contact parameters such as the rolling and sliding velocities, radii of curvature, and normal load such that a single continuous analysis of a tooth contact from its root to tip can be performed under any lubrication condition ranging from full-film to mixed EHL or boundary lubrication conditions. Predicted transient pressure and film thickness distributions are used to determine the instantaneous rolling and sliding shear within the lubricant, from which the gear mesh mechanical power losses are determined. Correction factors are introduced for the power losses to account for the thermal effects. The accuracy of the proposed model is assessed through comparisons to published spur gear efficiency experiments. The contribution of the rolling action to the total power loss is quantified to show that the rolling losses are indeed not negligible.

KEY WORDS
Automotive; Gears; Mixed EHL; Traction; Roughness Effects

INTRODUCTION
Power losses of gear pairs have become a critical research topic in recent years due to renewed interest in improving power train efficiency and reducing carbon emissions, both stemming from environmental concerns. The research on this topic focused on both load-dependent (mechanical) and load-independent (oil churning, windage, etc.) power loss components of gear pairs and geared transmission systems. Though load-independent losses are associated with fluid–gear interactions (Seetharaman and Kahraman (1); Seetharaman, et al. (2)), the mechanical losses are defined primarily by the friction of lubricated contacts of rough gear tooth surfaces that are in combined sliding and rolling. This article focuses on the development of a new mechanical power loss model for spur gear pairs.

Earlier literature on gear power losses and efficiency consisted of a number of studies that employed either a user-defined uniform friction coefficient or a particular empirical friction relation derived from curve-fitting twin-disk-type traction measurements. Xu, et al. (3) provided a detailed review of these earlier gear efficiency studies, indicating that (a) the friction coefficient along the tooth surface is not constant but varies significantly, and (b) the empirical friction formulae are usually valid within narrow ranges that the experimental conditions and lubricants represent. Xu, et al. (3) provided a methodology for the prediction of gear mechanical efficiency. They devised a large parametric study of line contacts within certain ranges of rolling and sliding velocities, radii of curvature, normal force, lubricant viscosity (temperature), and surface roughness amplitudes. They used the model by Cioc, et al. (4) to predict the friction coefficient values for each combination of these contact parameters and applied a linear regression analysis to derive a single friction coefficient formula. This formula was then combined with a gear contact analysis model similar to the one proposed in Contry and Seireg (5) to predict the mechanical power losses of the gears used in the experiments of Pertry-Johnson, et al. (6). They concluded that the predictions match reasonably well with the experiments. This hybrid methodology was reported to be reasonably accurate and fast because all elastohydrodynamic lubrication (EHL) simulations are done up-front deriving the friction coefficient formula.

Because the EHL model of Cioc, et al. (4) allowed only a few asperity contacts at a time, the model of Xu, et al. (3) was rather limited in its ability in predicting the mechanical losses of gear pairs operating under very undesirable mixed EHL or boundary lubrication conditions. Such contact conditions are of common occurrence, especially in automotive gearing where speeds are low and the operating temperatures, loads, and surface roughness amplitudes are all high. In addition, most of the transmission fluids are low-viscosity ones designed to reduce churning losses and ensuring proper operations of other transmission components such as wet clutches.

Li and Kahraman (7) have recently developed a mixed EHL model of point contacts where fluid interfaces and asperity contacts of any severity can be modeled simultaneously using a unified Reynolds equation system proposed by Hu and Zhu (8) and Zhu (9). This unified approach employs a reduced Reynolds equation, which is mathematically equivalent to the description of dry contact where asperity interactions occur, eliminating the
Numerical difficulties in the simulation of mixed lubrication conditions, Li, et al. (10) later reduced the model to mixed EHL analysis of line contacts. Employing the methodology of Xu, et al. (3), they performed a large parametric study representing wide ranges of contact and surface parameters representative of gear contacts to derive a new regression formula of friction suitable for gear contacts operating with an automatic transmission fluid. This formula was able to represent diverse lubrication conditions ranging from boundary contact to full-film micro-EHL accurately. They combined this friction formula with a gear load distribution model (Conry and Seireg (5)) to predict spur and helical gear efficiency. Li, et al. (5) also showed that the efforts to reduce power losses of a gear pair might often result in undesirable noise and durability issues and, hence, efficiency must be included in the design process together with the other functionality requirements.

One common shortcoming of the above models is that they assumed that the gear contacts can be modeled as contacts with constant speed, load, and geometry parameters. The continuous meshing action that moves the contact from the root to the tip of a gear tooth was discretized in these models to a number of constant-parameter contacts, each representing a specific tooth contact position. Recently, Li and Kahraman (11) proposed a transient gear EHL model that captures the time variations of several parameters unique to gear contacts, including rolling and sliding velocities, radii of curvature, and normal load, as well as tooth surface roughness profiles. They provided comparisons to earlier models to show that the contact conditions can indeed be altered by these transient effects, suggesting that they must be included in gear pair power loss predictions. The other apparent shortcoming of the previous gear efficiency models is in the way they included the rolling power losses. These models either neglected the rolling losses altogether or used approximate rolling traction formulae to estimate the rolling power loss, which is rather simplistic.

The main objective of this study is to develop a new mechanical power loss model that is capable of accurately predicting both the sliding and rolling losses of spur gears by using the transient gear EHL model of Li and Kahraman (11). The model is intended to (a) predict the mechanical power losses associated with the gear contacts by modeling them continuously with their time-varying parameters and (b) eliminate any need for large-scale up-front EHL parameter studies and the errors associated with the regression analyses for friction formula derivation. This model will be implemented to simulate the experiments of Petry-Newton, et al. (6) for a variety of gear designs, surface roughness, and operating conditions to assess its accuracy. The contributions of the sliding and rolling components to the total mechanical power loss will also be investigated along the gear mesh cycle.

