A Transient Mixed Elastohydrodynamic Lubrication Model for Spur Gear Pairs

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In this study, a transient, non-Newtonian, mixed elastohydrodynamic lubrication (EHL) model of involute spur gear tooth contacts is proposed. Unlike the contact between two cylindrical rollers, spur gear contacts experience a number of time-varying contact parameters including the normal load, radii of curvature, surface velocities, and slide-to-roll ratio. The proposed EHL model is designed to continuously follow the contact of a tooth pair from the root to the tip to capture the transient characteristics of lubricated spur gear contacts due to these parameter variations, instead of analyzing the contact at discrete positions assuming time-invariant parameters. The normal tooth force along the line of action is predicted by using a gear load distribution formulation and the contact radii and tangential surface velocities are computed from the kinematics and geometry of involute profiles. A unified numerical approach is adapted for handling asperity interaction in mixed EHL conditions. The differences between the transient and discrete EHL analyses are shown for a spur gear pair having smooth surfaces and different tooth profile modifications. The transient behavior predicted by the proposed model is found to be mainly due to the squeezing and pumping effects caused by sudden load changes. The lubrication behavior under rough conditions is also investigated at different operating conditions. [DOI: 10.1115/1.4000270]

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1 Introduction

Elastohydrodynamic lubrication (EHL) of the contacts formed by rough surfaces in combined rolling and sliding has been a major research topic over the years. Building on smooth surface EHL model formulations, contacts of rough surfaces were analyzed by either using microEHL models [1–5], where a continuous fluid film is maintained between the contacting surfaces, or using mixed EHL models [6–10] that are capable of handling the actual asperity contacts as part of the lubrication analysis. These models vary in several aspects in terms of their ability in handling line or point contacts, Newtonian or non-Newtonian fluids, and isothermal or thermal contact conditions. As such analyses are computationally demanding and often subject to numerical difficulties, these models tend to differ in their discretization schemes (symmetric [1,2,4–8] or asymmetric [10] contact volumes) as well as in their solution methodologies (fast Fourier transforms [6,9,10], multilevel multi-integration [1–3,5,7,8], etc.). Additional differences can be noted among the mixed EHL models in the way they handle asperity contacts, including separate treatment of wet and dry areas [6,9] and the unified schemes [7,8,10], where the asperity contact regions are handled simultaneously with the lubricated regions by employing a reduced form of the Reynolds equation. In spite of such differences, these models are all designed to analyze the contact of two rough surfaces having constant speeds, a constant normal load, and a time-invariant geometry. This is sufficient for many fundamental EHL problems such as the contacts of two cylinders, two balls, or a ball and a disk. These models are also transient in the sense that the movement of surface roughness profiles across the contact is captured. However, these models might have certain shortcomings in handling lubricated gear contacts. Gear contacts experience a number of time-varying contact parameters, making the EHL conditions transient beyond the roughness related transient effects mentioned above. With the aid of Fig. 1, these unique differences can be listed as follows:

- The radii of curvature of the contacting tooth surfaces vary in time, i.e., \( r_1 = r_1(t) \) and \( r_2 = r_2(t) \). When a tooth of the driving gear initiates the contact with its mating tooth, it has the lowest radius of curvature \( r_1 = (r_1)_{min} \), which increases gradually as the contact moves up, reaching its maximum at the tip. The opposite is true for the mating contact surface on the driven gear. This variation should be taken into account in the undeformed gap term of the film thickness equation.

- The change in radii of curvature with time (or rotation) also causes the tangential surface velocities at the contact point to vary, i.e., \( u_1 = u_1(t) \) and \( u_2 = u_2(t) \). For any contact taking place below the pitch point of the driving gear, \( u_1 < u_2(t) \), where the sliding velocity \( u_1(t) < 0 \). At the pitch point, \( u_1(t) = u_2(t) \) and \( u_1(t) = 0 \) (pure rolling). Meanwhile the surface velocity of the driven gear tooth is lower when the contact is above the pitch point of the driving gear \( u_1(t) > 0 \). This might influence the EHL formulation in many aspects. First of all the rolling velocity \( u_2(t) \) in the Reynolds equation is time-varying. Second, the viscous shear stress required to determine the flow coefficient of the non-Newtonian fluid must use this time-dependent \( u_2(t) \). Finally, the time variance of the surface roughness profiles must be applied in accordance with the instantaneous surface velocities.

