Delta & Y Configurations, Principles of Superposition, Resistor Voltage Divider Designs

Use following techniques to solve for current through and voltage across a network resistor:

- Equivalent Resistances
- Kirchoff's Voltage Law
- Kirchoff's Current Law
- Principle of Superposition
- Thevenin Equivalent
- Norton Equivalent

Sketch Series Resistors Voltage Divider (including voltage source)
Calculate voltage across load resistor
Voltage Divider (Resistors in Series with Voltage Source) $V_2 = V (R_2 / (R_1 + R_2)$

Sketch Parallel Resistors Current Divider (including current source)
Calculate current through load resistor
Current Divider (Resistors in Parallel with Current Source) $I_2 = I (R_1 / (R_1 + R_2)$

Design Voltage Divider Network (10% rule for two resistors and voltage source) given output requirements

Convert Delta configuration to equivalent Wye configuration
Convert Wye configuration to equivalent Delta configuration

Solve Voltage Source & Current Source Circuit Network Problems (using Superposition)

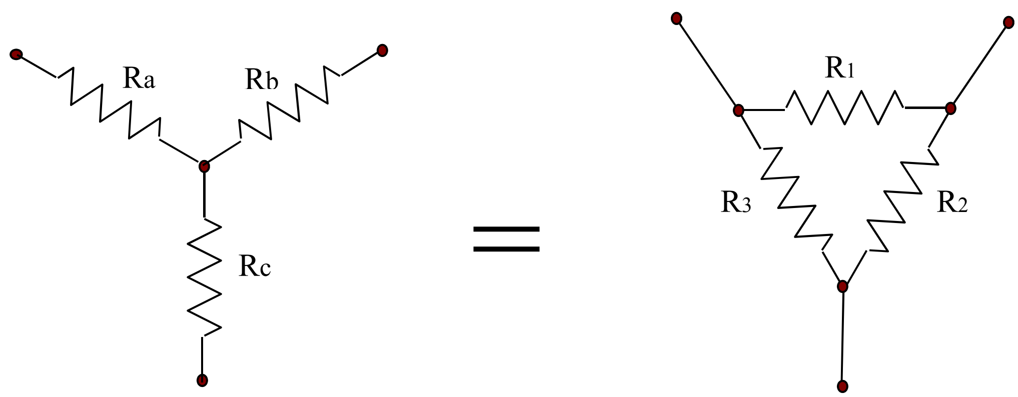
Attachments:

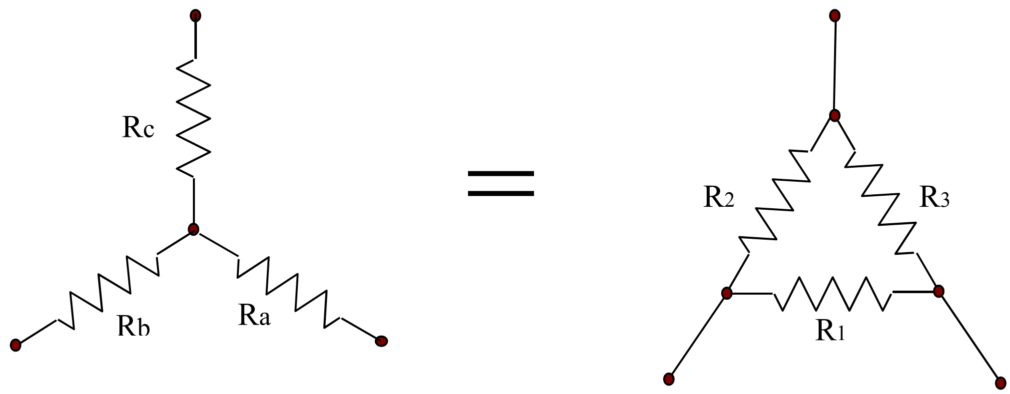
Delta & Y Configurations and Conversions (Figures & Equations)
Five Resistor Equivalent
Y to Delta & Delta to Y Practice Conversion Problems

Principle of Superposition Example Problem
Principle of Superposition (Figure 30-3 Giorgio Rizzoni 2009).pdf

Voltage Divider Design Notes & Practice Problem

Y to Delta & Delta to Y Conversions





Y to Delta Conversion

$$R_1 = \frac{R_a\,R_b + R_b\,R_c + R_a\,R_c}{R_c}$$

$$R_2 = \frac{R_a\,R_b + R_b\,R_c + R_a\,R_c}{R_a}$$

$$R_3 = \frac{R_a\,R_b + R_b\,R_c + R_a\,R_c}{R_b}$$

Delta to Y Conversion

$$R_a = \frac{R_1\,R_3}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_1\,R_2}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_2\,R_3}{R_1 + R_2 + R_3}$$

Calculate the equivalent resistance for a five resistor network (See Figure 1.)

