ACOUSTIC PARAMETER ESTIMATION BASED ON ATTENUATION AND DISPERSION MEASUREMENTS

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Abstract — In the measurement of acoustic attenuation that obeys a power-law $a = \beta f^n$, the traditional through-transmission method uses only the amplitude information of the recorded pulses to estimate the two parameters, $\beta$ and $n$. In this paper, we propose a new method that utilizes both the amplitude and phase information of the pulses to estimate the two parameters by simultaneously least-squares fitting the attenuation curve and the dispersion curve. Because the parameters are estimated under additional constraints, the estimation uncertainty is reduced. Experimental results from castor oil demonstrate the advantage of this new method.

I. INTRODUCTION

Determination of the acoustic attenuation has important applications in ultrasound tissue characterization and non-destructive material testing [1], [2]. For a wide variety of materials including soft tissues, the attenuation increases with frequency according to a power-law relation: $a(f) = \beta f^n$, where $\beta$ and $n$ ($1 \leq n \leq 2$) are two material-dependent parameters. Based on such a power-law relation, the measurement of the acoustic attenuation reduces to the estimation of the two parameters: $\beta$ and $n$. The value of $n$ also has a special “amplitude modulation” effect on the material’s dispersion: when $n$ increases from 1 to 2, the amount of dispersion gradually diminishes [3].

In vivo measurement of acoustic attenuation is traditionally performed using a so-called through-transmission method. Two transducers, one for transmitting an ultrasound pulse and one for receiving the pulse, are placed in a water tank. A pulse is first recorded when there is only a water path between the two transducers. The specimen is then inserted in between the two transducers and a second pulse is recorded. From the amplitude spectra of the two pulses, the attenuation function $\alpha(f)$ is first determined, and the two parameters, $\beta$ and $n$, can then be estimated by least-squares fitting a power-law function to $\alpha(f)$ [4]. Although this widely used method is quite simple and reasonably accurate in measuring $\alpha(f)$, it is often difficult to provide the precise value of each of the two parameters: the same $\alpha(f)$ may be fitted almost equally well by curves associated with different values of $\beta$ and $n$. In addition, ultrasound reflection at the water-specimen interface may produce additional measurement uncertainty [4]. In this paper, we propose a new method for estimating the two acoustic parameters. This method uses both the amplitude and phase information of the two pulses recorded in a through-transmission experiment. The two parameters, $\beta$ and $n$, are estimated by simultaneously least-squares fitting the attenuation curve which is derived from the amplitude spectra of the two pulses, and the dispersion curve which is derived from the phase spectra of the two pulses. Because the parameters are estimated under additional constraints, the estimation uncertainty is reduced. In addition, this method eliminates the effects of the acoustic reflection at the water-specimen interface and therefore increases the estimation accuracy.

II. METHOD

A) The Measurement

![Fig. 1: Experimental setup for attenuation and dispersion measurements.](image)

Fig. 1 shows the experimental setup for the through-transmission measurement. $P_w(t)$ represents the received pulse with the water path only, and $P_s(t)$ represents the received pulse with the specimen inserted. If we neglect the attenuation of water, the attenuation of the specimen can be found as [4]:

$$a(f) = \beta f^n$$
\[ \alpha(f) = \frac{1}{L} \ln(1 - R^2) + \frac{1}{L} \ln \left[ \frac{A_w(f)}{A_s(f)} \right] \]  
(1)

where \( R \) is the reflection coefficient at the water-specimen interface, \( L \) is the thickness of the specimen, and \( A_w(f) \) and \( A_s(f) \) are the amplitude spectra of \( P_w(t) \) and \( P_s(t) \), respectively. (1) is the basic equation for measuring the attenuation using the traditional method. To accurately determine the attenuation however, one needs to know the exact value of \( R \). Since the exact value of \( R \) is difficult to know, we define a new function \( P(f) \) to eliminate the effects of \( R \). We define the function \( P(f) \) as the difference between the attenuation value at frequency \( f \) and the attenuation value at frequency \( f_0 \):

\[ P(f) = \alpha(f) - \alpha(f_0) = \frac{1}{L} \ln \left[ \frac{A_w(f)A_s(f_0)}{A_s(f)A_w(f_0)} \right] \]  
(2)

where \( f_0 \) is a reference frequency within the useful frequency range of \( A_w(f) \) and \( A_s(f) \). In the actual experiment, \( A_w(f) \) and \( A_s(f) \) are obtained by Fast Fourier Transform. As a result, in the actual calculation, the variable \( f \) in (2) will be replaced by discrete frequencies \( f_i \) (\( i = 1, 2, \ldots, m \)) which cover the entire useful frequency range of the signal.