- The normal load of the gear tooth contact is not constant, i.e., \( W = W(t) \). The number of tooth pairs that are loaded to transmit the applied torque fluctuates between two integers, potentially causing sudden and drastic changes in the normal force experienced by each tooth pair in contact. The
intentional tooth profile modifications and unavoidable manufacturing errors also add more variations to the contact pressure.

A small number of published studies aimed at investigating such time-varying effects of spur gear contacts. In one such study, Wang and Cheng [11,12] predicted the minimum film thickness and thermal characteristics of spur gears having smooth tooth surfaces. In their model, the elastic deformation was approximated by that of a smooth dry Hertzian contact. Larsson [13] and Wang et al. [14] proposed involute spur gear models for isothermal non-Newtonian and thermal Newtonian fluids, respectively, by employing an assumed time-varying normal tooth force as the contact moves along the line of action. They showed certain transient variations in minimum film thickness that are attributable to the change in the normal load. These three studies, while establishing the need for a specialized EHL model for spur gears, lacked the ability to handle rough surface conditions. In analysis of gear contacts, boundary and mixed EHL conditions are rather common, especially in high-load and low-speed automotive applications [15]. Inclusion of rough tooth surfaces in the EHL analysis is a must for gear contact fatigue (such as pitting and scuffing failures) and efficiency (mechanical power loss) simulations. In addition, these studies limited their treatment of the gear mesh deformations to Hertzian effects. Yet, other effects due to tooth bending, base rotation and shear deformations were shown to be equally important in defining gear mesh compliance [16]. These models were also not able to include any deviations from involute tooth profiles due to intentional tooth modifications or unavoidable manufacturing errors. Such deviations are common in real-life gear systems.

The main objective of this paper is to develop a transient mixed EHL model that includes all time-varying effects continuously. This model will consider instantaneous \( r_i(t) \), \( u_i(t) \) (where \( i = 1 \) and 2 for the driving and driven gear, respectively), and \( W(t) \), computing the first two from the gear involute geometry while using a gear load distribution model for prediction of \( W(t) \). This non-Newtonian model will employ the “unified” formulation of Hu et al. [7,8] to be able to handle excessive asperity contact conditions experienced commonly by spur gears. It was pointed out by Venner [17] that the solutions obtained by using conventional second order discretization of the governing equations might be highly dependent on the mesh density. Paying special attention to such discretization errors, the governing equations will be solved by using a second order discretization scheme [2] with a sufficient finely meshed grid. For the surface elastic deformation, the FFT convolution method [6,10] will be implemented to minimize the computational time required. The model will be used to demonstrate the transient behavior associated with the time-varying contact parameters. Rough surface simulations will also be provided to demonstrate the ability of the proposed model in handling asperity contacts.

This work will be kept limited to spur gears, where the contact lines are parallel to the gear rotation axes and the load distributions are quite uniform along the axial direction. This allows a line contact formulation with the one-dimensional Reynolds equation to be suitable for this problem. For other types of gear pairs such as helical and hypoid gears, additional complications are present as, at any given time instant, velocity, radii, and the normal pressure are not uniform across the contact zone. Analysis of such gear contact problems are beyond the scope of this study.

In Secs. 2 and 3, the term transient EHL will be used to represent the EHL model with time-varying \( r_i(t) \), \( u_i(t) \), and \( W(t) \), while discrete EHL will mean analyses at any given tooth profile location with the geometry, speed, and load parameters treated as constants.

### 2 Transient, Non-Newtonian, Mixed EHL Model of Rough-Surface Spur Gear Contacts

#### 2.1 Gear Contact Parameters

The transverse plane of a spur gear pair is shown in Fig. 2. A local coordinate frame \( xy \) is
located at the transient contact point C, with the x-axis along the rolling direction (tangential to the contacting surfaces) and the y-axis on the plane of action along the line $B_1B_2$. The dashed circles denote the base circles, defined by the radii $r_{g1}$ and $r_{g2}$. At this mesh position, instantaneous local contact has the radii of curvature of

