Bernoulli Processes and Binomial Distributions

Conditions:
1. n independent trials
2. Only two possible outcomes per trial ("success" or "failure")
3. Probability of success on any one trial is constant = p
4. The random variable X is the number of successes in n trials, such that

Probability Mass Function $p(X = x) = C^n_x \, p^x \, (1-p)^{(n-x)}$ where $C^n_x = \dfrac{n!}{x!(n-x)!}$

Mean: $\mu = E(X) = np$ Variance: $\sigma^2 = V(X) = np(1 - p)$

Poisson Processes and Poisson Distributions

Poisson Process - Deals with the number of occurrences per interval.

Examples
Number of phone calls per minute
Number of cars arriving at a toll both per hour
Number of failures per 1000 hours
Number of bumps along a road per mile
Number of flaws in fabric per square yard
Number of pieces of debris per cubic meter of sea water

Conditions
For any interval over the real numbers:
1. Assume the occurrences happen at random throughout the interval.
2. Partition the interval into subintervals such that,
   a. The probability of more than one occurrence per subinterval is zero.
   b. The probability of one occurrence in a subinterval is the same for all the other subintervals, and is proportional to the length of the subinterval.
   c. An occurrence in any one subinterval is independent of an occurrence in any of the other subintervals.
3. Then the random occurrences are said to be a Poisson Process, such that;

If the mean number of occurrences in the interval is $\lambda > 0$, then the random variable X, (where X equals the number of occurrences in the interval) has a Poisson Distribution with parameter λ such that;

Probability Mass Function $f(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$ for x = 0, 1, 2, ...

Mean: $\mu = E(X) = \lambda$ Variance: $\sigma^2 = V(X) = \lambda$