In this study, the model formulations will be kept limited to spur gears with no axial variations. In cases where axial effects such as misalignments and gear lead errors are not negligible or gears are of helical type, this methodology can easily be expanded by simply modeling the gear in hand as a sum of narrow spur gear slices at certain relative positions from each other, carrying a certain portion of the load as predicted by the load distribution model (Conry and Seireg (5)). Such cases will be kept outside of the scope of this study.

It is assumed that the immediate failure mode of scuffing is not present. In terms of surface wear, any measured or predicted wear profiles (Ding and Kahraman (12)) can be fed into the load.

**NOMENCLATURE**

- \( a \) = Half Hertzian contact width
- \( a_{\text{max}} \) = Maximum half Hertzian contact width along the line of action
- \( C_a \) = Area contact ratio
- \( C_l \) = Load contact ratio
- \( E_1, E_2 \) = Young’s modulus of gears 1 and 2, respectively
- \( E' \) = Equivalent Young’s modulus
- \( E' = \frac{E}{2\left[(1-v_1^2)/E_1 + (1-v_2^2)/E_2\right]}^{-1} \)
- \( f \) = Flow coefficient
- \( g_0 \) = Geometry gap before deformation
- HPSTC = Highest point of single tooth contact
- LPSTC = Lowest point of single tooth contact
- \( P \) = Frictional power loss
- \( P_m \) = Instantaneous mechanical power loss
- \( P_{\text{mesh}} \) = Gear mesh power loss
- \( p \) = Pressure
- \( q \) = Viscous shear within the lubricant
- \( R_1, R_2 \) = Roughness profiles of surfaces 1 and 2, respectively
- \( R_{q1}, R_{q2} \) = Root-mean-square roughness amplitudes of surfaces 1 and 2, respectively
- \( r_1, r_2 \) = Contact radii of curvature of gears 1 and 2, respectively
- \( r_{b1}, r_{b2} \) = Base circle radius of gears 1 and 2, respectively
- \( r_{eq} \) = Equivalent radius of curvature \( r_{eq} = r_1 r_2 / (r_1 + r_2) \)
- \( \alpha_1, \alpha_2 \) = Pressure viscosity coefficients within low and high pressure ranges, respectively
- \( \eta \) = Lubricant viscosity
- \( \eta_0 \) = Lubricant viscosity at ambient pressure
- \( \eta' \) = Lubricant effective viscosity
- \( \theta_1, \theta_2 \) = Roll angle of gears 1 and 2, respectively
- \( \rho \) = Lubricant density
- \( \rho_0 \) = Lubricant density at ambient pressure
- \( \zeta \) = Profile contact ratio
- \( \tau_0 \) = Reference shear stress of the lubricant
- \( \upsilon_1, \upsilon_2 \) = Poisson’s ratio of gears 1 and 2, respectively
- \( \Omega_1 \) = Rotational speed of gear 1
- \( \omega_1 \) = Angular velocities of gear 1

**SAP** = Start of active profile
**SR** = Slide-to-roll ratio \( SR = u_1 / u_2 \)
**T** = Input torque
**t** = Time
**u_1, u_2** = Surface velocities in the direction of rolling of gears 1 and 2, respectively
**u_r** = Rolling velocity, \( u_r = \frac{1}{2}(u_1 + u_2) \)
**u_s** = Sliding velocity, \( u_s = u_1 - u_2 \)
**V** = Elastic surface deformation
**W** = Normal tooth force
**x** = Coordinate along the rolling direction
**Z_1, Z_2** = Number of teeth of gears 1 and 2, respectively

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distribution model (Conry and Seireg (5)) as the tooth profile deviations. With this, the proposed model can effectively predict the influences of surface wear on power losses. Additionally, tooth-to-tooth variations in roughness and dimensions are assumed to be negligible. Considering that the finishing processes such as shaving and grinding result in consistent roughness amplitudes and forms (especially after the run-in of the surfaces), this assumption should be reasonable.

MODEL FORMULATION

Determination of Gear Tooth Contact Parameters

As mentioned earlier, a variety of gear contact parameters vary as the contact moves along a given tooth surface within the meshing zone. Referring to Fig. 1(a), it is first noted that the radii of curvature of a given mating tooth pair vary in time; that is, \( r_1 = r_1(t) \) and \( r_2 = r_2(t) \). The radii of curvature at the contact point \( C \) in Fig. 1(a) are defined from the basic gear geometry and kinematics as

\[
\begin{align*}
  r_1(t) &= \frac{b_1}{2} C = r_{a1} \theta_1(t), \quad [1a] \\
  r_2(t) &= \frac{b_2}{2} C = r_{a2} \theta_2(t) \quad [1b]
\end{align*}
\]

where \( r_{a1} \) and \( r_{a2} \) are the base circle radii of gears 1 and 2, and the roll angles \( \theta_1(t) \) and \( \theta_2(t) \) are as defined in Fig. 1(a). The contact radius of the driving gear (gear 1) has its minimum at its start of active profile (SAP) and increases gradually to reach its maximum at the tip, whereas the opposite is true for the mating surface of the driven gear (gear 2). The changes in radii of curvature with time (or rotation) also cause the tangential surface velocities at the contact point to change with time; that is,

\[
\begin{align*}
  u_1(t) &= \omega_1 r_1(t), \quad [1c] \\
  u_2(t) &= \frac{Z_1}{Z_2} \omega_1 r_2(t) \quad [1d]
\end{align*}
\]

where \( Z_1 \) and \( Z_2 \) are the number of teeth of gears 1 and 2, and \( \omega_1 \) is the angular velocity of gear 1. Here, \( u_1(t) < u_2(t) \) when a tooth contact is in the dedendum region of the driving gear such that the sliding velocity \( u_1(t) = u_1(t) - u_2(t) < 0 \). Meanwhile \( u_1(t) = u_2(t) \) \( (u_1(t) = 0, \text{pure rolling}) \) at the pitch point and \( u_1(t) > 0 \) along the addendum of the tooth of gear 1. In the process, the rolling velocity \( u_r(t) = \frac{1}{2} [u_1(t) + u_2(t)] \) also becomes variable.

