Hypothesis Testing Simplified Algorithm

1. Decide what you what to "prove" and state it as the **Claim/Research Hypothesis**.

   Two-Sided Alternate Hypothesis
   For Claim/Research Hypotheses where no direction is implied or if stated as "equal to" or even in the case of "not equal to" then use the Two-Sided Alternate Hypothesis H_1: $\mu \neq \mu_0$.

   One-Sided Alternate Hypothesis
   For Claim/Research Hypotheses such as "greater than", "less than", "exceeds", "at least", "no more than", "no less than", etc, use a One-Sided Alternate Hypothesis H_1: $\mu < \mu_0$, or H_1: $\mu > \mu_0$.

   Null Hypothesis
   For both one-sided and two-sided hypotheses, use the Null Hypothesis H_0: $\mu = \mu_0$.

   Note: The Alternate hypothesis always points in the direction of significance.

2. Decide on the type of **Test Statistic** Z_0, t_0, χ_0^2, F_0

3. Decide on the **Level of Significance** α and the **Number of Tails** and the **Degrees of Freedom**.
   Determine the **Critical Value** and identify the **Rejection Region**.
   Note: P-Values may also be used as rejection criteria.

4. Conduct the experiment and collect the sample data; compute the sample statistics and the **Test Statistic**.

5. Conclusions:

   Rejecting the Null Hypothesis

   If the **Test Statistic** falls within the **Rejection Region** (or if the p-value is less than α), then **Reject the Null Hypothesis**.
   If the Claim/Research Hypothesis was $\mu = \mu_0$, then Reject the Claim/Research Hypothesis as well.
   If the Claim/Research Hypothesis was $\mu \neq \mu_0$, then Accept the Claim/Research Hypothesis.

   If the Claim/Research Hypothesis is stated in terms of the Alternate Hypothesis, and if we Reject the Null Hypothesis, then we can conclude that the sample data provides sufficient evidence to warrant accepting the Claim/Research Hypothesis.

   If the Claim/Research Hypothesis is stated in same terms as the Null Hypothesis, and if we Reject the Null Hypothesis, then we can conclude that the sample data provides sufficient evidence to warrant rejecting the Claim/Research Hypothesis.

   Failing to Reject the Null Hypothesis

   On the other hand, if we Fail to Reject the Null Hypothesis, then we can only conclude that the sample data does NOT provide sufficient evidence to either warrant rejecting the Null Hypothesis or to warrant accepting the Alternate Hypothesis.

Forming the Null Hypothesis

Research Hypothesis (Claim) H_R:	Null Hypothesis H_0:	Alternative Hypothesis H_1:
$\mu = \mu_0$	$\mu = \mu_0$	$\mu \neq \mu_0$
$\mu \neq \mu_0$	$\mu = \mu_0$	$\mu \neq \mu_0$
$\mu \leq \mu_0$	$\mu \leq \mu_0$	$\mu > \mu_0$
$\mu \geq \mu_0$	$\mu \geq \mu_0$	$\mu < \mu_0$
$\mu < \mu_0$	$\mu \geq \mu_0$	$\mu < \mu_0$
$\mu > \mu_0$	$\mu \leq \mu_0$	$\mu > \mu_0$

Notice, the Null Hypothesis always has the sense of equality $(=, \leq, \geq)$.
The Alternative Hypothesis is the opposite of the Null Hypothesis $(\neq, >, <)$.
Match the Research Hypothesis to either the Null or the Alternative Hypothesis.

Statement About the Research Hypothesis (Claim)

If H_R has the same form as H_0 and Reject H_0, then there IS sufficient sample evidence to warrant rejecting the research hypothesis.

If H_R has the same form as H_0 and Fail to Reject H_0, then there is NOT sufficient sample evidence to warrant rejecting the research hypothesis.

If H_R has the same form as H_1 and Fail to Reject H_0, then there is NOT sufficient sample evidence to warrant supporting the research hypothesis.

