

Measurement of acoustic dispersion using both transmitted and reflected pulses

Ping He

Department of Biomedical, Industrial and Human Factors Engineering, Wright State University, Dayton, Ohio 45435

(Received 10 August 1999; accepted for publication 10 November 1999)

Traditional broadband transmission method for measuring acoustic dispersion requires the measurements of the sound speed in water, the thickness of the specimen, and the phase spectra of two transmitted ultrasound pulses. When the sound speed in the specimen is significantly different from that in water, the overall uncertainty of the dispersion measurement is generally dominated by the uncertainty of the thickness measurement. In this paper, a new water immersion method for measuring dispersion is proposed which eliminates the need for thickness measurement and the associated uncertainty. In addition to recording the two transmitted pulses, the new method requires recording two reflected pulses, one from the front surface and one from the back surface of the specimen. The phase velocity as well as the thickness of the specimen can be determined from the phase spectra of the four pulses. Theoretical analysis and experimental results from three specimens demonstrate the advantages of this new method. © 2000 Acoustical Society of America.

[S0001-4966(00)04002-9]

PACS numbers: 43.35.Cg, 43.35.Yb [HEB]

INTRODUCTION

Determination of acoustic dispersion (phase velocity as a function of frequency) in the frequency domain using a broadband transmission method has recently found many applications. Since introduced by Sachse and Pao in 1978, this method has been used to study the dispersion properties of metal (Sachse and Pao, 1978), epoxy resin (Rokhlin *et al.*, 1986), paper materials (Brodeur *et al.*, 1993), polymeric materials (Zellouf *et al.*, 1996), and ultrasound contrast agent (Mobley *et al.*, 1998), as well as to verify Kramers–Kronig relationship for acoustic waves (Lee *et al.*, 1990; Brodeur *et al.*, 1993).

Determination of dispersion using the above transmission method requires the measurements of a reference velocity—usually the sound speed in water, the thickness of the specimen, and the phase spectra of two transmitted ultrasound pulses. Since the magnitude of dispersion for most materials is very small (less than 1% within the frequency range of 1–10 MHz), minimizing measurement uncertainties is particularly important in such applications. In many cases, the overall accuracy of the dispersion measurement is limited by the uncertainty of the thickness measurement. In some cases, the specimen may have irregular surfaces; in other cases, the specimen may not be rigid enough that the action of measuring its thickness causes a certain degree of deformation. Even under ideal surface conditions, it has been shown that the overall uncertainty of the dispersion measurement is dominated by the uncertainty of the thickness measurement when the sound speed in the specimen is significantly different from that in water (Mobley *et al.*, 1998).

In this paper, a new water immersion, broadband method for determining dispersion in the frequency domain is described which does not require measuring the specimen's thickness. This new method is essentially an extension of a time-domain method, first proposed by Kuo *et al.* (1990)

and later expanded by Hsu and Hughes (1992), which determines the sound speed in the specimen using times-of-flight data. The instruments and setup used in the new method are the same as that used in the traditional immersion, broadband transmission method for measuring dispersion. In addition to recording two transmitted pulses, the new method requires recording two reflected pulses, one from the front surface and one from the back surface of the specimen. The phase velocity of the specimen is determined from the phase spectra of the four recorded pulses. In addition, using an approach parallel to the one used by Hsu and Hughes (1992), it is shown that the thickness of the specimen can also be determined from the phase spectra of the four pulses. Results from three specimens are reported and an error analysis is presented.

I. THEORY

Figure 1 shows the signal paths in the immersion experiment for the dispersion measurement. $P_0(t)$ is the initial pulse launched by the transducer labeled as T_1 . $P_s(t)$ and $P_w(t)$ represent the transmitted pulses with and without the specimen inserted, respectively, and $P_1(t)$ and $P_2(t)$ represent the pulses reflected back from the front and back surfaces of the specimen, respectively. If we use $A(f)e^{-j\theta(f)}$ to represent the Fourier transform of a pulse $P(t)$, and assume that the dispersion of water is negligible (Rokhlin *et al.*, 1986), the phase velocity of the specimen can be obtained from the phase spectra of the two transmitted pulses, $P_s(t)$ and $P_w(t)$ (Wu, 1996; He, 1999)

$$\frac{1}{V_p(f)} = \frac{\theta_s(f) - \theta_w(f)}{2\pi fL} + \frac{1}{c_w}, \quad (1)$$

where $V_p(f)$ is the phase velocity in the specimen, c_w is the sound speed in water, $\theta_s(f)$ and $\theta_w(f)$ are the phase spectra

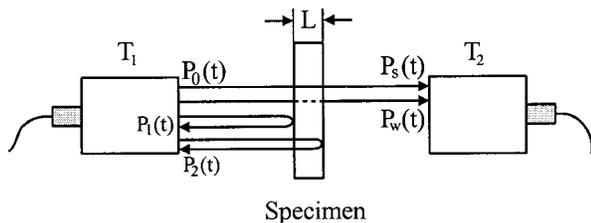


FIG. 1. Signal paths in the immersion experiment for the dispersion and thickness measurements.

of $P_s(t)$ and $P_w(t)$, respectively, and L is the thickness of the specimen.