We then define another function, \( Q(f) \), which describes the change in phase velocity, or the dispersion [5]:

\[ Q(f) = \frac{1}{V_p(f_0)} - \frac{1}{V_p(f)} = \frac{\phi_w(f_0) - \phi_s(f_0) - \phi_w(f) + \phi_s(f)}{2\eta_0 L} \]  
(3)

where \( V_p(f) \) is the phase velocity, \( \phi_w(f) \) and \( \phi_s(f) \) are the phase spectra of \( P_w(t) \) and \( P_s(t) \), respectively. Again, the variable \( f \) in (3) will be replaced by discrete frequencies \( f_i \) (\( i = 1, 2, \ldots, m \)) in the actual calculation.

B) The Models for Attenuation and Dispersion

According to the power-law relation \( \alpha(f) = \beta f^n \), the model for \( P(f) \) in (2) is:

\[ P^*(f) = \beta (f^n - f_0^n) \]  
(4)

To model \( Q(f) \) in (3), a time causal model developed by Szabo [3] is adapted. Szabo’s model is a time-domain expression of causality analogous in function to the Kramers-Kronig relations in the frequency domain that link together the attenuation and dispersion. Using Szabo’s model, we have:

\[ Q^*(f) = -\frac{\beta}{2\pi} \tan \left( \frac{n\pi}{2} \left( f^n - f_0^n \right) \right) \]  
(5)

C) Least-Squares Fit

By comparing the measured attenuation and dispersion data [i.e. (2) and (3)] with the model predictions [i.e. (4) and (5)], we define the following total squared error (TSE) function:

\[ TSE = k \sum_{i=1}^{m} \left[ P(f_i) - P^*(f_i) \right]^2 + (1 - k) \sum_{i=1}^{m} \left[ Q(f_i) - Q^*(f_i) \right]^2 \]  
(6)

where \( 0 \leq k \leq 1 \) is a weighting factor. When \( k = 0.5 \) which is the case used in this study, the attenuation and dispersion have the same weight in parameter estimation. The right side of (6) contains two unknown parameters, \( \beta \) and \( n \). The best estimates of these two parameters are defined as the \( \beta \) and \( n \) values that minimize \( TSE \). To perform the least-squares estimation, one may use the standard grid-search algorithm [6]. On the other hand, it can be shown that for a given value of \( n \), the estimate of \( \beta \) that minimizes \( TSE \) has an analytic solution. As a result, the two-variable grid-search is reduced to a one-variable (\( n \)) search, and the speed of the estimation process is significantly increased.

III. EXPERIMENT

To test the new method, we conducted a through-transmission experiment using castor oil as the specimen. The oil is contained in a Plexiglas tube that has an inner diameter of 8 cm and a length of 5.5 cm (\( L \) in Fig. 1). Each end of the tube is covered by a thin polyethylene film (GLAD Cling Wrap) that encloses and seals the container while providing a window for the ultrasound measurement. The transmitting and receiving transducers used in this study are Panametrics V382 (3.5MHz, 13-mm aperture) and V384 (3.5MHz, 6.35-mm aperture), respectively. The distance between the two transducers is 15 cm. A Panametrics 5052PR pulse/receiver is used to drive the transmitting transducer and receives the transmitted pulse. The output from the receiver is digitized by a SONY/TEK 390AD programmable digitizer which has a 10-bit resolution and a sampling frequency of 60MHz. The sampled data are then transmitted to a PC for further process.

IV. RESULTS

From the amplitude spectra \( A_w(f) \) and \( A_s(f) \), the frequency range of interest is chosen as from 0.5 to 4.5 MHz, and the reference frequency \( f_0 \) is 1 MHz. Fig. 2(a) shows the measured values of \( P(f) \) (small circles) and the fitted curve
P*(f) (solid line) using the proposed method. Fig. 2(b) shows the measured values of Q(f) (small circles) and the fitted curve Q*(f) (solid line). The frequency range for the least-squares fit is 1 – 4 MHz, and the estimated values are β = 0.7082 (dB cm⁻¹ MHz⁻¹) and n = 1.7178.