$$r_1(t) = B_1C = r_{g1}\theta_1(t)$$  \hspace{1cm} (1a)$$

$$r_2(t) = B_2C = r_{g2}\theta_2(t)$$  \hspace{1cm} (1b)$$

where $\theta_1(t)$ and $\theta_2(t)$ are the roll angles of the driving and driven gears, as shown in Fig. 2. As the gears rotate in mesh, the contact point C is first initiated at the start of active profile (SAP) of the driving gear 1 in the dedendum region. It moves upward toward the tip, passing through the pitch point and leaving at the tip. For this driving gear, the instantaneous radius of curvature $r_1(t)$ is at its minimum when $C$ is at the SAP and maximum when $C$ is at the tip of the tooth. Meanwhile, the contact of the tooth of the driven gear 2 is initiated at its tip and moves toward its SAP as the gears rotate in mesh. With this, $r_2(t)$ is maximum initially and is reduced gradually until the tooth leaves the contact at its own SAP. With the instantaneous contact radii of curvature, the tangential surface velocities at point C also vary as

$$u_1(t) = \omega_1 r_1(t)$$  \hspace{1cm} (2a)$$

$$u_2(t) = \frac{N_2}{N_1} \omega_1 r_2(t)$$  \hspace{1cm} (2b)$$

where $N_1$ and $N_2$ are the number of teeth of gears 1 and 2, and $\omega_1$ is the angular speed of gear 1. With this, the time-varying rolling velocity of the contact is $u_1(t) = 1/2[u_1(t) + u_2(t)]$. It is also noted that $u_1(t) < u_2(t)$ when the contact point C is below the pitch point in the dedendum region of gear 1. As a result, the slide-to-roll ratio $SR(t) = u_1(t)/u_2(t)$ is negative in this region, having its maximum absolute value when the contact is at the SAP of gear 1. When the contact arrives at the pitch point, $u_1(t) = 0$, resulting in pure rolling with $SR = 0$ and $u_2(t) = u_1(t) = u_2(t)$. As the contact moves up toward the tip in the addendum region, $SR(t) > 0$, reaching its maximum positive value at the tip.

For a gear pair having an involute (profile) contact ratio of $\varepsilon$ (meaning, on average, $\varepsilon$ tooth pairs are loaded), the number of loaded tooth pairs fluctuates between two integer values bounding $\varepsilon$. For instance, if $\varepsilon = 1.6$, there are two tooth pairs in contact sharing the load for about 60% of the meshing cycle, while a single tooth pair must carry the entire load for the remaining 40% of the mesh cycle. When the contact is near SAP of the driving gear, another contact of the preceding tooth pair exists near the tip of the driving gear tooth. As point C moves up, it reaches the lowest point of single tooth contact (LPSTC) when the preceding tooth pair leaves contact. Between the LPSTC and the highest point of single tooth contact (HPSCT) above the pitch point, there is only a single tooth pair in contact. Beyond HPSCT, another contact is initiated by the next tooth pair to help share some of the load. This fluctuation in the number of loaded tooth pairs in mesh together with the intentional tooth profile modifications (or manufacturing errors) and tooth deflections cause the normal load acting on the contact point C to vary with the mesh position. For the computation of the normal contact loads of the spur gear pair, a load distribution model similar to the one proposed by Conry and Seireg [16] will be used here. It computes elastic deformations at any point of the gear surface given the tooth compliance, applied torque, and initial tooth separations under no load, by considering the conditions of compatibility and equilibrium. The compatibility condition states that the sum of elastic deformations of the two bodies and the initial separation must be greater than or equal to the rigid body displacement for every point $q$ within the contact zone.
practical definition of an asperity contact is required. In this study, a threshold film thickness value \( e \) was used to distinguish between the use of Eq. (4a) for a fluid contact and Eq. (5) for an asperity contact. With the consideration that physical gap between the mating surfaces cannot accommodate less than two layers of lubricant molecules for any hydrodynamic flow to happen [19], an asperity contact condition is assumed at any grid point if \( h \) is less than the threshold value of \( e=2.5 \) nm.