Finally, the normal tooth force varies in time (\( W = W(t) \)). Focusing on the quasistatic contact condition, the number of the loaded tooth pairs \( N_t \) fluctuates between two integers (typically \( N_t = 1 \) or \( 2 \)), causing position (and hence time)-dependent loading changes to each tooth pair in contact. Additional time variation of the tooth force can be observed, especially near the gear mesh resonance frequencies, when dynamic effects are considered. In this study dynamic effects are assumed to be negligible and can be incorporated by combining a dynamic model of the gear pair with the proposed formulation. As in Li and Kahraman (11), a quasistatic gear load distribution model (Conry and Seireg (5)) is used here to compute \( W(t) \), including all essential components of the gear tooth compliance (bending, shear, base rotation, and Hertzian) as well as the intentional tooth modifications and any profile manufacturing errors.

Mixed EHL Analysis of a Tooth Contact

In the absence of shaft misalignments and lead errors or modifications, contacts of a spur gear pair can be characterized as one-dimensional line contact as shown in Fig. 1(b). The transient Reynolds equation governs the non-Newtonian fluid flow in the contact areas with no asperity interactions as

\[
\frac{\partial}{\partial x} \left[ f \frac{\partial p}{\partial x} \right] = \frac{\partial}{\partial x} \left[ \rho \left( \frac{\tau}{\tau_0} \right) \right] + \frac{\partial}{\partial x} \left[ \rho \frac{\partial h}{\partial x} \right] \quad [2]
\]

where \( p, h, \rho \) denote the pressure, thickness, and density of the fluid at coordinate \( x \) in the direction of rolling (profile direction) at time \( t \), respectively.

The Ree-Eyring fluid flow coefficient is defined as (Ehret, et al. (13)):

\[
f = \frac{\rho h^3}{12 \eta} \cosh \left( \frac{\tau_m}{\tau_0} \right) \quad [3a]
\]

where \( \eta \) is the lubricant viscosity, and \( \tau_m \) is the viscous shear stress determined by

\[
\tau_m = \tau_0 \sinh^{-1} \left( \frac{n_s(t)}{\tau_0 h} \right) \quad [3b]
\]

Here \( \tau_0 \) is the lubricant’s reference shear stress, which is linearly dependent on pressure (Hoglund and Jacobson (14); Evans and Johnson (15); Zhang (16)) when \( p \) exceeds a certain lubricant solidification pressure \( p_s \), and it is slightly influenced by \( p \) below \( p_s \). It is also noted that when \( p \) goes beyond an upper limit value of \( p_c \), that is, \( p > p_c \), this relationship becomes nonlinear (Zhang (16)). In this work, the relationship between \( \tau_0 \) and \( p \) within the entire loading range is assumed to take the form of

\[
\tau_0 = \tau_{0a} + \tau_e \left[ 1 + \tanh \left( \frac{2p}{p_c + p_e - 1} \right) \right] \quad [3c]
\]

where \( \tau_{0a} \) is the reference shear stress at the ambient pressure, and \( \tau_e \) is a shear parameter determined by \( \tau_e = \tau_{0a} + \kappa (p_c - p_e) \) and...
\[ \Theta = \kappa (p_{s} + p_{r})/t_{o}. \]

Because the hyperbolic function \( \tan \Theta \) is approximately linear when \( |\Theta| < 1 \), the proportional variation of \( t_{o} \) with \( p \) within the range \( p_{s} < p < p_{r} \) with a slope \( \kappa \) can be represented by Eq. [3c]. For 75W90 oil (a typical gear oil used in this study), \( t_{o0} = 5 \text{ MPa}, p_{s} = 0.5 \text{ GPa}, p_{r} = 3.5 \text{ GPa}, \text{ and } \kappa = 0.035. \]

Equation [2] describes the lubricant flow within the contact regions where the fluid film thickness is greater than zero such that the surfaces are separated. In the regions where asperity contacts occur (i.e., \( h \) is infinitesimally small), the reduced Reynolds equation applies (Li and Kahraman \( \gamma \) \( \delta \), Hu and Zhu (8); Zhu (9); Li, et al. (10); Li and Kahraman (11)).

\[ \frac{\partial [u_{t}(t) \rho h]}{\partial x} + \frac{\partial (p h)}{\partial t} = 0 \]  

[4]

With the assumption of a smooth transition between the fluid and asperity contact areas, Eqs. [2] and [4] govern the mixed EHL behavior of the contact, with the asperity contact and fluid regions handled simultaneously.