If H_R has the same form as H_1 and Reject H_0, then there IS sufficient sample evidence to warrant supporting the research hypothesis.

Only the last case, (H_R has the same form as H_1 and Reject H_0) leads to the conclusion that the sample data actually supports the research hypothesis.

Therefore, in order to present the strongest statistical conclusion, one should state the Research Hypothesis in the form of an Alternative Hypothesis (and hope the evidence supports rejecting the Null Hypothesis).

One-Sided & Two-Sided Hypothesis Testing

Left-Side (less than, not less than, equal to or greater than, etc)

H_1: $\mu < \mu_0$
H_0: $\mu \geq \mu_0$ (logical opposite)
H_0: $\mu = \mu_0$ (Montgomery & Runger)

If Test Value < -Critical Value then
Reject H_0,
Accept H_1,
Conclude $\mu < \mu_0$

If Test Value > -Critical Value then
Fail to Reject H_0,
Conclude: Insufficient evidence to warrant rejecting the null hypothesis.

Right-Side (more than, not more than, equal to or less than, etc)

H_1: $\mu > \mu_0$
H_0: $\mu \leq \mu_0$ (logical opposite)
H_0: $\mu = \mu_0$ (Montgomery & Runger)

If Test Value > +Critical Value then
Reject H_0,
Accept H_1,
Conclude $\mu > \mu_0$

If Test Value < +Critical Value then
Fail to Reject H_0,
Conclude: Insufficient evidence to warrant rejecting the null hypothesis.

Two-Sided Hypothesis Testing

H_1: $\mu \neq \mu_0$
H_0: $\mu = \mu_0$ (logical opposite)
H_0: $\mu = \mu_0$ (Montgomery & Runger)

If |Test Value| > Critical Value
that is to say if Test Value < -Critical Value or Test Value > +Critical Value then
Reject H_0,
Accept H_1,
Conclude $\mu \neq \mu_0$

If |Test Value| < Critical Value then
Fail to Reject H_0,
Conclude: Insufficient evidence to warrant rejecting the null hypothesis.

Hypothesis Testing (Strong Conclusions, Rejecting the Null, and P-Values)

The Null Hypothesis must always contain a sense of equality (=, ≤, ≥).
The Alternate Hypothesis is the logical opposite of the Null Hypothesis (≠, >, <).
The Claim must match one of the six conditions (=, ≤, ≥, ≠, >, <).

Select the Null / Alternate Hypotheses pair that matches the Claim.

Calculate the Test Statistic (Z_0, t_0, F_0, χ^2, etc).

The Alternate Hypothesis points to the direction of significance.

- < > Two-Tail
- < One-Tail Left
- > One-Tail Right

To estimate the associated p-value, interpolate the appropriate probability distribution table by:

One-Tail Left	$H_0: \mu \geq \mu_0$	$H_1: \mu < \mu_0$	p-value = $\Phi(z_0)$
One-Tail Right	$H_0: \mu \leq \mu_0$	$H_1: \mu > \mu_0$	p-value = $1 - \Phi(z_0)$
Two-Tail	$H_0: \mu = \mu_0$	$H_1: \mu \neq \mu_0$	p-value = $2[1 - \Phi(\lvert z_0\rvert)]$

Draw a conclusion regarding the null hypothesis based on the magnitude of the p-value.

P-Value (Probability Value) is the probability of obtaining a value of the sample test statistic that is at least as extreme as the one found from the sample data, assuming the null hypothesis is true.

P-Value	Interpretation
Less than 0.01	Highly statistically significant; very strong evidence for rejecting the null hypothesis.
0.01 to 0.05	Statistically significant; adequate evidence to warrant rejecting the null hypothesis.
Greater than 0.05	Insufficient evidence to warrant rejecting the null hypothesis.