The film thickness of a contact point at any coordinate \( x \) at time \( t \) is defined as

\[
h(x,t) = h_0(t) + g_0(x,t) + V(x,t) - R_1(x,t) - R_2(x,t)
\]

where \( h_0(t) \) is the reference film thickness, and \( R_1(x,t) \) and \( R_2(x,t) \) are the roughness profiles of the two surfaces at time \( t \). \( R_1(x,t) \) and \( R_2(x,t) \) in Eq. (6) move at different rates according to the time-varying surface velocities \( u_1(t) \) and \( u_2(t) \), which respond to transient slide-to-roll ratio \( SR(t) \), highlighting another difference from the discrete EHL models. The term \( g_0(x,t) \) is the gap between the two surfaces before any elastic deformation occurs, defined as

\[
g_0(x,t) = \frac{x^2}{2\epsilon_0(t)}
\]

where \( \epsilon_0(t) \) is the equivalent radius of curvature. The timedependence of the gap term is also unique to the contact of a spur gear pair. Additionally, \( V(x,t) \) represents the sum of the elastic deformations due to the normal load applied to the two contacting bodies, and is given as [20]

\[
V(x,t) = \int_{x_s}^{x_f} K(x-x')p(x',t)dx'
\]

where \( x_s \) and \( x_f \) are the start and end points of the computational domain of the contact zone and \( K(x) \) is the influence coefficient, given in the form of

\[
K(x) = -\frac{4 \ln |x|}{\pi E'}
\]

Here, \( E' \) is the equivalent modulus of elasticity. In order to avoid overestimating the lubricant viscosity when pressure is high, a refined two-slope viscosity-pressure model of Allen will be used in the form modified later by Goglia et al. [21]

\[
\eta = \begin{cases} 
\eta_0 \exp(\alpha_1 p), & p < p_a \\
\eta_0 \exp(\alpha_2 p + c_1 p^2 + c_2 p^3), & p_a \leq p \leq p_b \\
\eta_0 \exp(\alpha_2 (p - p_b)), & p > p_b 
\end{cases}
\]

Here, \( \alpha_1 \) and \( \alpha_2 \) are the viscosity-pressure coefficients for the low \((p < p_a)\) and high \((p > p_b)\) pressure ranges, respectively, \( p_a \) is the transition pressure value between these two ranges, and \( p_a \) and \( p_b \) are the threshold pressure values of the low and high pressure ranges, respectively. The constants \( c_i \) \((i \in [1,4])\) are determined such that both \( \eta \) and \( \partial \eta / \partial p \) are continuous at the two threshold pressures. In addition, considering a compressible lubricant, the density-pressure relationship can be modeled by [22]

\[
p = \rho_0 \frac{(1 + \gamma p)}{(1 + \lambda p)}
\]

where \( \gamma=2.266 \times 10^{-8} \text{ Pa}^{-1} \) and \( \lambda=1.683 \times 10^{-9} \text{ Pa}^{-1} \).

The last equation of interest here is the load balance equation, which states that the total contact force due to the pressure distribution (both hydrodynamic and asperity contacts) over the entire contact area must equal the normal tooth load applied at that instant, that is

\[
W(t) = \int_{x_1}^{x_2} p(x,t)dx
\]

Here, the tooth force intensity along the contact line (normal force per unit face width) \( W(t) \) is not constant, but varies in a specific way according to the tooth-to-tooth load sharing characteristics of the gear mesh. Equation (11) is not required for solution of Eqs. (4a) and (5), but rather acts as a check for the solution. If the predicted pressure distribution does not equal the applied normal force, the reference film thickness \( h_0(t) \) in Eq. (6) must be adjusted within a load iteration loop until Eq. (11) is satisfied.

2.3 Discretization. In order ensure the consistency of the EHL analysis along the line of action, the size of the computational domain is fixed through the entire simulation starting at SAP and ending at the tip of the tooth of gear 1. A refined contact grid consisting of \( N \) grid elements is used in order to capture the surface roughness geometry sufficiently. At a given time increment \( ts \), the lubricant viscosity, density, film thickness, and pressure are all assumed to be uniform within each contact element \( i \) \((i \in [1,N])\), represented by the values at the center point of the grid cell.