The film thickness of the contact point \( x \) at time \( t \) under elastic condition is defined as

\[ h(x, t) = h_{0}(t) + g_{0}(x, t) + V(x, t) - R_{1}(x, t) - R_{2}(x, t) \]  

[5]

where \( h_{0}(t) \) is the reference film thickness, and \( R_{1}(x, t) \) and \( R_{2}(x, t) \) are the measured roughness profiles of the two surfaces in the direction of rolling at time \( t \). \( R_{1}(x, t) \) and \( R_{2}(x, t) \) move with their respective surfaces at time-varying velocities of \( u_{1}(t) \) and \( u_{2}(t) \). In Eq. [5], the time-varying term \( g_{0}(x, t) \) represents the unloaded geometric gap between the two tooth surfaces, defined as

\[ g_{0}(x, t) = \frac{x^{2}}{2 r_{o0}(t)} \]  

[6]

where \( r_{o0}(t) = r_{1}(t) r_{2}(t)/[r_{1}(t) + r_{2}(t)] \). This time dependence of the gap term is also unique to the contact of a spur gear pair. Additionally, the elastic deformation \( V(x, t) \) due to the normal load \( W \) applied is given as \( V(x, t) = \int_{c}^{\pi} K (x - \eta)p(x', t) d\eta \), where \( x_{1} \) and \( x_{2} \) are the limits of the computational domain of the contact zone of Fig. 1(b), and \( K = -4 \ln |x|/(\pi E) \) is the influence function with \( E = 2\left[ (1 - v_{1}^{2})/E_{1} + (1 - v_{2}^{2})/E_{2}\right]^{-1} \), where \( v_{1} \) and \( E_{1} \) are the Poisson’s ratio and the Young’s modulus of contact body \( t \) (Johnson (17)).

A two-slope viscosity–pressure model of Allen is used in its modified form (Goglia, et al. (18)) given as

\[ \eta = \begin{cases} 
\eta_{1} \exp(\alpha_{1} p), & p < p_{a} \\
\eta_{2} \exp(\alpha_{2} p + c_{1} p^{2} + c_{2} p^{3}), & p_{a} \leq p \leq p_{b} \\
\eta_{2} \exp[\alpha_{0} p_{t} + \alpha_{2} (p - p_{b})], & p > p_{b} 
\end{cases} \]  

[7]

Here, \( \alpha_{1} \) and \( \alpha_{2} \) are the viscosity–pressure coefficients for the low \( (p < p_{a}) \) and high \( (p > p_{b}) \) pressure ranges, respectively; \( p_{t} \) is the transition pressure value between these two ranges; and \( p_{a} \) and \( p_{b} \) are the threshold pressure values of the low- and high-pressure ranges, respectively. Due to the lack of the data concerning the lubricant solidification effect on density, the lubricant compressibility is modeled through a simple density–pressure relationship of Dowson and Higginson (19):

\[ \rho = \rho_{0}(1 + \gamma p)/(1 + \lambda p) \]  

[8]

where \( \gamma = 2.266 \times 10^{-9} \text{ Pa}^{-1} \) and \( \lambda = 1.683 \times 10^{-9} \text{ Pa}^{-1} \).

The last equation of interest here is the load balance equation, \( W(t) = \int p(x, t) dx \), which states that the total contact force due to the pressure distribution including both the hydrodynamic and asperity contact regions over the entire contact zone must balance the normal load applied. Here \( W \) is the tooth load per unit gear face width. Though the load balance equation is not directly required for the solution of the EHL problem at hand, it provides a check for the solution. The value of \( h_{0}(t) \) in Eq. [5] needs to be adjusted within a load iteration loop until the load balance is achieved.

These governing equations above are solved by using a second-order discretization scheme that is described in Li and Kahraman (11), paying special attention to the spatial and time discretization parameters and grid sizes for avoiding any discretization errors.

**Mechanical Power Loss at Each Gear Tooth Contact**

The shear traction between the contact surfaces consists of (a) the viscous shear within the lubricated areas of the contact and (b) the contact friction due to any direct asperity interactions. Assuming no slip between the lubricant and the tooth surfaces and considering both the Poiseuille and Couette flows, the viscous shear stress \( q \) that varies linearly along the film thickness direction \( z \) is given as

\[ q(x, z, t) = \eta' \frac{\partial u}{\partial z} = \frac{(2z - h)}{2} \left( \frac{\nu}{\nu} \right) - \nu' \frac{u_{t}}{h} \]  

[9]

where \( \eta' = \eta/cosh(z_{m}/t_{o}) \) is the effective viscosity. The power loss due to viscous friction at a certain position \( x \) and time \( t \) is found by integrating the product of the shear stress \( q \) and the sliding velocity within the lubricant over the film thickness as

\[ P(x, t) = A \int_{0}^{h} q(x, z, t) \left( \frac{\partial u}{\partial z} \right) dz = A \left[ \frac{h^{3}}{12\nu} \left( \frac{\nu}{\nu} \right)^{2} + \nu' \frac{u_{t}^{2}}{h} \right] \]  

[10a]

where \( A \) is the area of the grid element at coordinate \( x \). The first and second terms of the right-hand side of Eq. [10a] are referred to as the rolling and sliding power losses, respectively. It is noted here that the use of the above effective viscosity expression is approximate and the viscous shear can be determined directly by using the Ree-Eyring model as \( q = t_{o0} \sinh^{-1}\left[ (t_{o0}/h_{0}) (\partial u/\partial z) \right] \). This would turn Eq. [10a] into the form of

\[ P(x, t) = A \int_{0}^{h} t_{o0} \sinh^{-1}\left[ \frac{u_{t}}{t_{o0}} \left( \frac{2h - 2z}{2h} \right) \left( \frac{\nu}{\nu} \right) - \frac{u_{t}^{2}}{h} \right] dz \]  