Using the same analysis (He, 1999), $V_p(f)$ can also be obtained from the phase spectra of the two reflected pulses

$$\frac{1}{V_p(f)} = \frac{\theta_2(f) - \theta_1(f)}{4\pi fL}, \quad (2)$$

where $\theta_1(f)$ and $\theta_2(f)$ are the phase spectra of $P_1(t)$ and $-P_2(t)$, respectively (to take account of the inherent extra 180° -phase shift between P_2 and P_1 , the recorded P_2 is always inverted first in all the data processing described in this paper). From the above two equations, we cancel L and obtain the phase velocity

$$V_p(f) = c_w \left[1 + 2 \times \frac{\theta_w(f) - \theta_s(f)}{\theta_2(f) - \theta_1(f)} \right]. \quad (3)$$

This is the basic equation for determining $V_p(f)$ without measuring the specimen's thickness. It should be noted that each $\theta(f)$ in Eq. (3) is the absolute phase calculated using an origin of time ($t=0$) that is the instant when $P_0(t)$ is launched. Due to the large time delays between $P_0(t)$ and the transmitted or reflected pulses, the absolute phases of these pulses change rapidly with frequency, and phase unwrapping is required. In order to accurately determine the true absolute phase, a large number of zeros are usually appended to the original pulse to increase the sample density of the phase function, and the $2m\pi$ phase ambiguity needs to be carefully account for (Wu, 1996; Mobley, 1998). In order to simplify the process of phase calculation and reduce the ambiguity, we employ the following technique to determine the absolute phase.

We first assume that each of $P_s(t)$, $P_w(t)$, $P_1(t)$, and $P_2(t)$ is acquired using a long sampling window that starts from $t=0$. Instead of directly performing Fourier transform to the recorded pulse, we first shift the center of the pulse to the beginning of the sampling window (by circularly rotating to the left all the samples within the sampling window) and then calculate the phase spectrum of the shifted pulse. The relation between the phase spectra of the original and shifted pulses is

$$\theta(f) = \phi(f) + 2\pi f t, \quad (4)$$

where $\phi(f)$ is the phase spectrum of the shifted pulse, and $t = n/f_s$ is the time shift, where f_s is the sampling frequency and n is the number of samples being shifted. Substituting Eq. (4) into Eq. (3), the equation for determining dispersion becomes

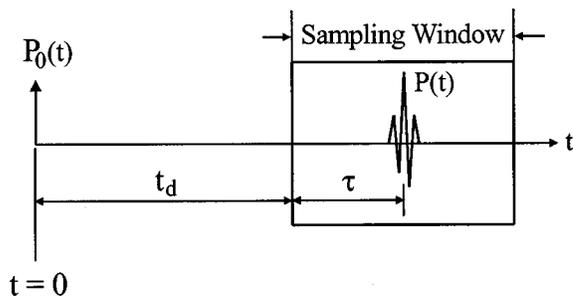


FIG. 2. Temporal relations of the initial pulse $P_0(t)$, sampling window, and recorded pulse $P(t)$.

$$V_p(f) = c_w \left[1 + 2 \times \frac{\phi_w(f) - \phi_s(f) + 2\pi f(t_w - t_s)}{\phi_2(f) - \phi_1(f) + 2\pi f(t_2 - t_1)} \right], \quad (5)$$

where $\phi_w(f)$, $\phi_s(f)$, $\phi_2(f)$, $\phi_1(f)$ are the phase spectrum of shifted $P_w(t)$, $P_s(t)$, $-P_2(t)$, $P_1(t)$, respectively, and t_w , t_s , t_2 , t_1 are the respective time shifts.

We next consider that in a typical experiment, the pulse is recorded using a relatively small sampling window that starts not at $t=0$ but after a trigger delay t_d , as shown in Fig. 2. In this case, the recorded pulse is still shifted to the beginning of the sampling window and the phase spectrum of the shifted pulse is then calculated as $\phi(f)$. The total time shift, t , used in Eqs. (4) and (5) in this case should include both the time shift, τ , within the sampling window and the trigger delay, t_d , of the sampling window

$$t = t_d + \tau. \quad (6)$$

In both cases, the change (with frequency) of the phase spectrum, $\phi(f)$, can be minimized by adjusting the amount of time shift, and phase unwrapping is normally not needed (or rather easy to perform) in determining $\phi_w(f)$, $\phi_s(f)$, $\phi_2(f)$, and $\phi_1(f)$ (as we will show later in Fig. 4). As a result, $V_p(f)$ can be determined accurately by Eq. (5).

Equations (3) and (5) indicate that the absolute phase velocity $V_p(f)$ determined by the above method is directly proportional to c_w which changes with temperature (Del Grosso and Mader, 1972). If we express dispersion as the relative change in phase velocity, then the need for measuring c_w can also be eliminated:

$$\frac{V_p(f) - V_p(f_0)}{V_p(f_0)} = \frac{F(f) - F(f_0)}{1/2 + F(f_0)}, \quad (7)$$

where f_0 is a reference frequency, and the function $F(f)$ is defined as

$$F(f) = \frac{\phi_w(f) - \phi_s(f) + 2\pi f(t_w - t_s)}{\phi_2(f) - \phi_1(f) + 2\pi f(t_2 - t_1)}. \quad (8)$$

In this case, the uncertainty of the dispersion measurement is solely determined by the uncertainty of the time measurement.

If we cancel V_p in Eq. (1) and Eq. (2), and make use of Eq. (5), we obtain an estimate for the thickness L :

$$\begin{aligned}\hat{L} &= \frac{c_w}{4\pi f} [\theta_2 - \theta_1 + 2(\theta_w - \theta_s)] \\ &= \frac{c_w}{4\pi f} [\phi_2 - \phi_1 + 2(\phi_w - \phi_s) \\ &\quad + 2\pi f(t_2 - t_1 + 2t_w - 2t_s)].\end{aligned}\quad (9)$$

In theory, the frequency dependence of all the components on the right side of Eq. (9) should be completely canceled out, making \hat{L} a perfect constant. In practice, however, \hat{L} calculated from the experimental data will not be a perfect constant, but will change with frequency. The mean of \hat{L} within the frequency range of the measurement system is used as the estimated thickness L and the standard deviation of \hat{L} can be used to judge the reliability of the new method in estimating thickness and measuring dispersion.

