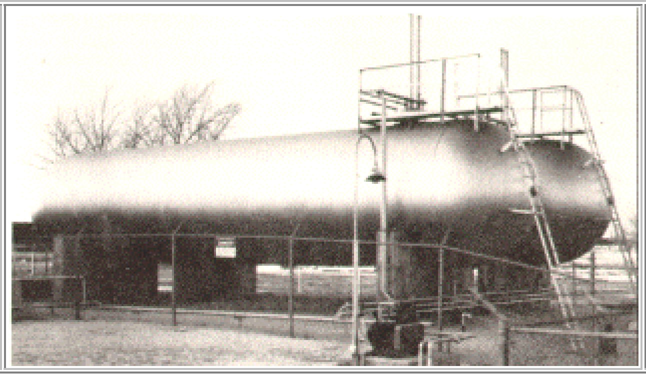
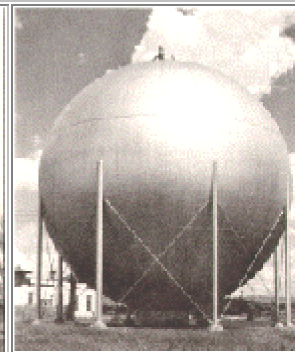
Pressure Vessels





Figures from Mechanics of Materials by Higdon et al. 4th edition, John Wiley and Sons, 1985

Educational Objectives

1. Introduce the concept of stress and strain.

2. Introduce Hooke's law, Young's modulus and Poisson's ratio.
3. Show basic equations for obtaining stresses in a thin walled pressure vessel.

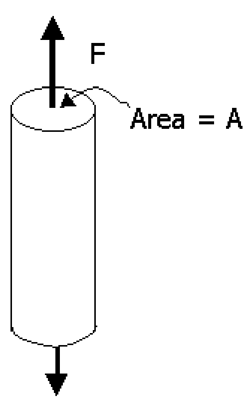
3. Introduce electrical resistance strain gauges
4. Measure internal pressure in a soda pop can using strain gauges.

Background Information

1. Stress

The ability of a structural component to support a load depends on two factors, the inherent strength of the material and the area over which the load is being supported. The term stress is used to identify the intensity of force.

$$\text{Stress } \sigma = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$



Stress is usually measured in units of N/mm^2 or Pascals (Pa) in SI units or in units of lb/in^2 or psi in the conventional system of units.

A component with a larger area can support more force even when the material that it is made of is not changed.

2. Strain

All objects deform when acted upon by a force. The deformation is called elastic if the object returns to its original size and shape when the force is removed. Permanent deformation is called plastic deformation. Generally, a longer the object greater is the amount of deformation it will undergo. Think about trying to stretch a piece of rubber band. Longer the rubber band more it stretches. Strain is a normalized measure of the amount of deformation. Longitudinal strain is defined as

$$Strain\ \varepsilon = \frac{Change\ in\ Length}{Original\ Length} = \frac{\Delta L}{L_0}$$

Since strain is a ratio of two lengths, it is unit-less, though sometimes it is measured in units or mm/mm or inch/inch.

3. Hooke's Law

Hooke's law states that during elastic deformation, stress and strain are proportional. The proportionality constant is called the Young's Modulus, and is usually indicated by the letter E. This is a property of the material.

$$\sigma = E\varepsilon$$

The units of E are Pascal in SI or psi in conventional units.

In the figure above, when a stress s is applied, the length of the cylinder will change by an amount

$$\Delta L = \varepsilon L_0 = \frac{FL_0}{AE}$$

During elastic deformation $\varepsilon < 0.001$. In other words, the length of the cylinder will change by less than about 1/1000 of the original length.

4. **Poisson' Ratio**

In the figure above, when the length of the cylinder increases due to the application of a stress along its axis, there is a decrease in diameter of the cylinder. In other words, the application of an axial stress causes an axial strain, as well as a transverse strain. The amount of transverse strain is proportional to the axial strain.

$$\text{Poisson's Ratio} = \nu = -\frac{\varepsilon_{transverse}}{\varepsilon_{axial}}$$

Poisson's ratio is unit-less.