[10b]

which is more complicated to evaluate. In addition, this form of power loss fails to give the contributions of the individual sliding and rolling components. The contribution of the rolling action to the total power loss is one of the stated objectives of this study. On the other hand, Eq. [10b] still requires the evaluation of \( \nu/\nu \), which is the solution of the non-Newtonian Reynolds equation [2] employing the effective viscosity concept in the flow coefficient term. For these reasons, the effective viscosity method is used here in the formulation of the power loss.
In the previous studies (Xu, et al. (3); Li, et al. (10); Wu and Cheng (20)), the power loss due to the rolling action was conventionally computed by using an approximate rolling traction formula for line contacts (Goksem and Hargreaves (21)):

\[
F_r = \frac{4.318}{\alpha_t} \left( \frac{\beta \varphi}{1-E} \right)^{0.658} \left( \frac{W}{W_{eq}} \right)^{0.012} \frac{\varphi}{\alpha_t} \frac{r_{eq}}{W_{eq}} \tag{11}
\]

where the nondimensional parameters \( \overline{C} = \alpha_t E \), \( \overline{T} = \frac{n_{eq}}{(\alpha_t E r_{eq})} \), and \( \overline{W} = W/W_{eq} \). Goksem and Hargreaves (21) assumed full-film, smooth contact conditions where the rolling traction is mostly contributed from the inlet region \( x_s < x < -a \) (a is the half-Hertzian width). This is because \( \varphi/\alpha \) changes its sign across the contact within the Hertzian zone of \( -a < x < a \) (positive when \( x < 0 \) and negative when \( x > 0 \)) such that a cancellation of the traction within the Hertzian zone takes place. As a result, the total rolling friction acting on the contact surface can be written as \( F_r = \frac{1}{2} f_r h \frac{\varphi}{\alpha} dx \), leading to the approximation formula of Eq. (11) (Goksem and Hargreaves (21)). In Eq. [10a], however, the rolling power loss is shown to originate from the entire computational domain, including the Hertzian zone where \( h \) is small, whereas \( \varphi/\alpha \) can be very large due to the surface roughness effects. Therefore, use of Eq. (11) might cause a significant underestimation of the rolling power losses, especially when the surfaces are rough, whereas Eq. [10a] is a more accurate way to include the rolling losses.

Due to shear heating, the reduction in lubricant viscosity and consequently the film thickness are expected. This effect can be modeled by incorporating an energy equation with the other governing equations to form a thermal EHL model. In such a thermal model, one would also require including the temperature effects in Eqs. (7) and (8). Due to the computational burden caused by such a thermal analysis, an approximate method will be used here based on a thermal reduction factor proposed by Gupta, et al. (22) for the correction of the film thickness

\[
\phi_T = \frac{1}{1 + 0.213 \left( 1 + 2.23 \left( \frac{n_{eq}}{n_t} \right)^{0.83} \right)^{2}} \frac{\varphi}{h_T} \tag{12a}
\]

Here, \( L_T = \frac{n_{eq}}{k} \) is the thermal loading parameter, where \( \beta \) and \( k \) are the temperature–viscosity coefficient and the fluid thermal conductivity, respectively. With the assumption that the reduction in film thickness is mainly due to the decrease in viscosity, the smooth surface central film thickness formulation of Dowson and Hamrock for line contacts (Hamrock, et al. (23)) can be corrected as

\[
h_t = \frac{2.922^{0.470} r_{eq}}{W^{0.1166}} \left( \frac{\beta \varphi}{1-E} \right)^{0.062} \tag{12b}
\]

By defining a viscosity thermal correction factor \( C_t = \phi_T^{4.445} \), the rolling and sliding power loss thermal correction factors can then be obtained respectively as \( C_s = \phi_s^{1.55} \) and \( C_r = \phi_r^{0.695} \), such that the power loss of Eq. [10a] is corrected as

\[
P(x, t) = A \left[ C_s \frac{h^3}{12 \pi} \left( \frac{\varphi}{\alpha} \right)^2 + C_r \frac{\varphi}{\nu_s} \frac{n_{eq}}{h} \right] \tag{13}
\]

Meanwhile, within the asperity contact regions, the shear stress due to sliding is defined as \( q = \mu d p \), and the corresponding friction power loss is

\[
P(x, t) = A \mu d p \left[ t_n \right] \tag{14}
\]

where \( \mu_d \) is the dry contact coefficient of friction. In this study, a typical \( \mu_d \) value of 0.1 is used under the boundary lubrication condition (Wu and Cheng (20); Lai and Cheng (24)). Dependency of the power loss solutions to this user-defined value of \( \mu_d \) increases as the lubrication condition moves from full-film or mixed EHL to boundary lubrication, with the limiting case of dry gear contact solely dictated by this value. In cases where the fluid film carries a sizable portion of the load (as is the case in most medium-speed gear applications), the influence of the dry friction on the power losses is rather limited.

The total instantaneous mechanical power loss over the entire contact area with both the fluid and asperity contact regions is then found as

\[
P_m(t) = \sum_{i=1}^{M} P(x, t) \tag{15}
\]

where \( M \) is the number of the discretized elements of the computational domain.

**Overall Gear Mesh Mechanical Power Loss**

Equation [15] applies to a single contact of a particular tooth pair. Consider a spur gear having a profile contact ratio of \( \varsigma \) (1 < \( \varsigma \) < 2), indicating that the gear pair has two tooth pairs in contact for \((\varsigma - 1)\) of the entire gear mesh cycle and the remaining \((2 - \varsigma)\) of the mesh cycle is defined by only one loaded tooth pair. In the single-tooth contact portion of the mesh cycle, the contact is between the highest and lowest points of single-tooth contact (HPSTC and LPSTC) as illustrated in Fig. 2(a). Meanwhile, under the double-tooth contact condition as illustrated in Fig. 2(b), one tooth is loaded below LPSTC to share the load with the proceeding tooth, whose contact is above HPSTC. If there is only a single-tooth contact at a given rotational position angle \( \psi \) at time \( t \), then the gear mesh power loss is defined by the loss of this single-tooth contact; that is, \( P_{mesh}(\psi) = P_m(\psi) \) where \( P_m \) is defined by Eq. [15]. However, if \( \psi \) corresponds to a rotational position when there are two tooth pairs in contact, \( P_{mesh}(\psi) = P_m(\psi) + P_{roll} \) where \( P_{roll} \) are the power losses of the two loaded tooth contacts at this position, each defined again by Eq. [15]. For a continuous gear EHL analysis performed for \( N \) number of rotational gear mesh cycle increments corresponding to a rotational increment of \( \Delta \psi \) the average gear mesh mechanical power loss is found as

\[
\mathcal{P}_{mesh} = \frac{1}{N} \sum_{n=1}^{N} P_{mesh}(n \Delta \psi) \tag{16}
\]