For the Poiseuille term of Eq. (4a), a second order central finite difference scheme is applied as

\[
\frac{\partial}{\partial x} \left( \frac{\partial p(x,t)}{\partial x} \right) = \frac{f_{i-1/2,t} p_{i-1/2,t} - f_{i+1/2,t} p_{i+1/2,t}}{\Delta x^2}
\]

with the flow coefficients approximated by

\[
f_{i-1/2,t} = \frac{1}{2} \left[ f_{i-0,t} + f_{i+0,t} \right]
\]

\[
f_{i+1/2,t} = \frac{1}{2} \left[ f_{i-0,t} + f_{i+0,t} \right]
\]

Since the central difference discretization might introduce oscillations to the solution [23], a second order backward scheme is employed for the Couette term as

\[
\frac{\partial (p h)}{\partial x} = \frac{3}{2} \left( \frac{p_{i+1,t} h_{i+1,t} - 2 p_{i-1,t} h_{i-1,t} + p_{i-2,t} h_{i-2,t}}{\Delta x} \right)
\]

The squeeze term is linearized using the second order backward scheme as well as

\[
\frac{\partial (p h)}{\partial t} = \frac{3}{2} \left( \frac{p_{i+1,t} h_{i+1,t} - 2 p_{i,t} h_{i,t} + p_{i-1,t} h_{i-1,t}}{\Delta x} \right)
\]

3 Example Results

In this section, several simulation results will be presented to (i) highlight the differences between the proposed transient EHL model and a discrete EHL equivalent, (ii) to show the impact of tooth profile modifications on transient behavior through its influence on the normal tooth load, and (iii) to demonstrate the effectiveness of the model in handling mixed EHL conditions.

3.1 Discrete Versus Transient Spur Gear EHL Models. First, this example gear pair with no tooth modifications (perfectly involute tooth surfaces) will be considered. At a representative
input speed of 2000 rpm, the variation in the instantaneous radii of curvature $r_1$ and $r_2$, as well as the equivalent radius of curvature $r_{eq}$ of this example gear pair are plotted in Fig. 3(a) as the functions of the roll angle $\theta_1$ of gear 1. Here, $r_1$ and $r_2$ vary linearly with $\theta_1$. $r_{eq}$ increases as the contact moves up along the tooth of gear 1, reaching its maximum value when $r_1=r_2=2r_{eq}$ then starts to reduce. In relation to this, the surface velocities $u_1$ and $u_2$ also vary linearly with $\theta_1$ and $u_1=u_2=ur$ when the contact is at the pitch point, as shown in Fig. 3(b). The variations in $u_s$ and $u_r$ are illustrated in the same figure as well. The value of $u_r$ at the rotational speed considered changes from $-3$ m/s at the SAP to 0 m/s at the pitch point and 3 m/s at the tip, corresponding to a SR range of about $-1.0$ to $1.0$. Likewise, $u_s$ experiences a slight linear increase as the gears roll in mesh. It is noted here that these variations can be more or less significant in other gear sets having different speed ratios, tooth size, and involute geometry parameters.

In Fig. 3(c), the variation in the normal tooth contact force $W$ with $\theta_1$ computed from the load distribution model presented in Sec. 2.1 is illustrated at an input torque of 250 Nm. Here, $W$ increases linearly from 2.3 kN at SAP to about 4 kN right before the LPSTC, where it experiences a sudden jump up to 7 kN as the gear mesh transitions from double- to single tooth contact. A similar sudden jump down occurs at HPSTC, now with a transition from single- to double-tooth contact. Beyond HPSTC, $W$ decreases linearly as the contact moves toward the tip.

A typical automatic transmission fluid is used as the lubricant in these simulations. At 95 °C, this lubricant has the pressure-viscosity coefficients of $\alpha_1=12.3$ GPA$^{-1}$ and $\alpha_2=1.2$ GPA$^{-1}$, the transition pressure value of $p_s=0.9$ GPA with $p_a=0.63$ GPA and $p_p=1.26$ GPA, the ambient dynamic viscosity of $\eta_0=0.0054$ Pa s, and the ambient density of $\rho_0=820.95$ kg/m$^3$. Here, the value of $\alpha_2$ is assumed. All the analyses consider a fixed computational domain of $-2.5a_{max}\leq x\leq 1.5a_{max}$, where $a_{max}$ is the maximum value of the half-Hertzian width of the contact during its travel from SAP to tip. The number of grid nodes in the profile direction is chosen to be $N=512$, corresponding to a mesh size of 1.3 $\mu$m. The line of action is discretized by 1000 contact positions (in comparison to 200 in Ref. [13] and 180 in Ref. [14]), which corresponds to a temporal resolution of $\Delta t=\Delta \theta_1/\omega_1=2.1 \times 10^{-6}$ s at this speed value.