A comparison between the method presented here and the methods proposed earlier by Kuo *et al.* (1990) and Hsu *et al.* (1992) can be made at this point. The method presented here is based on the frequency-domain analysis and it measures the phase velocity of the specimen as a function of frequency. The methods proposed by Kuo *et al.* and Hsu *et al.* are performed in the time domain, and both measure the speed of sound of the specimen at unspecified frequency. If the time shift, t , in Eq. (4) is adjusted in such a way that $|\phi(f)|$ is minimized within the frequency range of interest, the time shifts, t_w , t_s , t_2 , and t_1 in Eq. (5) and Eq. (9) then represent the gross propagation delay of P_w , P_s , P_2 , and P_1 , respectively. Then, if one ignores dispersion by assuming $\phi_w(f) = \phi_s(f)$, and $\phi_2(f) = \phi_1(f)$, Eq. (5) is reduced to Eq. (6) in the paper by Kuo *et al.* (1990) and Eq. (9) is reduced to Eq. (10) in the paper by Hsu *et al.* (1992). In other words, the time-domain methods proposed by Kuo *et al.* and Hsu *et al.* can be considered as approximations to the frequency-domain method presented here.

II. EXPERIMENT AND RESULTS

A typical setup for immersion experiment is used to test the new method (He, 1999). A focused transducer (Panametrics V382, 3.5 MHz, 13-mm aperture, 8.9-cm focal distance) is used as the transmitter/receiver (T_1 in Fig. 1) and a non-focused transducer (Panametrics V384, 3.5 MHz, 6.35-mm aperture) is used as the receiver (T_2 in Fig. 1). This pair of transducers gives a useful frequency range of 1–5 MHz in all the experiments reported in this paper. The distance between T_1 and the front surface of the specimen is approximately 10 cm. The pulser/receiver used in the experiment is Panametrics 5052 PR. The amplified pulse is A/D converted by a SONY/TEK 390 AD programmable digitizer which has an adjustable digital delay for triggering the sampling window. Each sampling window contains 512 samples and the sampling frequency is 60 MHz. The samples are transferred to a PC and processed using a software package MATLAB (Math Works, MA). The water temperature is 21.2 °C, which gives $c_w = 1485.968$ m/s (Del Grosso and Mader, 1972).

A Plexiglas, a high-density polyethylene, and a low-density polyethylene sample are used in the experiment. The origin of the Plexiglas sample is unknown. The two polyeth-

TABLE I. Measured thickness using a digital caliper (average at five locations) and estimated thickness using Eq. (9) within the frequency range of 1 to 5 MHz for three specimens.

	Measured thickness (mm) (five locations)	Estimated thickness (mm) (1–5 MHz)
Plexiglas	39.942 ± 0.008	39.888 ± 0.004
High-density polyethylene	18.76 ± 0.03	18.706 ± 0.004
Low-density polyethylene	3.156 ± 0.009	3.149 ± 0.006

ylene samples are both obtained from Poly Hi Solidur (Scranton, PA). The densities of the three samples are 1.176, 0.966, and 0.918 g/cm³, respectively. The thickness of each sample is measured at five locations using a digital caliper (model 599-571-4, Brown & Sharpe, North Kingstown, RI), and the mean and standard deviation are listed in Table I. The mean thickness is used for calculating dispersion using the traditional method as well as for comparing with the thickness estimated using the new method.

Figure 3 shows the four pulses obtained using the Plexiglas sample and Fig. 4 shows their phase spectra (P_2 is first reversed) after being shifted to the beginning of the respective sampling window. As shown in the figure, all the phase spectra change slowly with frequency. As a result, phase unwrapping is not needed in determining these spectra.

Figure 5(a) compares the phase velocity of the Plexiglas sample determined using the new method (solid line) and the traditional method (dashed line). The solid line is obtained using Eq. (5) based on the phase spectra (ϕ_1 , ϕ_2 , ϕ_w , and ϕ_s) shown in Fig. 3. In this particular experiment, the time shifts t_1 , t_2 , t_w , and t_s are 96.517, 125.450, 108.150, and 75.783 μ s, respectively. Only four representative error bars at frequencies 1.5, 2.5, 3.5, and 4.5 MHz are presented in the figure. These error bars are calculated using Eq. (12), which will be derived later in Sec. III, based on an uncertainty of 0.1 °C in the temperature measurement and an uncertainty of 8.33×10^{-9} s in the time measurement. The dashed line in the figure is calculated using Eq. (1) with a measured $L = 39.942$ mm, as listed in Table I.

Figure 5(b) plots the estimated thickness using Eq. (9). As expected, \hat{L} calculated from the experimental data is not a perfect constant, but the change (with frequency) is very small. Since the apparent flatness of this curve can be arbitrarily changed by changing the vertical scale of the plot, we chose the vertical plot range to be approximately $\pm 1\%$ around the mean value (in this case, it is from 39.9–0.4 mm to 39.9+0.4 mm). The mean and standard deviation of the estimated thickness are calculated within the frequency range of 1–5 MHz, and the results are listed in Table I.

Figure 6 shows the experimental results from the high-density polyethylene sample, and Fig. 7 shows the experimental results from the low-density polyethylene sample. Again, the mean and standard deviation of the thickness estimation for each specimen are listed in Table I.