**COMPARISONS TO EXPERIMENTS AND NUMERICAL RESULTS**

In this section, the model predictions will be compared to the published spur gear power loss measurements of Petry-Johnson, et al. (6) to assess the accuracy of the proposed model, meanwhile demonstrating the impact of gear module, surface roughness...
amplitude, lubricant parameters, and operating (speed and torque) conditions on the power losses of jet-lubricated spur gear pairs. The detailed design parameters of the unity-ratio test gear pairs used in Petry-Johnson, et al. (6), 23-tooth gears with the module of 3.95 mm and 40-tooth gears with the module of 2.32 mm, are listed in Table 1. Two levels of surface roughness were considered by Petry-Johnson, et al. (6), representing ground gear surfaces of $R_q = 0.3 \, \mu m$ and chemically polished gears with $R_q = 0.08 \, \mu m$, which were measured after the run-in process. One of the ground surface measurements is illustrated in Fig. 3, showing the asperity peaks are rounded off while the asperity valleys remain steep. In this work, only the 75W90 gear oil (lubricant A in Petry-Johnson, et al. (6)) is considered. Table 2 lists its properties at four different temperature levels. At the operating temperature of 100°C in Petry-Johnson, et al. (6), the second viscosity–pressure coefficient $\alpha_2 = 1.2 \, GPa^{-1}$ with the threshold and transition pressures taking the values of $p_a = 0.42 \, GPa$, $p_b = 0.84 \, GPa$, and $p_t = 0.6 \, GPa$.

### Table 1—Basic Design Parameters of the Example Spur Gears Considered (6)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>23-Tooth</th>
<th>40-Tooth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module</td>
<td>3.95</td>
<td>2.32</td>
</tr>
<tr>
<td>Pressure angle ($^\circ$)</td>
<td>25.0</td>
<td>28.0</td>
</tr>
<tr>
<td>Pitch diameter</td>
<td>90.86</td>
<td>92.74</td>
</tr>
<tr>
<td>Base diameter</td>
<td>82.34</td>
<td>81.89</td>
</tr>
<tr>
<td>Outside diameter</td>
<td>100.34</td>
<td>95.95</td>
</tr>
<tr>
<td>Root diameter</td>
<td>81.30</td>
<td>85.80</td>
</tr>
<tr>
<td>Start of active profile</td>
<td>85.38</td>
<td>87.73</td>
</tr>
<tr>
<td>Circular tooth thickness</td>
<td>6.435</td>
<td>2.925</td>
</tr>
<tr>
<td>Root fillet</td>
<td>1.63</td>
<td>0.83</td>
</tr>
<tr>
<td>Face width</td>
<td>19.5</td>
<td>26.7</td>
</tr>
<tr>
<td>Center distance</td>
<td>91.5</td>
<td>91.5</td>
</tr>
</tbody>
</table>

Units are in millimeters unless otherwise specified.

In Li and Kahraman (11), extensive comparisons were presented between the sequential steady-state solutions and the solutions with the transient effects associated with gear contacts as proposed here. A quantitative comparison between the two methods is not possible when the surfaces are rough because one cannot simulate the relative motions of the roughness profiles in a steady-state manner. The comparisons in Li and Kahraman (11) therefore focused on the corresponding smooth surfaces to show considerable differences in the film thickness as well as the pressure and film ripples due to the sudden load changes at the transition points between the single- and double-tooth contact regions.

In order to avoid any redundancies with Li and Kahraman (11), the smooth surface minimum film thickness predictions between the two approaches along a tooth contact surface are compared in Fig. 4 for the 23-tooth example gear pair operating at $T = 413 \, Nm$ and $\Omega_1 = 6,000 \, rpm$. It is seen that the transient method yields thicker fluid film for contacts below LPSTC, whereas the opposite is true for contacts above HPSTC. The sudden tooth load changes at LPSTC and HPSTC introduce fluctuations to $h_{min}$ that are not captured by the sequential steady-state solutions.

By using the proposed transient gear EHL model, the mechanical power losses of the 23-tooth gear pair at the torque value of $T = 413 \, Nm$ are predicted within the gear 1 rotational speed range of $\Omega_1 = 2,000$ to 10,000 rpm. The size of the computational domain, whose origin coincides with the contact point C (Fig. 1(a)), is fixed through the entire gear mesh cycle and is adjusted such that $2.5d_{max} \leq x \leq 1.5d_{max}$, where $d_{max}$ is the maximum half-Hertzian width as C moves from SAP to the tip of the tooth of gear 1. At this loading level, the computational domain is discretized into 512 elements, resulting in the spatial resolution of $\Delta x = 1.87 \, \mu m$. Likewise, the line of action is divided into 500 increments, which corresponds to a time resolution of $\Delta t = \Delta t_1/\omega_1 = 1.3 \times 10^{-6} \, s$ at $\Omega_1 = 6,000 \, rpm$. For the other $\Omega_1$ values, the number of line of action increments is adjusted such that same temporal resolution is achieved. Such a grid density

### Table 2—Basic Parameters of the 75W90 Gear Oil Used in This Study

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>$\alpha_1$ (GPa$^{-1}$)</th>
<th>$\eta_0$ (Pa s)</th>
<th>$\rho_0$ (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>18.0</td>
<td>0.1626</td>
<td>844.30</td>
</tr>
<tr>
<td>50</td>
<td>13.9</td>
<td>0.0499</td>
<td>829.30</td>
</tr>
<tr>
<td>75</td>
<td>11.4</td>
<td>0.0208</td>
<td>814.30</td>
</tr>
<tr>
<td>100</td>
<td>9.68</td>
<td>0.0106</td>
<td>799.30</td>
</tr>
</tbody>
</table>
and time discretization are found to be sufficient for this example analysis.