With these operating conditions and lubricant, two sets of analyses were performed next: one with the proposed transient EHL model and one with a discrete equivalent of the same model. Here, the transient EHL analysis consists of a single simulation that follows the contact through its travel along the tooth surfaces, while the discrete EHL analyses were performed at a large number of discrete roll angle ($\theta_1$) values. The minimum film thickness $h_{min}$ values predicted from these two analyses are compared in Fig. 4 for this unmodified gear pair having smooth tooth surfaces. It is seen that the predicted $h_{min}$ values of the discrete and transient EHL models differ very little at the double-tooth contact regions when $\theta_1<18.5$ deg and $\theta_1>28$ deg. In between these two positions, however, significant differences are observed between the predictions of discrete and transient EHL models. The
dicts a fluctuation of h noted from the p zone of the contact results in the changes shown in Fig. 4. It is points A–G. At point A passes through the LPSTC. These points are marked in Fig. 4 as Fig. 5 at seven discrete roll angles right after the tooth contact.

In order to better describe this transient behavior, snapshots of the transient pressure and film thickness distributions are shown in Fig. 5 at seven discrete roll angles right after the tooth contact passes through the LPSTC. These points are marked in Fig. 4 as points A–G. At point A (Fig. 5(a)) before the contact reaches the LPSTC, typical steady-state smooth surface h and p distributions are evident with a contact width of nearly 0.25 mm. Small pressure ripples are initiated at the inlet and outlet of the contact at point B (Fig. 5(b)) right after the sudden increase in W. This is due to the strong transient squeezing effect caused by this sudden increase. The ripple at the inlet propagates to the right as the contact proceeds to the other locations, as shown in Figs. 5(c)–5(g), while the right-hand side ripple leaves the contact zone right away. As a result, a well-defined ripple in the film thickness distribution is generated in Fig. 5(b) at point B, which is shown to move toward the outlet of the contact. The size of this ripple in comparison to that of the conventional h_{min} located at the outlet zone of the contact results in the changes shown in Fig. 4. It is noted from the p and h distributions in Fig. 5(e) at point G (before the HPSTC when W decreases suddenly) that the contact width is wider than the one shown in Fig. 5(a) for a double-tooth contact condition.

Similar transient behavior is evident after the contact passes the HPSTC, as shown in Fig. 6, for the points marked as H to L in Fig. 4. The sudden reduction in W causes a drastic contraction of the contact width and entrainment of more oil into the contact. This pumping effect, however, is less evident compared with the squeezing effect right after the LPSTC. In Fig. 6(a), a ripple in the h distribution originate on the left edge of the contact (point H), which progresses to the right before dissipating fully at $\theta_1 = 28$ deg.

Here, the transient pumping phenomena shown in Fig. 6 is rather independent from the squeezing phenomena illustrated in Fig. 5. This is partly because the duration of the single tooth contact regime was long enough in this gear set for transient behavior due to squeezing to dissipate. However, if a gear pair with a higher involute contact ratio (higher than $e=1.744$ of this example gear set) is considered for improving the noise characteristics, the gear pair would then spend a shorter period of time in the single tooth contact regime, potentially initiating the pumping transients before squeezing effects die out.

The example gear set used in Figs. 4–6 had unmodified (purely involute) profiles such that the changes in W observed in Fig. 3(c) were very drastic and discontinuous. It is common in gear engineering to modify tooth profile surfaces through removal of additional material to compensate for tooth deflection to reduce motion transmission errors and vibration excitations as well as minimizing such sudden impacts. Two common forms of such tooth modifications will be considered next to assess the accuracy of the proposed transient EHL model against the discrete EHL solutions. Figure 7(a) defines linear modifications (commonly known as a tip relief) of 15 μm that starts at a roll angle values of 23 deg for both gears. Meanwhile, in Fig. 7(b), a profile crown (a circular profile modification) of 15 μm is applied to both gears. At the same input torque value of 250 Nm, the corresponding
changes to $W$ are shown in Fig. 8 in comparison to the unmodified gears. As shown in this figure, application of the linear tip relieves shown in Fig. 7(a) help reduce the sudden tooth load changes at the LPSTC and HPSTC, while the circular involute crown in Fig. 7(b) literally eliminates any sudden jumps of $W$. The corresponding $h_{\text{min}}$ values of these modified gear profiles predicted by the transient and discrete EHL models are compared in Figs. 9(a) and 9(b). In Fig. 9(a) for the gear pair with the linear tip relief, the transient effects right after the LPSTC and HPSTC remain the same qualitatively, but with much lower ripple amplitudes. In return, the differences between the discrete and transient EHL models at the double-tooth contact regions are more pronounced, with discrete EHL model predicting slightly lower $h_{\text{min}}$ below the LPSTC and slightly higher above the HPSTC. On the other hand, fluctuations of $h_{\text{min}}$ values with the rotation of gears is eliminated almost entirely with the gears having the involute crown modifications, as shown in Fig. 9(b), while the discrete and transient EHL models deviate slightly more from each other.