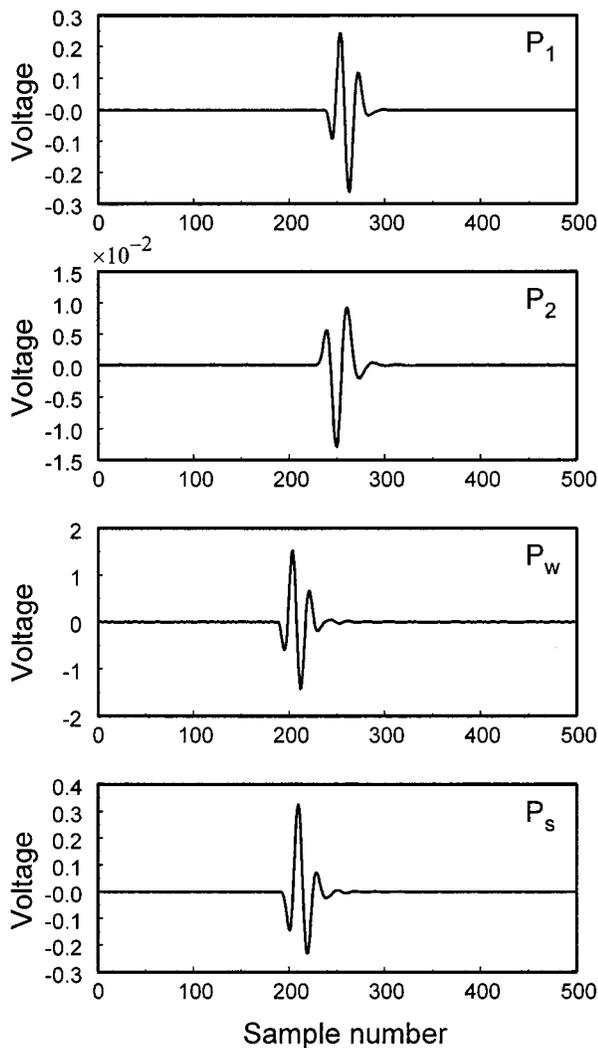


FIG. 3. Actual waveforms of the four recorded pulses: P_1 and P_2 are the pulses reflected back from the front and back surfaces of the specimen, respectively, and P_s and P_w are the transmitted pulses with and without the specimen inserted between the two transducers, respectively. Notice the extra 180° -phase shift of P_2 with respect to P_1 . The time interval per sample is $1/60 \mu\text{s}$.

III. DISCUSSION

The most directly noticeable fact exhibited by the experimental results is the excellent agreement between the measured and estimated thickness of the samples, as seen in Table I. The discrepancy between the measured and estimated thickness for each sample is 0.14% (Plexiglas), 0.29% (high-density polyethylene), and 0.22% (low-density polyethylene), respectively. This accuracy of thickness estimation is slightly better than the accuracy (from 0.16% to 0.80%) obtained by Hsu and Hughes using the time-domain, pulse-overlap method (1992). The results in Table I also show that within the frequency range of 1–5 MHz, deviation of \hat{L} from a perfect constant is very small: 0.01% (Plexiglas), 0.02% (high-density polyethylene), and 0.19% (low-density polyethylene), respectively. On the one hand, this high degree of flatness of \hat{L} obtained in the present experiment provides a strong support to the proposed method of using mixed spectra (of transmitted and reflected pulses) to determine dispersion. On the other hand, the degree of flatness of

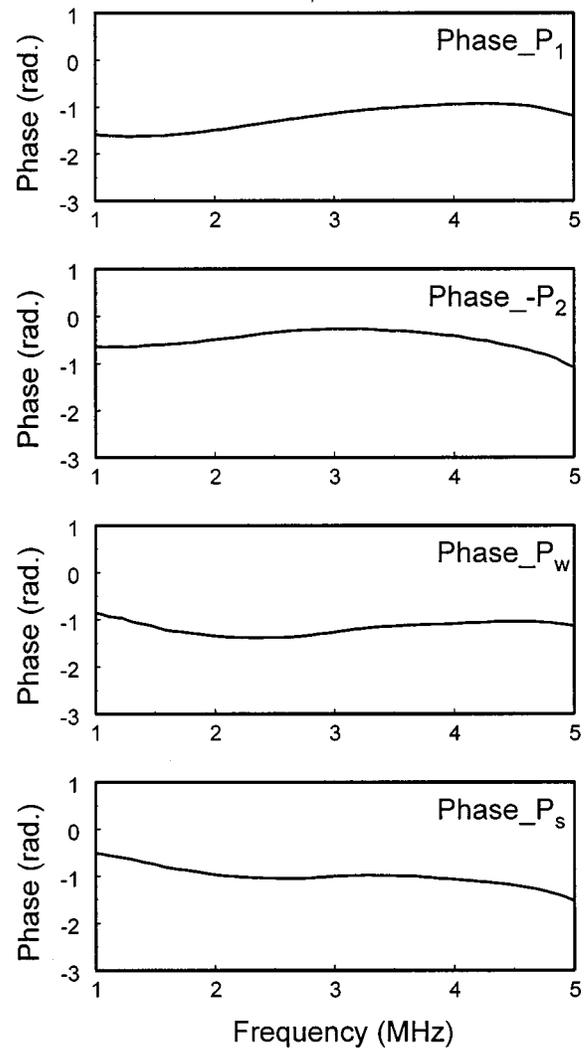


FIG. 4. The phase spectra of the four pulses shown in Fig. 3 after their centers being shifted to the beginning of the respective sampling window. Notice the second curve from the top is the phase of the inverted P_2 .