In Fig. 5(a), the predicted $P_{\text{mesh}}$ is compared to the measurements (Petry-Johnson, et al. (6)) for ground ($Rq = 0.3 \mu m$) and polished ($Rq = 0.08 \mu m$) tooth surfaces, with the roughness profiles fed into the model measured from the actual gear test specimens (after the run-in process). No scuffing and surface wear conditions were allowed during the gear efficiency experiments. A very good agreement between the predicted and measured $P_{\text{mesh}}$ values is observed, as they are well within 0.1 kW of each other (less than 5% relative error). Figure 5(b) shows a similar comparison for the same gear pairs within the torque range of $T = 140$ to 684 Nm at the constant rotational speed of $\Omega_1 = 6,000$ rpm. The agreement between the predictions and measurements (Petry-Johnson, et al. (6)) is still good, especially at low to moderate torque ranges, say $T \leq 450$ Nm. However, at higher torque values, the model predicts higher losses than the measurements, especially for the ground gears with rougher surfaces, as shown in the upper plot of Fig. 5(b). One possible reason for this difference could be the thermal effects. At the rotational speed of 6,000 rpm (corresponds to $u_1 = 12.5 m/s$), with the slide-to-roll ratio ($SR$) as high as 0.85 and pitch-line Hertzian pressure in excess of 1.6 GPa for $T \geq 546$ Nm, the shear heating between the contact surfaces can be expected to be more significant, impacting the lubricant viscosity and friction shear to cause such deviations. Under these conditions, the thermal reduction factor of Eq. [12a], which was regressed from the numerical solutions considering perfectly smooth surfaces for one type of lubricant under certain operating conditions (Gupta, et al. (22)), may not be sufficiently accurate. A thermal version of the proposed EHL formulation could remedy this at the expense of a significant increase in computational time. It should also be noted that Petry-Johnson, et al. (6) measured the gear box power losses, from which the bearing losses were removed using a simplified bearing torque loss formula (Khonsari and Booser (25)) to find $P_{\text{mesh}}$. Therefore, some of the differences could also be attributed to the potential errors in the estimation of the bearing mechanical power losses in the experiments of Petry-Johnson, et al. (6).

In Figs. 6(a) and 6(b), the same type of comparisons between the predictions of the proposed model and the published experiments (Petry-Johnson, et al. (6)) are shown for the finer pitch, 40-tooth gear pairs specified in Table 1. Here, the predictions are in very good agreement with the measured data within the entire ranges of speed and torque for both ground and chemically polished gear tooth surfaces. All of the predicted values are within 0.07 kW of the measured ones (less than 4% relative error). Because these 40-tooth spur gears had wider face width and higher contact ratio compared to the 23-tooth gears, the pitch line Hertzian pressures were lower (below 1.5 GPa for $T = 684$ Nm).
Fig. 7—Instantaneous film thickness (dashed line) and pressure (solid line) distributions of ground (a) 23-tooth and (b) 40-tooth gear pairs at an instant when $SR = -0.2$ for $\Omega_1 = 2,000, 6,000, \text{and} 10,000 \text{ rpm and } T = 413 \text{ Nm}$.

and the maximum slide-to-roll ratio was less than half of that of the 23-tooth gears, resulting in moderate thermal conditions.

Based on the comparisons of Figs. 5 and 6, it can be stated that the predicted influences of the gear module, surface roughness amplitudes, and the operating conditions (speed and torque) on mechanical power loss all match the measurements quantitatively. For instance, reducing the module from 3.95 to 2.32 mm by increasing the number of teeth from 23 to 40 is predicted to reduce the mechanical power losses by 41–48% for the ground gears and 48–54% for the chemically polished gears, which are both in agreement with the experiments. This corresponds to an increase of the gear mesh mechanical efficiency from 99.6 to 99.75% for the ground gear surfaces and from 99.65 to 99.85% for the chemically polished gears, which are both in agreement with the experiments. This corresponds to an increase of the gear mesh mechanical efficiency from 99.6 to 99.75% for the ground gear surfaces and from 99.65 to 99.85% for the chemically polished gears, which are both in agreement with the experiments. This corresponds to an increase of the gear mesh mechanical efficiency from 99.6 to 99.75% for the ground gear surfaces and from 99.65 to 99.85% for the chemically polished gears, which are both in agreement with the experiments. This corresponds to an increase of the gear mesh mechanical efficiency from 99.6 to 99.75% for the ground gear surfaces and from 99.65 to 99.85% for the chemically polished gears, which are both in agreement with the experiments.