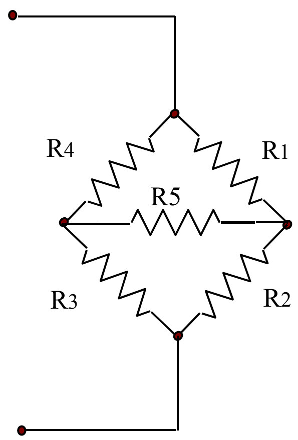


Figure 1.

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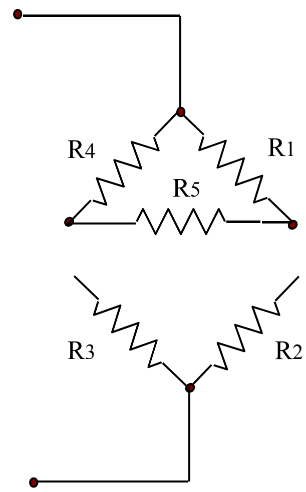
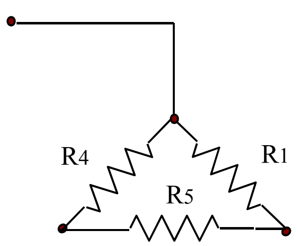
Step 1. Divide into two parts.
Delta(R1 R5 R4) and "V"(R2 R3)



Step 2. Convert Delta(R1 R5 R4)



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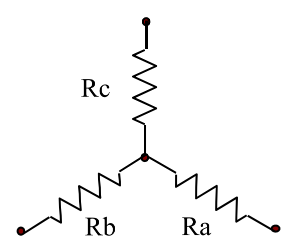
to Y(Rc Ra Rb)



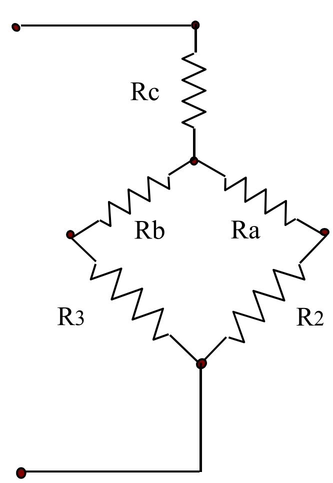
Step 2. Delta(R1 R5 R4) to Y(Rc Ra Rb) - continued
Note: Rotated and Reflected Labeling

$$Rc = \frac{R1\ R4}{R1 + R4 + R5}$$

$$Rb = \frac{R4\ R5}{R1 + R4 + R5}$$

$$Ra = \frac{R1\ R5}{R1 + R4 + R5}$$

Step 3. Combine Delta(R1 R5 R4) with "V"(R2 R3)



Step 4.
Rb in Series with R3 therefore RbR3 = Rb + R3
Ra in Series with R2 therefore RaR2 = Ra + R2

Step 5.
RbR3 in Parallel with RaR2 $= \frac{(RbR3) \times (RaR2)}{(RbR3) + (RaR2)}$