\hat{L} obtained in a particular experiment may be used to judge the validity of the dispersion measurement for that experiment. The relatively large fluctuation of \hat{L} from the low-density polyethylene sample is associated with the low amplitude of P_2 , which is in turn caused by the large attenuation of the material (measured as 21 dB/cm at 3 MHz, as compared with 3.5 dB/cm for the Plexiglas sample and 5.2 dB/cm for the high-density polyethylene sample). It is expected that if the signal-to-noise ratio of the reflected signal were too weak, the fluctuation of \hat{L} would be very large, indicating that the calculated dispersion using the mixed spectra would be no longer reliable.

Direct validation of the new method for measuring dispersion is more difficult because there are no standard dispersion data for the three samples used in this experiment. Kuo *et al.* (1990) measured the sound speed in three Plexiglas samples and obtained values ranging from 2750 to 2766 m/s. The solid curve shown in Fig. 5 is in the same range. In addition, the attenuation and dispersion properties of the same Plexiglas sample have been carefully studied before and the results presented in this paper are in excellent agree-

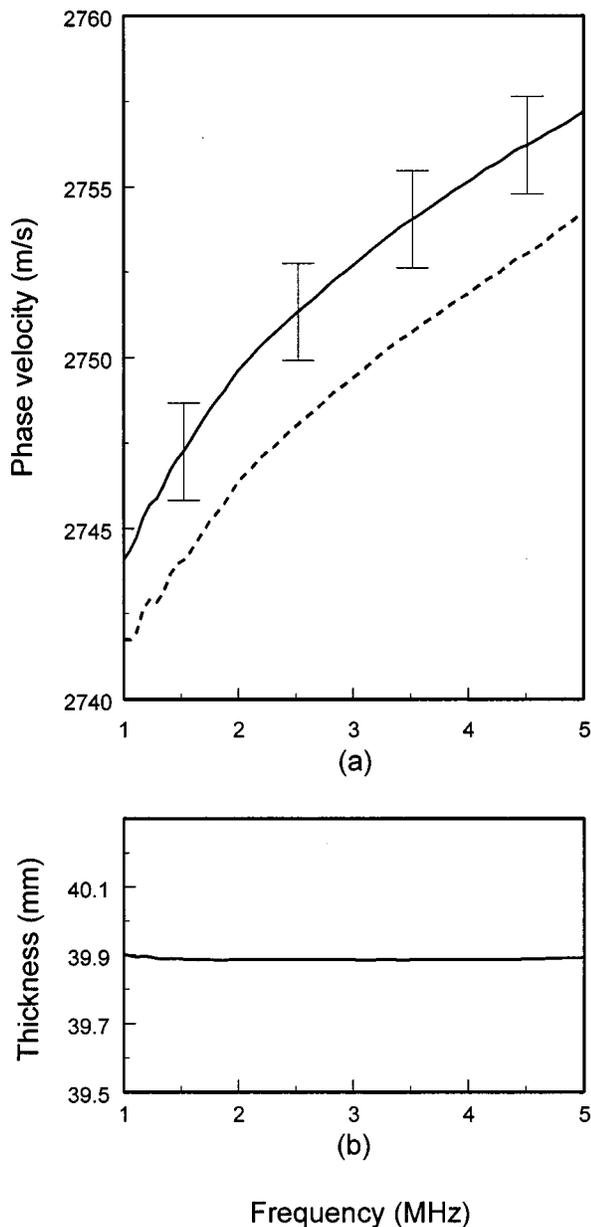


FIG. 5. (a) Solid line—phase velocity of the Plexiglas sample determined using the new method. Representative error bars are shown at 1.5, 2.5, 3.5, and 4.5 MHz. Dashed line—phase velocity determined using the traditional method with a thickness of 39.942 mm. (b) Thickness of the same sample determined using the proposed method. The plot range in the vertical direction is approximated $\pm 1\%$ around the mean value.

