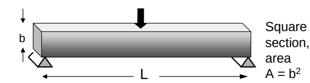
Case Studies in Materials Selection

- Light stiff beam
- Support structure for concave parabolic mirrors
- Flywheel Energy Storage systems

More info: "**Materials Selection in Mechanical Design**", **Chapters 5 and 6**

Example: Light-stiff Beam

Select the best material for a light and stiff column of length L supports a load of F



- For this problem
  - Function: Beam
  - Objective: Minimize mass
  - Constraints: Length L, Bending Stiffness S
  - Free variables: Material, cross-sectional area
- During the deflection of the beam under three-point loading is
- The stiffness is

Analysis

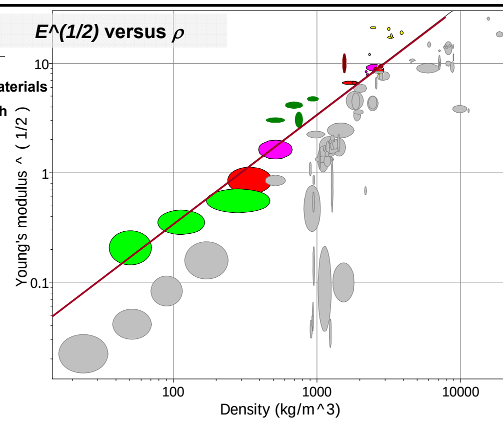
- To minimize mass:

  Eliminate A to get

- To minimize mass, the material performance index to be maximized is

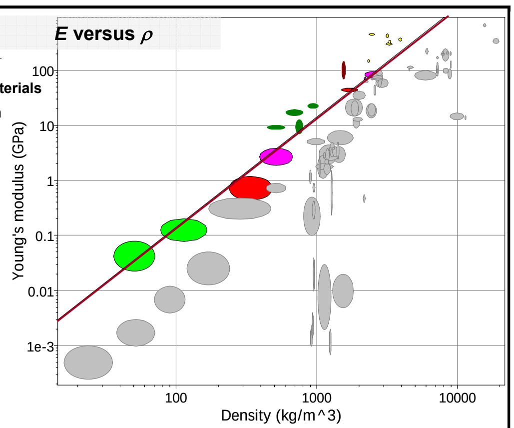
E^(1/2) versus ρ

To compare materials
Draw a line with
Slope = 1

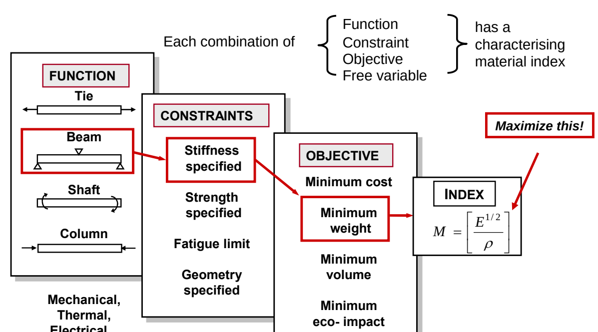


E versus ρ

To compare materials
Draw a line with
Slope = 2

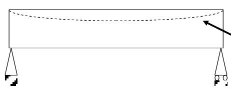


Material "indices"



Support structure for concave parabolic mirrors

- Concave parabolic mirrors are used in telescopes – both optical and radio
- The shape is critical. Therefore the material used should be relatively stiff and dimensionally stable.
  - A ground based telescope will be subject to gravity
  - A space telescope will have to be launched
  - Mass is also an important factor



Reflective surface

Support structure for parabolic mirrors

Model the support structure as a circular disk of radius R and thickness t

Function	Precision mirror
Objective	Minimize mass
Constraints	•Radius R is specified •Must not distort more than δ under self weight •High dimensional stability – no creep, low thermal expansion
Free variables:	Material Thickness

Support structure for parabolic mirrors

- Appendix A lists relationships among various types of loading and the response of different basic structures.
- For this problem, we need Appendix A-7
- The deflection of a disk under a pressure differential Δp is
- For deflection under self load, replace Δp by the weight per unit area $\rho g t$ to get

for a material with a Poisson's ratio of 0.3

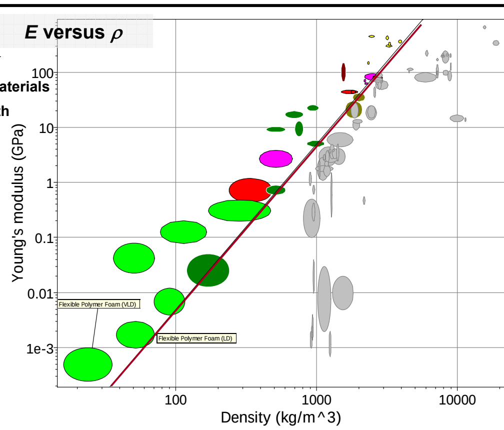
- For an optical mirror, this deflection should be of the order of $10\mu m$ or less
  - This depends on the wavelength

Support structure for parabolic mirrors

- The mass of the mirror is
- The lightest mirror is the one with the greatest value of M
- Other properties of importance could include the thermal expansion coefficient, high melting point, low moisture sensitivity, etc.