It is customary to call the test statistic significant when our conclusion warrants rejecting the null hypothesis. Therefore, we can consider the p-value to be the smallest level at which the data is significant.

Note: *Power of the Test* is a measure of the statistical test *sensitivity*; that is to say, it is the probability of detecting a *significant difference* between the *true mean* and the *hypothesized mean*.

Rejecting the null hypothesis leads to a strong conclusion. If the claim has the same form as the null hypothesis, we can also strongly conclude that we reject the claim. If the claim has the same form as the alternate hypothesis and we reject the null hypothesis, we can strongly conclude that we accept the claim. The risk of rejecting a true null hypothesis (Probability of Type I Error) is limited to the magnitude of the p-value.

On the other hand, if we fail to reject the null hypothesis, it is proper to state that "there is insufficient evidence to warrant rejecting the null". Likewise, similar statements apply to the claim regarding insufficient evidence to warrant rejecting the claim (if it has the same form as the null hypothesis) or insufficient evidence to warrant accepting the claim (if it has the same form as the alternate hypothesis).

Remember, we either Reject the Null Hypothesis or Fail To Reject the Null Hypothesis; we should never accept the null hypothesis in lieu of failing to reject it. To do so, is to risk the Probability of a Type II Error (accepting a false null hypothesis) of unknown magnitude.

Failing to Reject the Null Hypothesis

If we *Fail to Reject the Null Hypothesis*, then we should conclude there is "*insufficient evidence to warrant rejecting the H_0*", or equivalently, "*insufficient evidence to warrant accepting H_1*".

In other words, for example:

One Sample

if H_0: $\mu = \mu_0$ there is insufficient evidence to say $\mu \neq \mu_0$
if H_1: $\mu < \mu_0$ there is insufficient evidence to say $\mu < \mu_0$
if H_1: $\mu > \mu_0$ there is insufficient evidence to say $\mu > \mu_0$

Two Sample

if H_0: $\mu_1 = \mu_2$ there is insufficient evidence to say $\mu_1 \neq \mu_2$
or there is insufficient evidence to say the samples are not the same
or there is insufficient evidence to say the samples are different
if H_1: $\mu_1 < \mu_2$ there is insufficient evidence to say $\mu_1 < \mu_2$
if H_1: $\mu_1 > \mu_2$ there is insufficient evidence to say $\mu_1 > \mu_2$

Matched Pairs (paired *t* test)

if H_0: $\mu_d = 0$ there is insufficient evidence to say $\mu_d \neq 0$
or there is insufficient evidence to say the samples are not the same
or there is insufficient evidence to say the samples are different

etc., etc., etc.

To *accept the null hypothesis* in lieu of *failing to reject the null hypothesis*, is to risk a Type II Error of unknown magnitude.