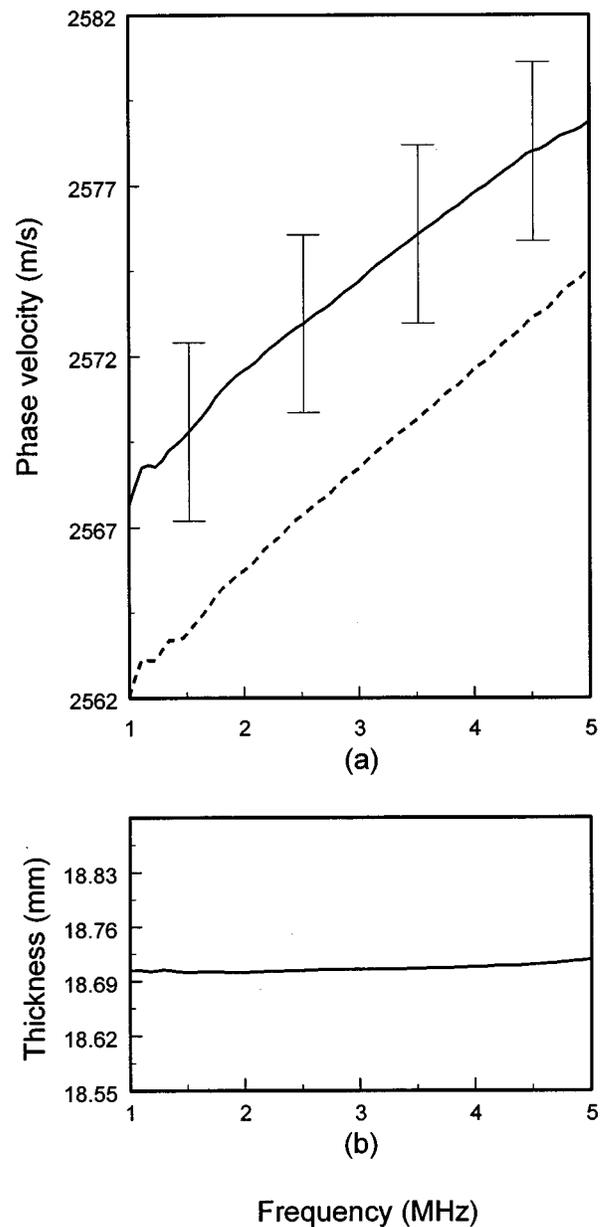


FIG. 6. (a) Solid line—phase velocity of the high-density polyethylene sample determined using the new method. Representative error bars are shown at 1.5, 2.5, 3.5, and 4.5 MHz. Dashed line—phase velocity determined using the traditional method with a thickness of 18.76 mm. (b) Thickness of the same sample determined using the proposed method. The plot range in the vertical direction is approximated $\pm 1\%$ around the mean value.

ment with the results obtained in the previous experiments (He, 1990). O'Donnell *et al.* (1981) measured the attenuation and dispersion of polyethylene in the frequency range of 1 to 10 MHz but did not specify the density of the specimen. From the large attenuation value (16 dB/cm at 3 MHz) reported by these investigators, it is judged that the acoustic properties of their specimen are close to that of the low-density polyethylene sample used in this study. Their dispersion curve shows a 45-m/s change in the phase velocity when the frequency is changed from 1 to 5 MHz. The dispersion curve plotted in Fig. 7(a) shows a similar change in the phase velocity. Wu (1996) measured dispersion of both high-density and low-density polyethylene. The dispersion curve of the high-density polyethylene sample plotted in Fig.

6 is in good agreement with the dispersion curve reported by Wu [Fig. 5(a), 1996]. However, that curve in Wu's paper is labeled as from the low-density polyethylene. It is noticed that the phase velocity of the low-density polyethylene reported by Wu is higher than that of the high-density polyethylene, which is in contradiction with the results obtained by this author. To further examine this discrepancy, we used the time-domain, pulse-overlap method (Hsu and Hughes, 1992) to measure the sound speed in these two samples, and obtained a speed of 2560 m/s for the high-density polyethylene and a speed of 2066 m/s for the low-density polyethylene. This independent measurement verified that the high-density polyethylene used in this study indeed has a higher sound speed than the low-density polyethylene. The exact source of

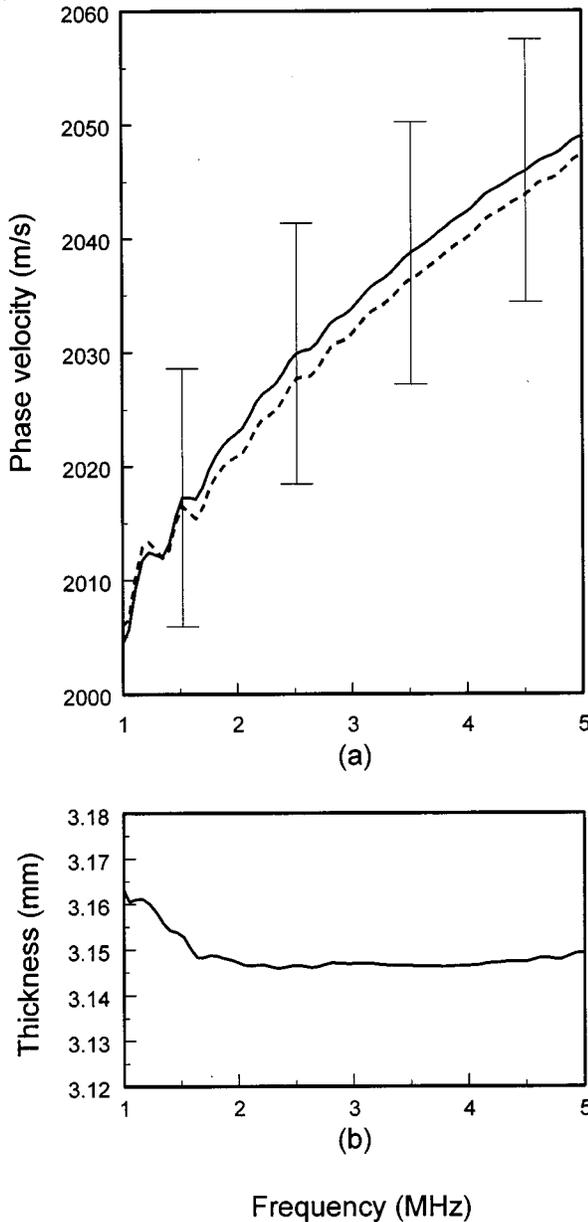


FIG. 7. (a) Solid line—phase velocity of the low-density polyethylene sample determined using the new method. Representative error bars are shown at 1.5, 2.5, 3.5, and 4.5 MHz. Dashed line—phase velocity determined using the traditional method with a thickness of 3.156 mm. (b) Thickness of the same sample determined using the proposed method. The plot range in the vertical direction is approximated $\pm 1\%$ around the mean value.

the discrepancy between Wu's results and our results is not known at this time.