To examine the stability of the new method in parameter estimation, we apply the least-squares fit in four different frequency ranges: 0.5 – 4 MHz, 0.5 – 4.5 MHz, 1 – 4 MHz, and 1 – 4.5 MHz. For each frequency range, a set of β and n estimates is obtained. The mean and standard deviation of each estimate for the four fitting ranges are then calculated.

As a comparison, we then use only the amplitude information to estimate the two parameters based on (1). We first ignore the first term on the right side of (1) (offset = 0) and apply the least-squares fit to the four frequency ranges. Again, a set of β and n estimates is obtained for each frequency range, and the mean and standard deviation of each estimate for the four frequency ranges are calculated. In the process of curve fit, it is noticed that there is a significant deviation between the measured attenuation values and the fitted curve. In other word, the reflection coefficient R has noticeable effects. We then add an adjustable offset to represent the first term on the right side of (1) and perform least-squares fit again. We found that an offset value of –0.13 dB/cm optimizes the performance of the method. With this optimal offset, the above estimation procedure is repeated.

Table 1 summarizes the test results. In the table, Method 1 refers to the proposed method for parameter estimation that utilizes both the amplitude and phase information of the received pulses, and Method 2 refers to the traditional method that uses only the amplitude information. The ‘offset’ in Table 1 represents the term [ln(1-R²)]/L in (1).

<table>
<thead>
<tr>
<th>Method 1</th>
<th>β</th>
<th>n</th>
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<tbody>
<tr>
<td>Method 1</td>
<td>0.7175 ± 0.011</td>
<td>1.7108 ± 0.009</td>
</tr>
<tr>
<td>Method 2 with offset = 0 dB/cm</td>
<td>0.8450 ± 0.065</td>
<td>1.5798 ± 0.077</td>
</tr>
<tr>
<td>Method 2 with offset = -0.13 dB/cm</td>
<td>0.7261 ± 0.040</td>
<td>1.6904 ± 0.055</td>
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</tbody>
</table>

The results in Table 1 show that the proposed method has the least standard deviations for both β and n estimates. Using Method 2, when the effects of the acoustic reflection at the water-specimen interface is ignored (offset = 0), the parameter β is significantly over-estimated and n is significantly under-estimated. In addition, the standard deviations of both estimates are the largest. When an offset value of –0.13 dB/cm is used, Method 2 produces a pair of β and n estimates that are much closer to the values obtained by Method 1. On the other hand, the standard deviations of both estimates are still much larger than that of Method 1.

V. CONCLUSION AND DISCUSSION

A new method is proposed for estimating two acoustic parameters, β and n, in a through-transmission experiment. The proposed method utilizes both the amplitude and phase information to estimate the two parameters, and therefore reduces the estimation bias and uncertainty. Since the phase information is already available in a through-transmission experiment, the improvement in accuracy and precision is achieved without additional cost.

Fig. 2 shows that the fit between the measured and predicted dispersion (Fig. 2-b) is noticeably worse than the fit between the measured and predicted attenuation (Fig. 2-a). There are two possible explanations for the less-than-perfect fit of dispersion. First of all, as seen in Fig. 2, the magnitude of dispersion is very small as compared with the magnitude of attenuation. As a result, the dispersion measurement may be more susceptible to experimental errors. Secondly, the dispersion model as expressed in (5) is not necessarily a
perfect model. It should be pointed out that even the power-law relation that describes the frequency-dependence of attenuation is just a phenomenological formulation, and therefore is not necessarily a perfect model. The dispersion model in (5) is built upon the power-law attenuation model. Consequently, it is expected that the dispersion model can’t be more accurate than the attenuation model.

Besides the Szabo’s model, there are several other models that describe the relation between the dispersion and attenuation. Based on the limited experimental results, it appears that Szabo’s model is more accurate than other models in predicting dispersion from attenuation [7]. On the other hand, all these models share the same premise that for a linear and causal system, the attenuation and dispersion are related to each other. It is the same premise that has inspired the development of the proposed method. Since there is an intrinsic relation between the attenuation and dispersion of the specimen, there must also be an intrinsic relation between the change in amplitude and change in phase of the pulse transmitting through the specimen. Based on this premise, the two parameters, $\beta$ and $n$, that determine both attenuation and dispersion, should be better determined when the amplitude and phase information are considered all together. The fit of the dispersion data will be improved when the experimental errors are further reduced and/or a more accurate dispersion model is developed. The general approach proposed in this paper for parameter estimation however, should remain valid.

REFERENCES