Regarding Hypothesis Testing

1. Hypotheses are always about the population (not about the sample).

2. We can never know whether or not, the null hypothesis is True or False.

3. We always make statistical inferences with respect to the **Null Hypothesis H0**.

2. If we **Reject the Null Hypothesis H0**, then we **Accept the Alternate Hypothesis H1**.

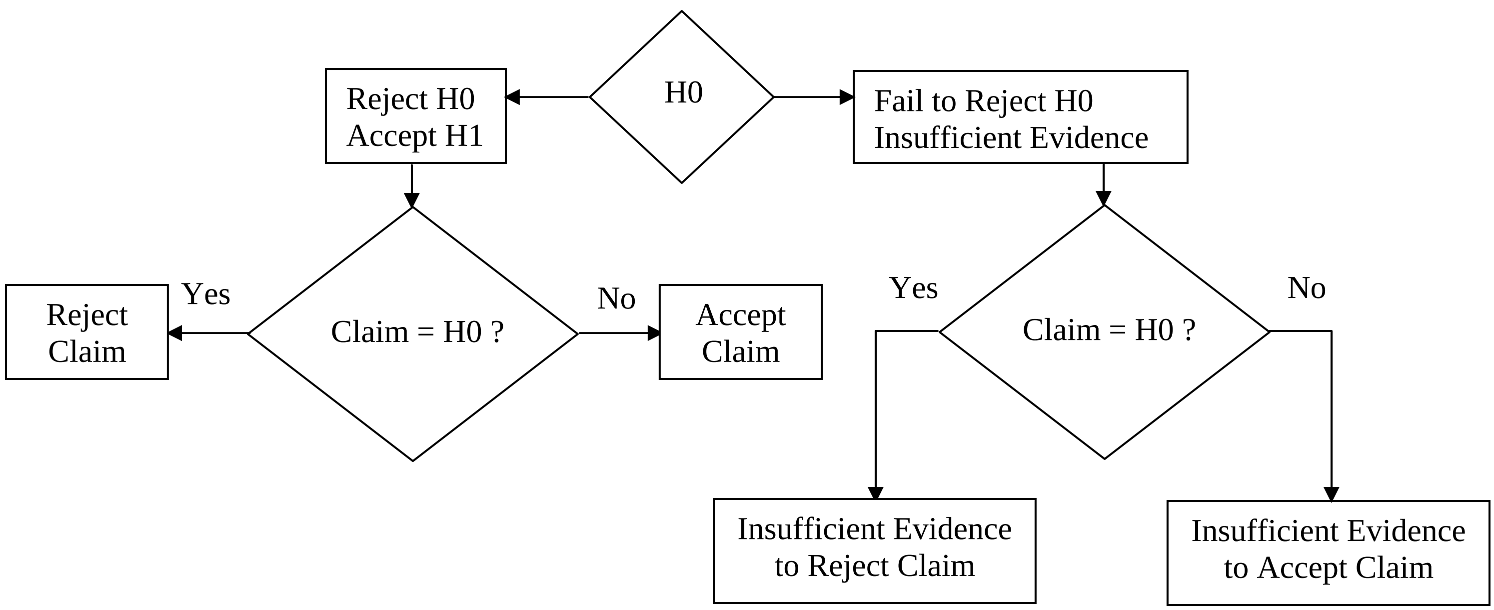
   If the **Claim** looks like the **Null**, then we **Reject the Claim**.
   If the **Claim** looks like the **Alternate**, then we **Accept the Claim**.

3. If we **Fail to Reject the Null Hypothesis H0**, then we say there is

   Insufficient Evidence to Warrant Rejecting the Null, which is the same as saying
   Insufficient Evidence to Warrant Accepting the Alternate.

   If the **Claim** looks like the **Null**, then we say there is
   Insufficient Evidence to Warrant Rejecting the Claim.

   If the **Claim** looks like the **Alternate**, then we say there is
   Insufficient Evidence to Warrant Accepting the Claim.



Hypothesis Testing Errors & Power of the Test

Case	Decision	Error
H_0 False	Reject H_0	None
H_0 True	Fail to Reject H_0	None
H_0 True	Reject H_0	Type I
H_0 False	Fail to Reject H_0	Type II

Decision	**Case**	
	Ho True	Ho False
Reject Ho	Type I Error	No Error
Fail to Reject Ho	No Error	Type II Error

Type I Error   Rejecting H_0 when H_0 is True
Type II Error   Failing to Reject H_0 when H_0 is False

Probability of Type I Error = α
Probability of Type II Error = β

Power of the Test = Probability of Correctly Rejecting a False Null Hypothesis

Probability of Rejecting H_0 when H_0 is False = $1 - \beta$

Probability of Accepting H_1 when H_1 is True = $1 - \beta$

Smaller Level of Significance (α)

Less Probability of Type I Error

Greater Probability of Type II Error

Larger Level of Significance (α)

Greater Probability of Type I Error

Less Probability of Type II Error

Increasing the sample size reduces the probability of both Type I and Type II Errors