Step 6.
Rc in Series with Parallel RbR3 || RaR2

Step 7. Req = Rc + Parallel RbR3 || RaR2

Y to Delta & Delta to Y Practice Problem Conversions

1. Write closed-form solutions for each of the following configurations.

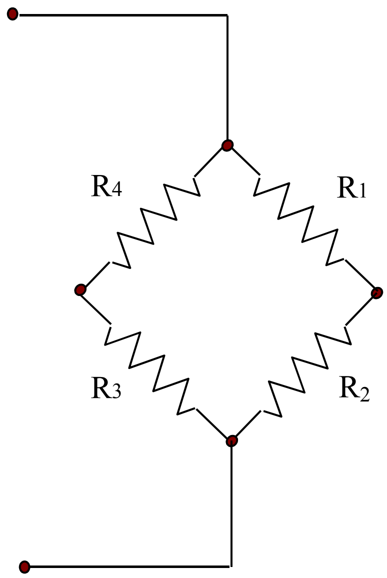


Figure A

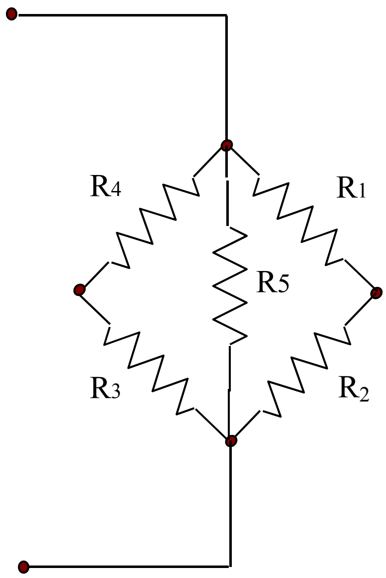


Figure B

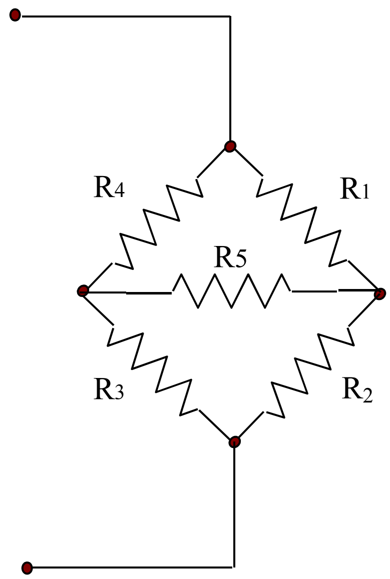


Figure C

2. For Figures A, B, C,
   Find single equivalent resistive values for $R_1 = 5$, $R_2 = 7$, $R_3 = 8$, $R_4 = 3$, $R_5 = 4$

3. Answers:
   Figure A. Req = 5.74 Figure B. Req = 2.36 Figure C. Req = 5.66

Solving Voltage and Current Circuit Problems Using the Theory of Superposition

The overall effect for two or more stimuli to a linear system is equal to the sum of individual effects.

Refer to Figure 1.

Objective:

a. Calculate V_{RLoad}
b. Calculate I_{RLoad}

Given:

$I = 3$ A
$V = 9$ V
$R_1 = 3\ \Omega$
$R_2 = 4\ \Omega$
$R_{Load} = 2\ \Omega$

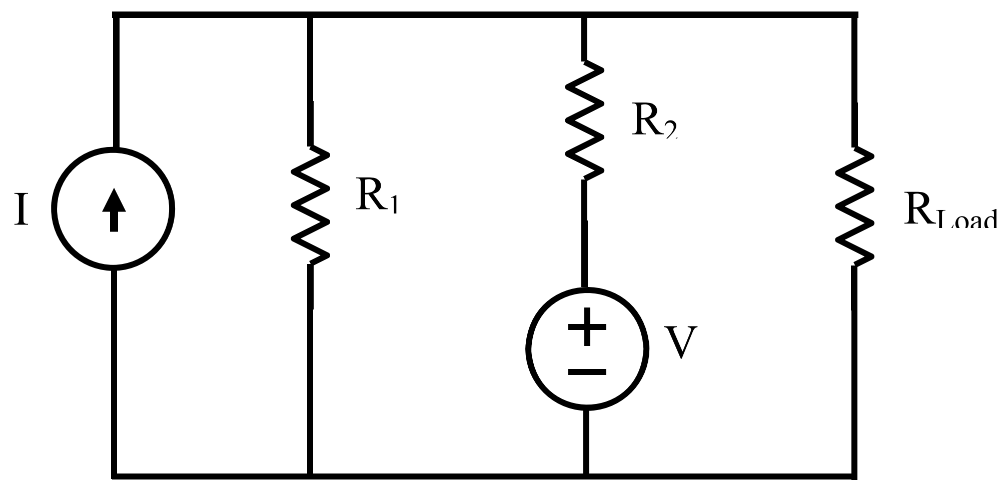


Figure 1.

