Weibull Distribution

1. The Exponential Distribution characterizes the probability of the time until the first occurrence of a Poisson Process event. The exponential random variable probability density function $f(x) = \lambda e^{-\lambda x}$.

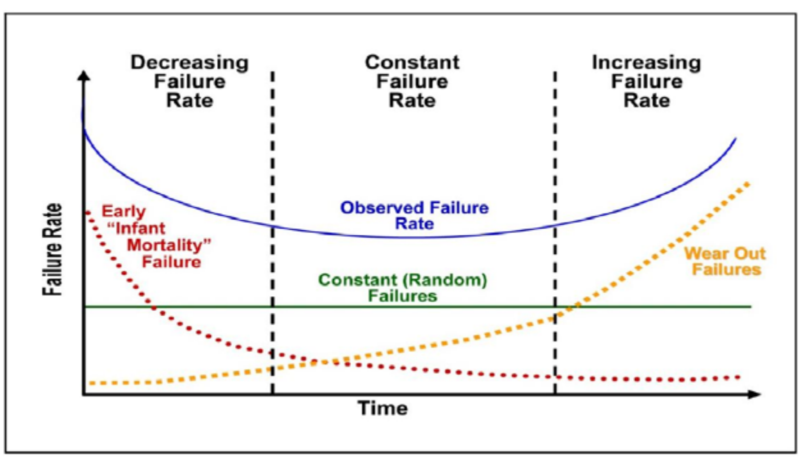
2. The Erlanger Distribution is a generalization of the exponential distribution for the probability of the time until the occurrence of exactly *r* events of a Poisson Process. The exponential random variable probability density function $f(x) = (\lambda^r x^{r-1} e^{-\lambda x}) / (r-1)!$ for $r = 1, 2, \ldots$.
For $r = 1$, $f(x) = \lambda e^{-\lambda x}$, i.e., the exponential probability density function.

The Gamma Function is a generalization for any non-negative value of r, such that $\Gamma(r) = (r - 1)\,\Gamma(r - 1)$ and if r is positive integer (r = n), then $\Gamma(n) = (n-1)!$

X	Gamma	Gamma
0	$(-1)!$	∞
0.5	$\sqrt{\pi}$	1.772
1.0	0!	1
1.5	$.5\sqrt{\pi}$	0.886
2.0	1!	1
2.5	$.75\sqrt{\pi}$	1.329
3.0	2!	2
3.5	$1.875\sqrt{\pi}$	3.323
4.0	3!	6

3. The Gamma Distribution has a pdf of $f(x) = (\lambda^r x^{r-1} e^{-\lambda x}) / \Gamma(r)$ for $x > 0$, and $\lambda > 0$ and $r > 0$.
The parameters λ and r are sometimes referred to as the **scale** and **shape** properties.
See Montgomery and Runger, Figure 4-25, (5ed, page 139; 6ed, page 141) for a depiction of Gamma probability density functions with selected values of λ and r.

4. The Weibull Distribution is often used to model the time until failure for many different physical systems.
The Weibull cumulative density function $P(X < x) = F(x) = 1 - e^{-(\frac{x}{\delta})^{\beta}}$, and $\mu = E(X) = \delta\Gamma(1 + 1/\beta)$ where x is the random variable, and $\delta > 0$ is the scale parameter and $\beta > 0$ is the shape parameter.
See Montgomery and Runger, Figure 4-26, (5ed, page 142; 6ed, page 144) for an illustration of Weibull probability density functions with selected values of δ and β.
The δ and β parameters provide a great deal of flexibility in modeling system failures; for example, where
probability of failure increases with time (wear-out, e.g. journals and bearings)
probability of failure decreases with time (infant mortality, e.g., electronic components
probability remains constant with time (external events such as shock).



http://allthingsnuclear.org/wp-content/uploads/2014/02/FS157-Figure-1-bathtub-nrc-ml13044a469.jpg

See Bathtub Curve
https://en.wikipedia.org/wiki/Bathtub_curve
http://www.weibull.com/hotwire/issue21/hottopics21.htm
http://www.weibull.com/hotwire/issue22/hottopics22.htm