

Homework #3 - Chapter 3 (Montgomery & Runger, 6ed)

Chapter 3 Discrete Random Variables and Probability Distributions

Problems:

Answers to odd-numbered problems can be found in Appendix B.

Answers to even-numbered problems are provided below.

3-4 Mean and Variance of Discrete Random Variable

Page 75, Example 3-10

Page 76, Example 3-11

Page 77, Problems 3-59, 61, 65

3-5 Discrete Uniform Distribution

Page 79, Problems 3-77, 79, 83

3-6 Binomial Distribution

Pages 84 - 85, Problems 3-93, 95, 101, 102, 103

3-9 Poisson Distribution

Pages 102 - 103, Problems 3-157, 160, 165, 167, 168

3-102. Let X denote the number of questions answered correctly. Then, X is binomial with $n = 25$ and $p = 0.25$.

$$a) P(X \geq 20) = \binom{25}{20} 0.25^{20} (0.75)^5 + \binom{25}{21} 0.25^{21} (0.75)^4 + \binom{25}{22} 0.25^{22} (0.75)^3 + \binom{25}{23} 0.25^{23} (0.75)^2 + \binom{25}{24} 0.25^{24} (0.75)^1 + \binom{25}{25} 0.25^{25} (0.75)^0 = 9.677 \times 10^{-10}$$

$$b) P(X < 5) = \binom{25}{0} 0.25^0 (0.75)^{25} + \binom{25}{1} 0.25^1 (0.75)^{24} + \binom{25}{2} 0.25^2 (0.75)^{23} + \binom{25}{3} 0.25^3 (0.75)^{22} + \binom{25}{4} 0.25^4 (0.75)^{21} = 0.2137$$

3-160. Let X denote the number of calls in one hour. Then, X is a Poisson random variable with $\lambda = 10$.

$$a) P(X = 5) = \frac{e^{-10} 10^5}{5!} = 0.0378$$

$$b) P(X \leq 3) = e^{-10} + \frac{e^{-10} 10}{1!} + \frac{e^{-10} 10^2}{2!} + \frac{e^{-10} 10^3}{3!} = 0.0103$$

c) Let Y denote the number of calls in two hours. Then, Y is a Poisson random variable with $\lambda = 20$.

$$P(Y = 15) = \frac{e^{-20} 20^{15}}{15!} = 0.0516$$

d) Let W denote the number of calls in 30 minutes. Then W is a Poisson random variable with $\lambda = 5$.

$$P(W = 5) = \frac{e^{-5} 5^5}{5!} = 0.1755$$

3-168. a) Let X denote the failures in 8 hours. Then, X has a Poisson distribution with $\lambda = 0.16$.

b) Let Y denote the number of failure in 24 hours. Then, Y has a Poisson distribution with $\lambda = 0.48$.