Fig. 7(a) that the 23-tooth gear pair has higher contact stresses and slightly more local asperity interactions than the 40-tooth gears with the distributions shown in Fig. 7(b). As the speed increases, asperity contacts are reduced as the film thickness increases. Defining an area contact ratio as the ratio of the total asperity contact area within the lubricated contact zone to the nominal Hertzian contact area (Li and Kahraman (7), (11)), the amount of asperity interactions can be quantified as the contact moves along the tooth mesh surface. In the example case, a minor level of asperity contacts is predicted. The average values of the area contact ratios at these three speed levels are found to be 0.09, 0.04, and 0.02 for the 23-tooth gear pair and 0.05, 0.02, and 0.01 for the 40-tooth gear pair, indicating at least 90% of the contact has fluid film in between. The power loss split between the asperity friction loss and the viscous shear loss is also quantified here. For the example 23-tooth ground gear pair (more severe asperity interactions), about 18% of the total power loss is found to be associated with the asperity contacts at an operating condition of $\Omega_1 = 6,000 \text{ rpm and } T = 413 \text{ Nm}$, whereas it is only slightly elevated to 20% when the torque is increased to 684 Nm at the same speed. Keeping the input torque constant at $T = 413 \text{ Nm}$ and varying the rotational speed from 2,000 to 10,000 rpm, the contribution of the asperity friction loss is found to decrease substantially from 38 to 9%, indicating that the asperity activities are more sensitive to the surface speed. Additionally, in Fig. 8, the variations of the total traction force (including both sliding and rolling) along the tooth surface of gear 1 are shown for the same example cases as in Fig. 7. Here, the active profile of the 40-tooth gear is seen to be much shorter than the 23-tooth gear, leading

It is seen from Fig. 7(a) that the 23-tooth gear pair has higher contact stresses and slightly more local asperity interactions than the 40-tooth gears with the distributions shown in Fig. 7(b). As the speed increases, asperity contacts are reduced as the film thickness increases. Defining an area contact ratio as the ratio of the total asperity contact area within the lubricated contact zone to the nominal Hertzian contact area (Li and Kahraman (7), (11)), the amount of asperity interactions can be quantified as the contact moves along the tooth mesh surface. In the example case, a minor level of asperity contacts is predicted. The average values of the area contact ratios at these three speed levels are found to be 0.09, 0.04, and 0.02 for the 23-tooth gear pair and 0.05, 0.02, and 0.01 for the 40-tooth gear pair, indicating at least 90% of the contact has fluid film in between. The power loss split between the asperity friction loss and the viscous shear loss is also quantified here. For the example 23-tooth ground gear pair (more severe asperity interactions), about 18% of the total power loss is found to be associated with the asperity contacts at an operating condition of $\Omega_1 = 6,000 \text{ rpm and } T = 413 \text{ Nm}$, whereas it is only slightly elevated to 20% when the torque is increased to 684 Nm at the same speed. Keeping the input torque constant at $T = 413 \text{ Nm}$ and varying the rotational speed from 2,000 to 10,000 rpm, the contribution of the asperity friction loss is found to decrease substantially from 38 to 9%, indicating that the asperity activities are more sensitive to the surface speed. Additionally, in Fig. 8, the variations of the total traction force (including both sliding and rolling) along the tooth surface of gear 1 are shown for the same example cases as in Fig. 7. Here, the active profile of the 40-tooth gear is seen to be much shorter than the 23-tooth gear, leading
to the maximum $SR$ value that is less than the half of that of the 23-tooth gear. Regardless of the speed levels, the 40-tooth gears are shown to have slightly lower friction force, resulting in lower sliding power losses.

In line with the stated objectives, the individual contributions of the sliding and rolling actions to the total instantaneous gear mesh power loss ($P_{\text{mesh}}(\psi) = P_{\text{mesh}}^{s}(\psi) + P_{\text{mesh}}^{r}(\psi)$) are demonstrated next. For this purpose, the sliding ($P_{\text{mesh}}^{s}(\psi)$) and rolling ($P_{\text{mesh}}^{r}(\psi)$) components of $P_{\text{mesh}}(\psi)$ of the 23-tooth gear pair with ground surfaces are shown in Fig. 9(a) at the example operating condition of $T = 413$ Nm and $\Omega_1 = 6,000$ rpm, and Fig. 9(b) does the same for the 40-tooth gear pair also with ground surfaces. It is observed from these figures that $P_{\text{mesh}}^{r}(\psi)$ remains relatively constant along the entire mesh cycle and constitutes an important portion of the total meshing power loss, especially around the pitch point where $P_{\text{mesh}}^{r}(\psi)$ is close to zero. On average, along one complete mesh cycle, neglecting the rolling power losses as in several previous studies would result in about 31.2 and 53.7% underestimation of $P_{\text{mesh}}$ for the example 23-tooth and 40-tooth gear sets operating at this condition, respectively. It is also observed from Fig. 9 that $P_{\text{mesh}}^{s}(\psi)$ becomes increasingly dominant on $P_{\text{mesh}}(\psi)$ as the loaded contacts move away from the pitch point position for both gear sets due to the increased sliding friction. Although the $P_{\text{mesh}}^{s}(\psi)$ of both example gear pairs are close, there is a significant difference between their $P_{\text{mesh}}^{s}(\psi)$ values. The $P_{\text{mesh}}^{r}(\psi)$ values of the finer pitch (40-tooth) gear set are about the half of those of the coarser-pitch (23-tooth) gear pair because the 23-tooth gear sets experience significantly more sliding than the 40-tooth gear pairs, as stated earlier.

CONCLUSION

A model was proposed in this study to predict load-dependent (mechanical) power losses of spur gear pairs by using a transient EHL model that was specifically developed for gear contacts. This EHL model included transient effects due to a number of time-varying contact parameters including the rolling and sliding velocities, radii of curvature of the tooth contact surfaces, and normal load, allowing a continuous analysis of the gear meshing contact as it moves from the root of a tooth to its tip. The pressure and film thickness predictions of the model were used to compute the instantaneous mechanical power losses at each tooth contact as well as the overall losses of the gear mesh. Correction factors were introduced for the sliding and rolling power losses to account for thermal effects. At the end, the capabilities and accuracy of the proposed model were demonstrated by comparing its predictions to published experimental data. The contributions of the sliding and rolling power loss components to the total gear mesh power loss were investigated to show the substantial influence of rolling action on mechanical efficiency, demanding the accurate formulation for rolling losses as proposed in this work.

REFERENCES