3.2 Influence of Tooth Surface Roughness Profiles. In this section measured tooth surface roughnesses shown in Fig. 10 are introduced to assess the ability of the transient EHL model in handling mixed EHL conditions in a transient manner. These measured profiles represent the run-in roughnesses of both gears in the profile direction due to hard gear finishing grinding process. The root-mean-square roughness amplitudes of these two surfaces are $R_q^1=0.55 \, \mu m$ and $R_q^2=0.32 \, \mu m$.

Figure 11 shows instantaneous $p$ and $h$ distributions of the example gear pair (with the profile crown modification shown in Fig. 7(b)) predicted by the proposed transient mixed EHL model with these ground surfaces at $\theta_i=16.6$ deg, 19.3 deg (at the LPSTC), 20.9 deg (at the pitch point), 22.9 deg (at the HPSTC), and 26 deg. In this simulation, the input torque was 250 Nm and the input speed is 2000 rpm. The transient effect of the surface
roughnesses on both $p$ and $h$ distributions are seen in this figure. The instantaneous pressure magnitudes change drastically along the $x$-axis due to the transient roughness effects, with the local pressures as high as 4.5 GPa (more than three times the maximum pressure under smooth condition in Figs. 5 and 6). One can also observe the evidence of multiple asperity contacts in the instantaneous $h$ distributions shown in the second column in Fig. 11, which exhibit several locations where $h=0$, indicating asperity interactions at these contact locations.

In order to quantify the severity of the occurrence of asperity contacts, first, an instantaneous asperity area contact ratio $C_a$ is defined as the ratio of the number of grid cells at asperity contact to the total number of loaded grid cells at each time instant. Figure 12(a) shows the variation in $C_a$ with $\theta_i$ for the same simulation shown in Fig. 11. It is noted that the variation in $C_a$ is rather transient, with its value varying between 0.2 (20% asperity contact area) and 0 (no asperity contact). Likewise, a second asperity contact parameter $C_{\ell}$ is defined as the ratio of the load carried by the asperities to the total normal contact load $W$. For the same simulation shown in Figs. 11 and 12(a), the variation in $C_{\ell}$ is shown in Fig. 13(a). Here, $C_{\ell} > 0.5$ at several instances near the SAP and tip of the tooth, indicating that the EHL conditions experienced by this gear pair under these conditions is truly mixed type. It is also clear that the proposed model is capable of handling such levels of asperity contact with no numerical difficulties.

Given the fact that the corresponding smooth surface film thickness value at the pitch point location is on the order of 0.1 $\mu$m and the measured roughness profiles shown in Fig. 10 correspond to a composite root-mean-square roughness amplitude of nearly 0.6 $\mu$m, one would expect more asperity interactions than those demonstrated in Figs. 11, 12(a), and 13(a). One reason for this is that the run-in surfaces shown in Fig. 10 have low-amplitude peaks with rounded tips, while the valleys into the material are deep and sharp. These sharp valleys, while increasing the $R_q$ values of the surfaces significantly, have little adverse effects on the lubricated contact conditions. As a result, $C_a < 0.2$ and $C_{\ell} < 0.5$ in Figs. 12(a) and 13(a), respectively.

In Figs. 12(b) and 13(b), $C_a$ and $C_{\ell}$ are plotted for the same gear set under the same conditions, except that the speed of gear 1 is now increased to 10,000 rpm. In comparison to the values at 2000 rpm, $C_a$ and $C_{\ell}$ amplitudes are reduced as expected, since an increase in film thickness with increased speed reduces asperity interactions.