Several observations can be made from the plots in Figs. 5(a), 6(a), and 7(a). In each figure, the solid line is obtained using both the transmitted and reflected pulses but without using the specimen's thickness, while the dashed line is obtained by the traditional method that uses the transmitted pulses only and a particular value for the sample's thickness. Although it appears that the two methods produce different dispersion curves, the discrepancy between the two curves in each figure is very small: 0.1% in Fig. 5(a), 0.2% in Fig. 6(a), and 0.1% in Fig. 7(a). In fact, if the thickness of the sample used to calculate the dashed curve in Fig. 5(a) is decreased by 0.054 mm (a mere 0.14% change), the new

dashed curve would move up and overlap almost exactly with the solid curve. The same results can also be produced with the curves in Fig. 6(a) and Fig. 7(a). Since the immersion broadband transmission method for dispersion measurement is a well-established method, the above observations validate the accuracy of the new method in measuring dispersion that does not need to know the specimen's thickness. On the other hand, the above observations also suggest that the traditional method is very sensitive to the choice of the specimen's thickness. By eliminating the need for measuring the specimen's thickness, it is expected that the overall uncertainty of the dispersion measurement will be reduced in the new method. We now perform an error analysis to quantitatively investigate the improvement of the new method in reducing the uncertainty of the dispersion measurement.

For the traditional method that determines the phase velocity based on Eq. (1), the variance of the phase velocity can be determined based on the theory of error propagation to first order in the measurement uncertainties (Bevington, 1969; Mobley *et al.*, 1998)

$$\begin{aligned} \sigma_{V_p}^2 &= \left(\frac{\partial V_p}{\partial L} \right)^2 \sigma_L^2 + \left(\frac{\partial V_p}{\partial \theta_s} \right)^2 \sigma_{\theta_s}^2 + \left(\frac{\partial V_p}{\partial \theta_w} \right)^2 \sigma_{\theta_w}^2 + \left(\frac{\partial V_p}{\partial c_w} \right)^2 \sigma_{c_w}^2 \\ &= V_p^4 \left[\frac{(\theta_s - \theta_w)^2}{\omega^2 L^4} \sigma_L^2 + \frac{1}{\omega^2 L^2} \sigma_{\theta_s}^2 + \frac{1}{\omega^2 L^2} \sigma_{\theta_w}^2 + \frac{1}{c_w^4} \sigma_{c_w}^2 \right], \end{aligned} \quad (10)$$

where $\omega = 2\pi f$, σ_L^2 , $\sigma_{\theta_s}^2$, $\sigma_{\theta_w}^2$, and $\sigma_{c_w}^2$ are the measurement uncertainties (variances) of L , θ_s , θ_w , and c_w , respectively. Using Eq. (4), we can separate θ into two parts: a linear-with-frequency part, ωt , where t is the gross propagation delay, and a nonlinear part, ϕ , which contains the dispersion effect. Since the first part (ωt) is much larger than the second part (ϕ), we can use the following approximations in estimating the measurement uncertainty: $(\theta_s - \theta_w) \cong \omega(t_s - t_w) = \omega L(1/V_p - 1/c_w)$, $\sigma_{\theta_s}^2 \cong \sigma_{\theta_w}^2 \cong \omega^2 \sigma_t^2$, where σ_t^2 is the variance of the time measurement. With these approximations, Eq. (10) can be reduced to

$$\sigma_{V_p}^2 = \frac{2V_p^4}{L^2} \sigma_t^2 + \left(\frac{V_p}{c_w} \right)^4 \sigma_{c_w}^2 + \left(1 - \frac{V_p}{c_w} \right)^2 \frac{V_p^2}{L^2} \sigma_L^2. \quad (11)$$

Equation (11) indicates that if $V_p \cong c_w$, the contribution of the uncertainty of the thickness measurement may not be significant. On the other hand, if V_p is significantly different from c_w , the contribution of the last term in Eq. (11) can be very large. To demonstrate this, we substitute in Eq. (11) the following values found in the Plexiglas experiment: $V_p = 2753$ m/s (at about 3 MHz), $L = 4$ cm, and $c_w = 1486$ m/s. For the sampling frequency $f_s = 60$ MHz, σ_t is estimated as $0.5 \times 1/f_s = 8.33 \times 10^{-9}$ s. For the temperature within the range of $19.2^\circ\text{C} \pm 0.1^\circ\text{C}$, c_w changes from 1479.546 m/s to 1480.174 m/s (Del Grosso and Meder, 1972). Consequently, σ_{c_w} is estimated as 0.3 m/s. With these data, the first two terms on the right side of Eq. (11) can be first determined as 4.97 (m/s)² and 1.06 (m/s)², respectively. In this case, if the fractional uncertainty of thickness, σ_L/L , is larger than 0.095%, or $\sigma_L > 0.0038$ cm, the last term in Eq. (11) will be dominant. Mobley *et al.* (1998) used a fractional uncertainty

$\sigma_L/L=7\%$ in their error analysis in measuring the phase velocity in Alburnex[®] suspensions. Such an uncertainty may be too large for solid specimens. If we use a more conservative $\sigma_L/L=0.5\%$, or $\sigma_L=0.02$ cm, the last term in Eq. (11) becomes 136.9 (m/s)², that is, 22 times larger than the sum of the first two terms on the right side of Eq. (11).

For the new method that determines phase velocity based on Eq. (3), the variance of phase velocity can be evaluated using the same analysis

$$\begin{aligned}\sigma_{V_p}^2 &= \left(\frac{\partial V_p}{\partial \theta_s}\right)^2 \sigma_{\theta_s}^2 + \left(\frac{\partial V_p}{\partial \theta_w}\right)^2 \sigma_{\theta_w}^2 + \left(\frac{\partial V_p}{\partial \theta_2}\right)^2 \sigma_{\theta_2}^2 \\ &\quad + \left(\frac{\partial V_p}{\partial \theta_1}\right)^2 \sigma_{\theta_1}^2 + \left(\frac{\partial V_p}{\partial c_w}\right)^2 \sigma_{c_w}^2 \\ &= \frac{2V_p^2}{L^2} \left[c_w^2 + \left(\frac{c_w - V_p}{2}\right)^2 \right] \sigma_t^2 + \left(\frac{V_p}{c_w}\right)^2 \sigma_{c_w}^2.\end{aligned}\quad (12)$$

Comparing Eq. (12) with Eq. (11), one notices that the potential major source of uncertainty associated with the thickness measurement is eliminated in the new method. The magnitudes of the remaining two terms associated with σ_t and σ_{c_w} are not significantly changed from that in Eq. (11) (in this particular example, they are slightly decreased to 1.71 and 0.31 (m/s)², respectively).