5. **Thin-walled Pressure Vessel**

A pressure vessel is any container that is used to store a liquid or gas at a high pressure. You have probably come across many pressure vessels in daily life, such as propane gas containers, a balloon, a soda can, the biscuit dough container, and an aircraft fuselage. The inside of the pressure vessel is at a higher pressure than the outside. This results in stresses in the container wall. The container wall stretches ever so slightly in response to the stresses.

There are two basic forms of pressure vessels; cylindrical pressure vessels are essentially long cylinders with their ends capped off, and spherical pressure vessels. A pressure vessel that has a wall thickness less than about 1/10 of the diameter of the container is called a thin-walled pressure vessel. The soda pop can as an example of a thin-walled cylindrical pressure vessel.

It can be shown that there are two principal stresses that develop in a thin-walled cylindrical pressure vessel: the axial stress σ_a and the hoop stress σ_h. The axial and hoop stresses are also called the longitudinal and circumferential stresses, respectively. It can also be shown that these stresses are related to the internal pressure in the pressure vessel through the following equations:

$$\sigma_a = \frac{pd}{4t}$$

$$\sigma_h = \frac{pd}{2t}$$

where p is the internal pressure, d is the diameter of the cylinder and t is the wall thickness. The hoop stress has twice the magnitude of the axial stress.

6. Electrical Resistance Strain Gauges

By now you must be familiar with the concept of electrical resistance. Ohm's law states that the current flowing through a metallic wire is proportional to the voltage applied across its ends. The electrical resistance R is simply the ratio of the voltage V and the current I.

$$R = \frac{V}{I}$$

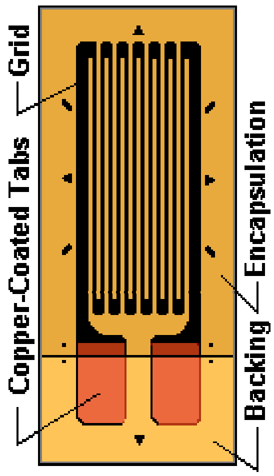
The resistance of the wire depends upon the length and the cross sectional area of the wire.

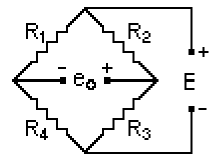
$$R = \frac{\rho L}{A}$$

where ρ is a property of the metal called the resistivity. If the length is increased or the cross sectional area is decreased, the resistance of the wire increases.

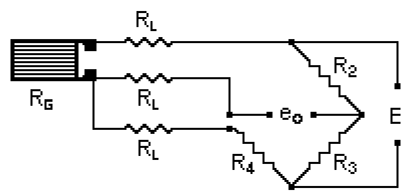
Electrical resistance strain gauges are essentially metallic resistors that can be pasted onto the surface of a part on which you want to measure strain. When the part deforms, in other words experience a strain, the strain gauge deforms along with it. This deformation is reflected as a proportional change in the resistance of the strain gauge.

Changes in electrical resistance can be accurately measured by using circuits such as a Wheatstone's Bridge circuit shown below.





Wheatstone's Bridge



Three-wire Quarter-Bridge Circuit

Figures from Micromeasurments Group.

When all the resistors in the Wheatstone's bridge circuit above have the same resistance, the output voltage e_0 will be zero. When one of the arms of the bridge is replaced by a strain gauge, then we have a Quarter Bridge circuit. Let us assume the lead wires that connect the strain gauge to the rest of the bridge have no resistance ($R_L = 0$). Now if the resistance of the strain gauge changes by ΔR, then the output voltage is

$$e_o = \frac{\Delta R}{4R_G} E$$

where R_G is the resistance of the strain gauge.

The corresponding strain is

$$\varepsilon = \frac{4}{GF} \frac{e_o}{E}$$

In the above equations, E is the bridge excitation voltage, GF is the gauge factor of the strain gauge and R is the nominal resistance of the strain gauge. In our experiments, we will set E = 10Volts, and for our gauges GF = 2.12 and R = 120 Ω.