It is noted here that each complete spur gear rough surface transient EHL analysis took about the same CPU time as a single discrete EHL analysis with the same roughness profiles at one certain contact position. Even when the film thickness differences demonstrated in Sec. 2 can be overlooked, one would require at least a dozen or more of discrete EHL analyses to describe the lubrication behavior of a gear pair in place of a single transient EHL analysis. This computational advantage of the proposed transient EHL model of a spur gear pair over a discrete EHL model is especially critical for other lubrication related analyses of contact fatigue and mechanical efficiency.

4 Conclusions

In this study, a transient, non-Newtonian, mixed EHL model for involute spur gear is proposed. The model accounts for the variations in several key contact parameters (normal contact force, radii of curvature, surface velocities, and slide-to-roll ratio) and continuously follows the contact of a tooth pair from the root to the tip, instead of analyzing it at discrete positions, assuming time-invariant parameters. Using the proposed continuous gear EHL model, the minimum film thickness variations along the line of action was found to be substantially altered from the steady-state solutions, primarily due to the sudden changes in the tooth loads, pointing to the necessity to take into account the time-dependence of the gear key contact parameters for accurate predictions. Properly modified tooth profiles with smoother single-to-double and double-to-single tooth contacts were shown to reduce the ripple effect caused by squeezing and pumping. At the end, analyses with measured gear surface roughnesses were shown to demonstrate the effectiveness of the proposed model in handling the excessive asperity contact conditions.

Nomenclature

\begin{itemize}
  \item $a = \text{half-Hertzian contact width}$
  \item $a_{\max} = \text{maximum half-Hertzian contact width along the line of action}$
  \item $C_a = \text{area contact ratio}$
  \item $C_{\ell} = \text{load contact ratio}$
\end{itemize}
\(E_1, E_2 = \) Young’s modulus of gears 1 and 2, respectively  
\(E' = \) equivalent Young’s modulus, \(E' = \frac{2(1 - v^2)}{E_1} + \frac{1 - v^2}{E_2}\)  
\(e = \) film thickness cutoff value  
\(f = \) flow coefficient  
\(g_0 = \) geometry gap before deformation  
\(h = \) film thickness  
\(N_1, N_2 = \) number of tooth of gears 1 and 2, respectively  
\(p = \) pressure  
\(R_1, R_2 = \) roughness profiles of surfaces 1 and 2, respectively  
\(R_{q1}, R_{q2} = \) root-mean-square roughness amplitudes of surfaces 1 and 2, respectively  
\(r_1, r_2 = \) contact radii of curvature of gears 1 and 2, respectively  
\(r_{b1}, r_{b2} = \) base circle radii of gears 1 and 2, respectively  
\(r_{eq} = \) equivalent radius of curvature, \(r_{eq} = r_1 r_2 / (r_1 + r_2)\)  
\(SR = \) slide-to-roll ratio, \(SR = u_s / u_t\)  
\(t = \) time  
\(u_{1}, u_{2} = \) surface velocities in the direction of rolling of gears 1 and 2, respectively  
\(u_r = \) rolling velocity, \(u_r = 1/2(u_1 + u_2)\)  
\(u_s = \) sliding velocity, \(u_s = u_1 - u_2\)  
\(V = \) elastic surface deformation  
\(W = \) normal tooth force  
\(W' = \) normal tooth force per unit face width  
\(x = \) coordinate along the rolling direction  
\(\alpha_1, \alpha_2 = \) pressure-viscosity coefficients within low and high pressure ranges, respectively  
\(e = \) involute contact ratio  
\(\eta = \) lubricant viscosity  
\(\eta_0 = \) lubricant viscosity at ambient pressure  
\(\theta_1, \theta_2 = \) roll angle of gears 1 and 2, respectively  
\(\nu_1, \nu_2 = \) Poisson’s ratio of gears 1 and 2, respectively  
\(\rho = \) lubricant density  
\(\rho_0 = \) lubricant density at ambient pressure  
\(\sigma_0 = \) reference shear stress of the lubricant  
\(\omega_1, \omega_2 = \) angular velocities of gears 1 and 2, respectively

References


