

ISE 2211 Test 5 (Normal Distribution) Course Notes and Course Activities

Course Notes:

Use Standard Normal Distribution Table or Excel Normal Distribution Functions (see below) to determine:

$$P(Z_a < Z < Z_b)$$

z given the probability of $P(Z < z) = \text{probability } p$

z given the probability of $P(Z > z) = \text{probability } p$

Given: Normal Distribution with Mean μ , Standard Deviation σ

Apply Z-Score, using the $Z = \frac{x - \mu}{\sigma}$ equation as appropriate.

Find: $P(a < X < b)$

Find: x given $P(X < x) = \text{probability } p$

Find: x given $P(X > x) = \text{probability } p$

Solve engineering problems using the above techniques.

Answer concept questions related to Normal Probability Distributions.

Course Activities:

The following activities will preferably be completed using the MS Excel formulas/functions: or a similar electronic spreadsheet.

STANDARDIZE, Z.TEST, NORM.DIST, NORM.INV, NORM.S.DIST, NORM.S.INV

References:

Standard Normal Distribution Table (Z Table)

<http://www.stat.ufl.edu/~athienit/Tables/Ztable.pdf>

Excel Normal Distribution Functions

Z - Score

STANDARDIZE(X, μ , σ)

Given a Normal Distribution with Mean μ , Standard Deviation σ , and some value X; Find $Z = \frac{X - \mu}{\sigma}$

Example: Normal Distribution Mean $\mu = 125$, Standard Deviation $\sigma = 15$, $X = 150$
=STANDARDIZE(150, 125, 15) = 1.67

Standard Normal Z Distribution (Mean $\mu = 0$ and Standard Deviation $\sigma = 1$)

NORM.S.DIST(Z, True/False)

Find $P(Z < z)$, given z .

Example: $z = 1.50$

=NORM.S.DIST(1.50) = 0.9332

Normal Distribution

NORM.DIST(X, μ , σ , True/False)

If *True*, returns Cumulative Distribution Function $P(X)$; i.e., integral from negative infinity to x .

Example: $\mu = 125$, $\sigma = 15$, $X = 150$, True
= NORM.DIST(150, 125, 15, True) = 0.9521

Note: This is the same as $P(Z) = P\left(Z = \frac{X - \mu}{\sigma}\right) = P\left(\frac{150 - 125}{15}\right) = P(1.667) = 0.9522$

Note: If $\mu = 0$ and $\sigma = 1$, this is the same as the Standard Normal Z Distribution see above NORM.S.DIST

$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

If *False*, returns the probability mass function

Example: $\mu = 125$, $\sigma = 15$, $X = 150$, False

=NORM.DIST(150, 125, 15, False) = 0.0066 Note: We will not use this function.

Z Inverse (Standard Normal Distribution)

NORM.S.INV(probability)

Find z , given $P(Z < z) = \text{probability } p$.

Example: $P(Z < z) = 0.9522$

=NORM.S.INV(0.9522) = 1.667

Example: $P(Z < z) = 0.0808$

=NORM.S.INV(0.0808) = -1.40

Z Inverse (Normal Distribution)

NORM.INV(probability, μ , σ)

Given a Normal Distribution with Mean μ , Standard Deviation σ , and probability p .

where $Z = \frac{X - \mu}{\sigma}$, Find X such that $P(X < x) = p$

Example: $p = 0.8413$ $\mu = 100$ $\sigma = 25$

=NORM.INV(0.8413, 100, 25) = 125

Note: $Z = 1.00$ from Table

Example: $p = 0.1131$ $\mu = 100$ $\sigma = 25$

=NORM.INV(0.1131, 100, 25) = 69.8

Note: $Z = -1.21$ from Table

Note:

$P(a < X < b) = P(X < b) - P(X < a)$

$P(Z > z) = 1 - P(Z < z)$

Be aware there will be some small rounding errors.