Procedures

In this lab, you will use electrical resistance strain gauges to measure the pressure inside a soda pop can. The procedure is as below:

1. Get your soda can from the TA.
2. Measure the diameter *d* of the can.
3. Use vernier calipers to measure the wall thickness *t* on the can that has been cut open.
4. Use sand paper to scrape of the paint on the can and mount two strain gauges, one in the axial direction and one in the hoop direction. Instructions on mounting strain gauges are in the appendix.

5. Connect the two gauges to two channels of the strain gauge amplifier (Vishay/Micromeasurment). The amplifier contains the rest of the Wheatstone's Bridge and the necessary amplifying circuit to convert resistance changes to strain.
6. The display shows the magnitude of strain in micro-mm/mm of one channel.
7. Turn the excitation voltage to each of the channels you are using.
8. Selecting each channel in turn, zero out the readings on the strain gauge indicators using the balance knob. If you are not able to zero them out, you have probably damaged the strain gauge during the mounting process or the wires have not been soldered properly. If this happens, use a multimeter to check the resistance across the wires and remount the gauges or resolder the lead wires if necessary.
9. After you have balanced both channels, **carefully** open the can. There will be change in the strain reading. Record the change in axial and hoop strains.
10. The measured strain values can be converted to the pressure inside the can before it was opened through the following equations:

$$p = \frac{4tE\varepsilon_a}{d(1-2\nu)} = \frac{4tE\varepsilon_h}{d(2-\nu)}$$

11. Use values of E = $70\text{x}10^9$ N/mm^2 and $\nu = 0.35$ to calculate the pressure inside the can. Since you have two independent strain measurements in the axial and hoop directions, you can determine the pressure two different ways. Comment on any differences you may see.

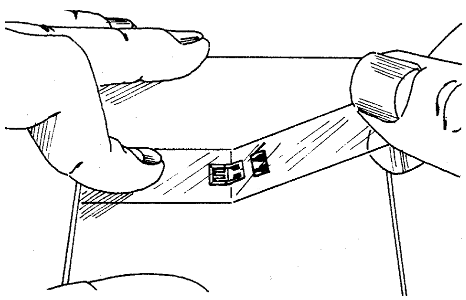
Food for Thought

When you microwave a hot dog why does the skin split along its length?

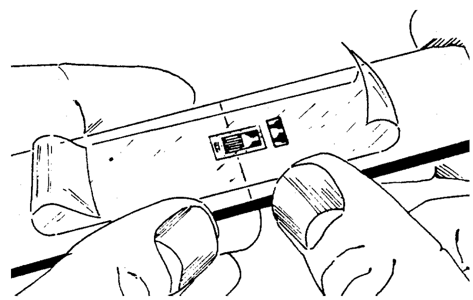
Appendix*

Strain gauge Mounting

1. Lightly draw alignment lines on the test surface on which the gauge is to be mounted.
2. Place the gauge with the "shiny" side up on a clean surface, such as the top of the plastic box in which the strain gauges are packed. Take a 4-inch piece of removable adhesive tape and lightly rub the non-sticky side of the tape against a metallic surface to discharge static electricity.
3. Carefully place the tape on the strain gauge.
4. Peal up the tape at a shallow angle (~30°). The strain gauge should come up with the tape.
5. Place the strain gauge and tape on the test surface at the desired location, using the previously marked lines for alignment. You will be able to peal back the tape and reposition the gauge if necessary.



Step (4)



Step (5)

6. After you are satisfied with the alignment of the strain gauge peal the tape back one more time to lift the gauge from the can surface. Peal the tape back an additional 1/2-inch.
7. Apply a small amount of catalyst on the exposed (dull) surface of the strain gauge. The catalyst is a blue liquid. Dip the brush that is attached to the cap of the catalyst bottle in the catalyst and then wipe it against the lip of the bottle about 10 times. Apply the catalyst on the gauge using smooth strokes, without stopping on the gauge.
8. Apply one drop of the adhesive (M-Bond 200) at the seam between the tape and the test surface, about 1/2 inch from where the gauge is to be attached (see item (6) above).