The proposed method can also be used to improve the accuracy of attenuation measurement. It is typical that in a broadband transmission experiment, both attenuation and dispersion are measured at the same time (Lee *et al.*, 1990; Wu, 1996; He, 1999). Just like dispersion, the measured attenuation is dependent on the thickness of the specimen. If the specimen's thickness cannot be determined accurately, a large error may occur in the attenuation measurement. Using the proposed method, the specimen's thickness can be first determined from the transmitted and reflected pulses. The estimated thickness is then used to determine the attenuation.

The method proposed in this paper may be particularly useful for real-time measurement of dispersion/attenuation when the thickness of the specimen changes with time. Rokhlin *et al.* (1986) reported a measurement of frequency dependence of ultrasound velocity and attenuation during the curing reaction of epoxy resin. According to the authors, the shrinkage of the sample during the curing reaction was 1%–2%. Based on Eq. (11) and the related arguments, one would expect that (relatively) significant errors would occur when a fixed initial thickness is used to calculate the phase velocity and attenuation. By eliminating the need for thickness measurement, the new method is immune from the error produced by the change in the sample's thickness.

There are several limitations of this new method; all are related to the requirement of obtaining two reflected pulses. First, in order to obtain two nonoverlapping echoes, the thickness of the specimen cannot be smaller than the axial resolution of the ultrasound system. Second, two surfaces of the specimen should be parallel to each other and should be kept normal to the beam. Otherwise, significant distortion in

echo waveform and phase spectrum may occur. Finally, if the acoustic impedance of the specimen is very close to that of the surrounding medium, or if the attenuation of the specimen is too large, the magnitude of the echo from the back surface may be reduced to the noise level. This last limitation may be the major obstacle in applying this method to certain biological tissues.

IV. CONCLUSIONS

In the measurement of acoustic dispersion using the traditional water-immersion broadband transmission method, if one can also reliably obtain reflected pulses from the front and back surface of the specimen, the phase velocity as well as the thickness of the specimen can be determined from the two transmitted and two reflected pulses. The degree of flatness of the estimated thickness within the useful frequency range of the system can be used to judge the reliability of the dispersion measurement. Experimental results from three specimens demonstrate the accuracy of the new method in determining the thickness and measuring the phase velocity. Theoretical analysis indicates that when the sound speed in the specimen is much different from that in water, the overall accuracy of the dispersion measurement can be significantly improved by eliminating the uncertainty associated with the measurement of the specimen's thickness.

- Bevington, P. R. (1969). *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, New York), Chap. 4, pp. 56–65.
- Brodeur, P. H., Hall, M. S., and Esworthy, C. (1993). "Sound dispersion and attenuation in the thickness direction of paper materials," *J. Acoust. Soc. Am.* **94**, 2215–2225.
- Del Grosso, V. A., and Mader, C. W. (1972). "Speed of sound in pure water," *J. Acoust. Soc. Am.* **52**, 1442–1446.
- He, P. (1999). "Experimental verification of models for determining dispersion from attenuation," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **46**, 706–714.
- Hsu, D. K., and Hughes, M. S. (1992). "Simultaneous ultrasonic velocity and sample thickness measurement and application in composites," *J. Acoust. Soc. Am.* **92**, 669–675.
- Kuo, I. Y., Hete, B., and Shung, K. K. (1990). "A novel method for the measurement of acoustic speed," *J. Acoust. Soc. Am.* **88**, 1679–1682.
- Lee, C. C., Lahham, M., and Martin, B. G. (1990). "Experimental verification of the Kramers–Kronig relationship for acoustic waves," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **37**, 286–294.
- Mobley, J., Marsh, J. N., Hall, C. S., Hughes, M. S., Brandenburger, G. H., and Miller, J. G. (1998). "Broadband measurements of phase velocity in Alburnex[®] suspensions," *J. Acoust. Soc. Am.* **103**, 2145–2153.
- O'Donnell, M., Jaynes, E. T., and Miller, J. G. (1981). "Kramers–Kronig relationship between ultrasonic attenuation and phase velocity," *J. Acoust. Soc. Am.* **69**, 696–701.
- Rokhlin, S. I., Lewis, D. K., Graff, K. F., and Adler, L. (1986). "Real-time study of frequency dependence of attenuation and velocity of ultrasonic waves during the curing reaction of epoxy resin," *J. Acoust. Soc. Am.* **79**, 1786–1793.
- Sachse, W., and Pao, Y. H. (1978). "On the determination of phase and group velocities of dispersive waves in solids," *J. Appl. Phys.* **49**, 4320–4327.
- Wu, J. (1996). "Determination of velocity and attenuation of shear waves using ultrasonic spectroscopy," *J. Acoust. Soc. Am.* **99**, 2871–2875.
- Zellouf, D., Jayet, Y., Saint-Pierre, N., Tatibouët, J., and Baboux, J. C. (1986). "Ultrasonic spectroscopy in polymeric materials. Application of the Kramers–Kronig relations," *J. Appl. Phys.* **80**, 2728–2732.