Set Notation and Axioms of Probability

Memory Hints:

$\cap$ looks like **A** for And

$\cup$ looks like **U** for Union

Intersection $\cap$ **AND** •

Union $\cup$ **OR** **+**

Complement: $\overline{}$ **'** **NOT** $\overline{\overline{Y}} = Y$

Null: $\varnothing$ $P(\varnothing) = 0$

Mutually Exclusive: $A \cap B = \varnothing$ If A and B are *mutually exclusive* then $P(A \cap B) = 0$

Commutative Law

A **AND** B = B **AND** A

$A \cap B = B \cap A$

A **OR** B = B **OR** A

$A \cup B = B \cup A$

Distributive Law

(A **AND** B) **OR** C = (A **OR** C) **AND** (B **OR** C)

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

(A **OR** B) **AND** C = (A **AND** C) **OR** (B **AND** C)

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

B = (A **AND** B) **OR** ($\overline{A}$ **AND** B)

$B = (A \cap B) \cup (\overline{A} \cap B)$

DeMorgan's Law

$\overline{(A \text{ AND } B)} = \overline{A}$ **OR** $\overline{B}$

$\overline{(A \cap B)} = \overline{A} \cup \overline{B}$

$\overline{(A \text{ OR } B)} = \overline{A}$ **AND** $\overline{B}$

$\overline{(A \cup B)} = \overline{A} \cap \overline{B}$

Axioms of Probability

$P(A) + P(\overline{A}) = 1$

$P(A) = 1 - P(\overline{A})$

$0 < P(A) < 1$

$\sum P(A_i) = 1$

Joint Probabilities

Addition Rule

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A and B are *mutually exclusive* then $A \cap B = \varnothing$ and $P(A \cap B) = 0$

therefore, if A and B are *mutually exclusive* then $P(A \cup B) = P(A) + P(B)$

Conditional Probability

The *conditional probability* of an event A given an event B, denoted P(A | B) is

$P(A \mid B) = P(A \cap B) / P(B)$

The *conditional probability* of an event B given an event A, denoted P(B | A) is

$P(B \mid A) = P(B \cap A) / P(A)$

Multiplication Rule

$P(A \cap B) = P(B \cap A)$

$P(A \cap B) = P(A \mid B) * P(B)$

$P(B \cap A) = P(B \mid A) * P(A)$

$P(A \mid B) * P(B) = P(B \mid A) * P(A)$

Bayes' Theorem

$P(A \mid B) = P(B \mid A) * P(A) / P(B)$

Total Probability Rule

$P(B) = P(B \cap A) + P(B \cap \overline{A}) = P(B \mid A) * P(A) + P(B \mid \overline{A}) * P(\overline{A})$

Joint Probabilities

Dependent (Without Replacement) $P(A \cap B) = P(A) * P(B \mid A)$

Independent (With Replacement) $P(A \cap B) = P(A) * P(B)$

Independence

Two events are *independent* if any one of the following equivalent statements is true.

$P(A \mid B) = P(A)$

$P(B \mid A) = P(B)$

$P(A \cap B) = P(A) * P(B)$

If any one of the above statements is true, then all of them are true.

Mutually Exclusive

If A and B are *mutually exclusive* then

$P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B)$

Independent, Dependent, & Mutually Exclusive

Multiplication Rule

$P(A \cap B) = P(B \cap A)$

$P(A \cap B) = P(A \mid B) * P(B)$

$P(B \cap A) = P(B \mid A) * P(A)$

$P(A \mid B) * P(B) = P(B \mid A) * P(A)$

Independent

$P(A \cap B) = P(A) * P(B)$

Mutually Exclusive

$P(A \cap B) = 0$

Hence independent events cannot be mutually exclusive.

Dependent

$P(A \cap B) = P(B \cap A) = P(A) * P(B|A) = P(B) * P(A|B)$

where $P(A) \neq P(A|B)$ and $P(B) \neq P(B|A)$, *then events A & B are dependent.*

If neither conditional probability $P(A|B)$ or $P(B|A)$ *equals zero, then events A & B are dependent, but not mutually exclusive.*

In either conditional probability equals zero $\{P(A|B) = 0$ or $P(B|A) = 0\}$, *the events A & B are dependent and mutually exclusive.*

Example: Probability Matrices for Independent, Dependent, & Mutually Exclusive

	Independent		
	A	$\bar{A}$	
B	0.12	0.28	0.40
$\bar{B}$	0.18	0.42	0.60
	0.30	0.70	1.00

P(A ^ B) = 0.12 = P(A) * P(B) hence Independent

	Dependent (Not Mutually Exclusive)		
	A	$\bar{A}$	
B	0.09	0.31	0.40
$\bar{B}$	0.21	0.39	0.60
	0.30	0.70	1.00

P(A ^ B) = 0.09 hence Dependent,
but not Mutually Exclusive

	Dependent (Mutually Exclusive)		
	A	$\bar{A}$	
B	0.00	0.40	0.40
$\bar{B}$	0.30	0.30	0.60
	0.30	0.70	1.00

P(A ^ B) = 0.00 hence Dependent,
and Mutually Exclusive