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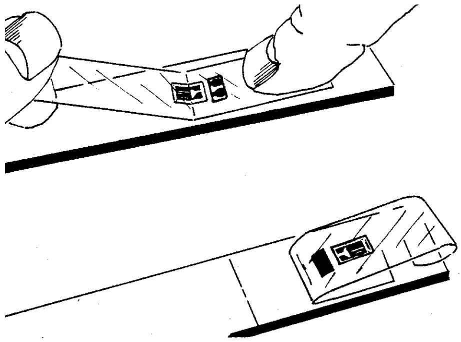
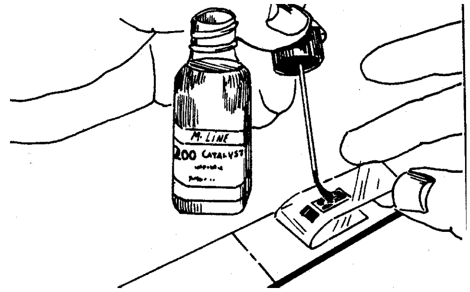
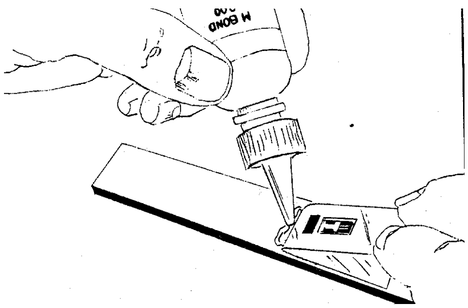
* Instructions and figures are based on information in Bulletin 309D Student Manual for Strain Gauge Technology, Measurements Group, 1992



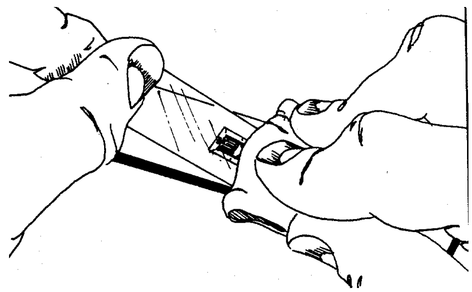
Step (6)



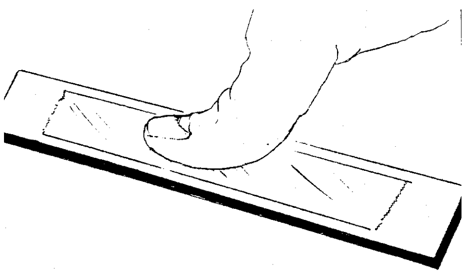
Step (7)



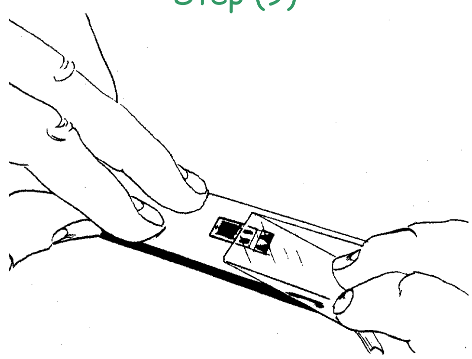
Step (8)



Step (9)



Step (10)



Step (11)

9. Quickly attach the gauge on to the test surface by a smooth wiping action on the tape. This will spread the glue uniformly over the gauge. Be careful not to get glue on the skin.
10. Apply gentle thumb pressure on the gauge for 30 seconds.
11. Wait 1 minute and then peal back the tape. The gauge should now stay attached to the test surface.
12. Repeat steps (1) through (11) for the other gauge.
13. Attach lead wires.
    (a) Apply flux to the terminals of the strain gauge and the lead wires.

(b) Apply a small drop of solder to the strain gauge terminals and solder the wires in place. Make sure the wires don't cross and the gauge terminals are not shorted out.

(c) Secure wires in place with adhesive tape.

14. Check the resistance across the ends of the lead wires using a digital multimeter. The resistance should be 120Ω. If you get a number much different from 120Ω, either you have damaged the strain gauge during the mounting process or the wires have not been soldered properly. Remount the gauge or resolder the lead wires if necessary.