Additional Normal Distribution Practice Problems

First, use either a Standard Normal Distribution (Z Table) or your calculator to solve the following problems. Then verify your answers using the Excel Normal Distribution Functions.

1. For a Standard Normal Distribution (i.e., Mean $\mu = 0$ and Variance $\sigma^2 = 1$ Standard Deviation $\sigma = 1$)

Find:	Find:	Find z, such that
$P(Z < 1.43)$	$P(-1.43 < Z < +1.37)$	$P(Z < z) = 0.98$
$P(Z > 1.43)$	$P(-1.37 < Z < +1.43)$	$P(Z > z) = 0.98$
$P(Z > -1.43)$	$P(-1.43 < Z < -1.37)$	$P(Z < z) = 0.02$
$P(Z < -1.43)$	$P(+1.37 < Z < +1.43)$	$P(Z > z) = 0.02$

2. Normal Distribution Mean = 120, Standard Deviation = 32
Find:
$P(X < 80)$
$P(X > 160)$
$P(80 < X < 160)$
x such that $P(X < x) = 50\%$
x such that $P(X > x) = 50\%$
x such that $P(X < x) = 25\%$
x such that $P(X > x) = 25\%$
x such that $P(X > x) = 98\%$
x such that $P(X < x) = 98\%$

3. Assume the detection of a digital signal imbedded in background noise is normally distributed with mean = 2.70 volts and standard deviation = 0.45 volts. If the signal level exceeds 3.60 volts, the system reports that a digital 1 has been transmitted. What is the probability of reporting a digital 1 if no digital signal was sent. Note: This is the probability of a false detection (false positive).

In simple terms, let v = the voltage level, find $P(v > 3.60)$
Note: $Z = (v - \mu) / \sigma$; so $Z = (3.60 - 2.70) / 0.45 = 0.90 / 0.45 = 2.00$
$P(v > 3.60) = P(Z > 2.00) = 1 - P(Z < 2.00) = 1 - 0.9772 = 0.0228$
So the probability of saying a digital signal 1 was sent when no digital signal was sent equals $0.0228 \approx 3\%$

4. Assume the life of a semiconductor laser at constant power is normally distributed with mean of 7000 hours and a standard deviation of 600 hours.

What is the probability that a laser fails before 6000 hours?
$P(X < 6000) = P(Z < [X - \mu] / \sigma) = P(Z < [6000 - 7000] / 600) = P(Z < -1.67) = 0.0475 \approx 5\%$

What is the laser operating life (in hours) for which 95% of all laser are expected to exceed?
$P(X > x) = 0.95$ which is the same as $P(X < x) = 1 - 0.95 = 0.05$
Find z such that $P(Z < z) = 0.05$ Answer $z = -1.645$
and from $Z = (X - \mu) / \sigma$ we have $X = \mu + Z\sigma = 7000 + (-1.645)600 = 7000 - 987 = 6013$ hours

What are the symmetrical lower and upper bounds on the 99% of laser operating life (in hours)?
Note: Since we are asked to find the ***symmetric*** lower and upper bounds $P(-z < Z < +z) = 0.99$;
z can be found by $P(Z < -z) = (1 - 0.99) / 2 = 0.005$; hence $z = -2.575$
And $X_{Lower} = \mu - |Z|\sigma = 7000 - (2.575)(600) = 7000 - 1545 = 5455$
And $X_{Upper} = \mu + |Z|\sigma = 7000 + (2.575)(600 = 7000 + 1545 = 8545$
QED the 99% symmetrical bounds on the laser operating life is 5455 to 8545 hours.
Would you expect the 95% bounds to be wider or narrower than the 99% bounds?
Is this counter-intuitive?

Additional Normal Distribution Practice Problems (Answers)

1. For a Standard Normal Distribution (i.e., Mean $\mu = 0$ and Variance $\sigma^2 = 1$ Standard Deviation $\sigma = 1$)

$P(Z < 1.43) = x$	$P(-1.43 < Z < +1.37) = x$	$P(Z < z) = 0.98 \quad z = +2.05$
$P(Z > 1.43) = x$	$P(-1.37 < Z < +1.43) = x$	$P(Z > z) = 0.98 \quad z = -2.05$
$P(Z > -1.43 = x)$	$P(-1.43 < Z < -1.37) = x$	$P(Z < -z) = 0.98 \quad z = -2.05$
$P(Z < -1.43) = x$	$P(+1.37 < Z < +1.43 = x)$	$P(Z > -z) = 0.98 \quad z = +2.05$

2. Normal Distribution Mean = 120, Standard Deviation = 32

Find:

$P(X < 80) = 0.1056$

$P(X > 160) = 0.1056$

$P(80 < X < 120) = 0.8944 - 0.1056 = 0.7888$

x such that $P(X < x) = 50\%$ $x = 120.0$

x such that $P(X > x) = 50\%$ $x = 120.0$

x such that $P(X < x) = 25\%$ $x = 98.4$

x such that $P(X > x) = 25\%$ $x = 141.6$

x such that $P(X > x) = 98\%$ $x = 54.3$

x such that $P(X < x) = 98\%$ $x = 185.7$