E versus *ρ*

To compare materials
Draw a line with
Slope = 3



- **Also specify E_{min} = 100GPa using Limit Stage**

Select Materials - All Stages

Name Identity

Alumina
Aluminum nitride
Aluminum/Silicon carbide composite
Boron carbide
CFRP, epoxy matrix (isotropic)
Silicon
Silicon carbide
Silicon nitride

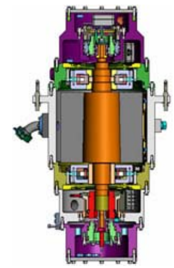
Materials for Energy Storage Flywheels

- A flywheel is an energy storage system in which energy is stored as the kinetic energy of a spinning mass.
  - Children's toy cars – typically made of lead
  - Gyrobus – Switzerland in the 1950s – large steel disks spinning at 3000 rpm
  - Uninterruptible power supply (UPS)
  - Regenerative braking systems
  - Smooth out uneven energy delivery (flywheels in IC engines)

http://en.wikipedia.org/wiki/Flywheel_energy_storage

Materials for Energy Storage Flywheels

- A typical flywheel energy storage system consists of rotor suspended by bearings inside a vacuum chamber to reduce friction, connected to a combination electric motor/electric generator
- The rotors are generally made of steel on smaller systems, but large systems use high-tensile-strength fibers (such as carbon fibers) embedded in epoxy resins, or some other high-strength composite material.
- Energy is stored by using an electric motor to increase the speed of the spinning flywheel. The system releases its energy by using the momentum of the flywheel to power the motor/generator.



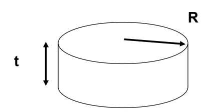
Materials for Energy Storage Flywheels

- There are a wide range of materials that have been used for making the rotors in flywheel systems
  - Lead (very high density)
  - Steel (high density)
  - Fiber reinforced composites (relatively light)
- What drives the choice of these diverse materials for similar applications?
- There are actually two different objectives for the above applications
  - Commercial and space based energy storage systems need to maximize energy stored per unit mass
  - Flywheels in toys are required to maximize energy storage per unit volume for a given angular velocity

Materials for Energy Storage Flywheels

- **Maximum energy storage per unit mass**

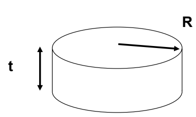
Function	Flywheel
Objective	Maximize kinetic energy per unit mass
Constraints	•Outer radius R is fixed •Must not burst •Must not crack
Free variables:	Material



Materials for Energy Storage Flywheels

- **Maximum energy storage per unit volume at fixed ω**

Function	Flywheel
Objective	Maximize kinetic energy per unit volume
Constraints	•Outer radius R is fixed •Must not burst •Must not crack
Free variables:	Material



Materials for Energy Storage Flywheels

- Kinetic energy of a spinning mass is given by

- Kinetic energy per unit mass is

Materials for Energy Storage Flywheels

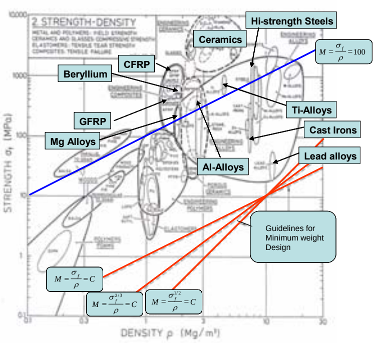
- Since KE is to be maximized
  - Increase angular velocity
  - Increase length of the shaft
  - Increase density of the material
- Constraints
  - Should not burst. Therefore the tensile hoop stresses that develop due to centrifugal forces should not exceed the failure stress
- The maximum principal stress that develops when a solid disk is spinning is

Materials for Energy Storage Flywheels

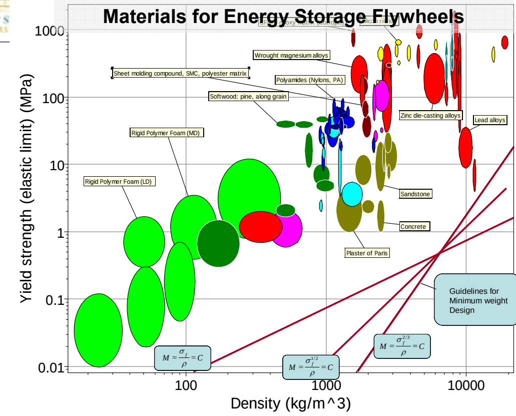
- Rearranging the terms in the above two equations gives
- The Poisson's ratio for most engineering materials is about (1/3). So in order to maximize the energy stored per unit mass, one needs to maximize the material index *M* given by

Materials for Energy Storage Flywheels

Similar to Figure 4.4



Materials for Energy Storage Flywheels



Materials for Energy Storage Flywheels

Candidate Materials for flywheels

Material	M (kJ/kg)	Comments
Ceramics	200 – 2000 (in compression only)	Brittle and weak in tension – unsuitable
Composites: CFRP	200 – 500	The best performance
Composites: GFRP	100 – 400	Good performance, Less expensive than CFRP
Beryllium	300	Good performance, expensive, toxic
High Strength Steels High Strength Al alloys High Strength Mg alloys Ti alloys	100 - 200	All about the same performance. Steel and Al alloys are less expensive than Mg or Ti alloys
Lead	3	Traditional
Cast iron	8-10	Traditional

Materials for Energy Storage Flywheels

- Lead and cast iron have very low material indices M compared to the other materials
- Why are they used in applications like toy cars?
- Commercial energy storage systems spin at very high angular velocities. Therefore the danger of developing tensile stresses of the order of the failure stress is real
- In a toy car, the lead flywheel is usually spun by a pulled string. The speeds are no where near that which can cause tensile failure.
- The volume of the toy is however limited. The objective then becomes to maximize energy storage for minimum volume, and the constraint of the flywheel falling apart can be removed

Materials for Energy Storage Flywheels

- The kinetic energy per unit volume is

- The material index now becomes simply

- So materials with the highest density are best – lead, cast iron (tungsten, molybdenum, gold, silver …?)
  - Cost eliminates the last few elements in the above list