Procedure:

I. Short all Voltage Sources — Calculate V_{RLoad} (Generated by Current Sources) — $V_{RLoad} = 2.77$ V

II. Open all Current Sources — Calculate V_{RLoad} (Generated by Voltage Sources) — $V_{RLoad} = 2.08$ V

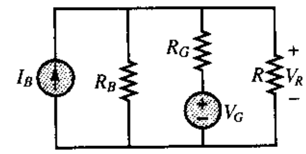
III. Calculate overall results by adding results from Step I and Step II — $V_{RLoad} = 2.77 + 2.08 = 4.85$ Volts

Calculate $I_{RLoad} = V_{Rload} / R_{Load} = 4.85\ V / 2\ \Omega = 2.43$ Amps

Principle of Superposition

Problem

Determine the voltage across resistor R in the circuit of Figure 3.30.



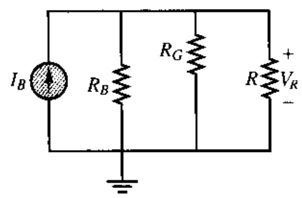
(a)

Figure 3.30 (a) Circuit used to demonstrate the principle of superposition

Solution

Known Quantities: The values of the voltage sources and of the resistors in the circuit of Figure 3.30 are $I_B = 12$ A; $V_G = 12$ V; $R_B = 1\ \Omega$; $R_G = 0.3\ \Omega$; $R = 0.23\ \Omega$.

Find: The voltage across R.



(b)

Figure 3.30 (b) Circuit obtained by suppressing the voltage source

Analysis: Specify a ground node and the polarity of the voltage across R. Suppress the voltage source by replacing it with a short circuit. Redraw the circuit, as shown in Figure 3.30(b), and apply KCL:

$$-I_B + \frac{V_{R-I}}{R_B} + \frac{V_{R-I}}{R_G} + \frac{V_{R-I}}{R} = 0$$

$$V_{R-I} = \frac{I_B}{1/R_B + 1/R_G + 1/R} = \frac{12}{1/1 + 1/0.3 + 1/0.23} = 1.38\ \text{V}$$

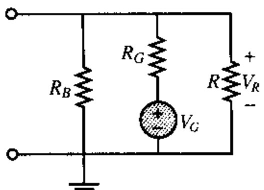
Suppress the current source by replacing it with an open circuit, draw the resulting circuit, as shown in Figure 3.30(c), and apply KCL:

$$\frac{V_{R-V}}{R_B} + \frac{V_{R-V} - V_G}{R_G} + \frac{V_{R-V}}{R} = 0$$

$$V_{R-V} = \frac{V_G/R_G}{1/R_B + 1/R_G + 1/R} = \frac{12/0.3}{1/1 + 1/0.3 + 1/0.23} = 4.61\ \text{V}$$

Finally, we compute the voltage across R as the sum of its two components:

$$V_R = V_{R-I} + V_{R-V} = 5.99\ \text{V}$$



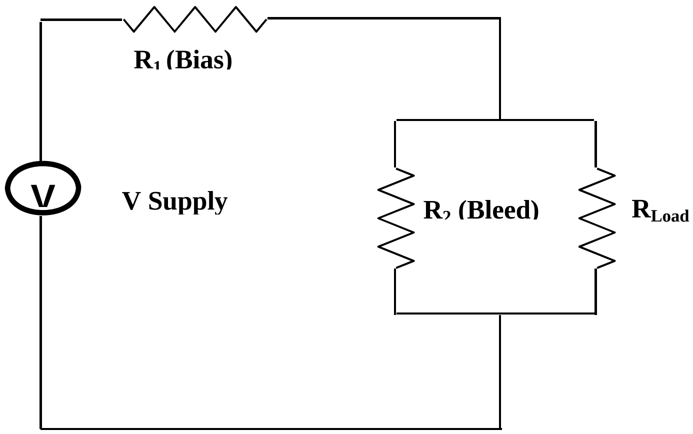
(c)

Figure 3.30 (c) Circuit obtained by suppressing the current source

Fundamental of Electrical Engineering, Giorgio Rizzoni, McGraw-Hill, 2009

Voltage Divider Design Note (10% Rule) - Minimizes I^2R Heat Loss

Given values for voltage source and R_{Load} voltage and current specifications (V_{Load} & I_{Load});
Determine values for Bias Resistor (R_1) and Bleed Resistor (R_2).



Step 1. Calculate Load Resistor (= V_{Load} / I_{Load})

Step 2. Calculate I_2 (Bleed Current) = 10% I_{Load}

Step 3. Select R_2 (Bleed Resistor) based on I_2 such that $R_2 = V_{R2} / I_2$ (Note: $V_{R2} = V_{RLoad}$)

Step 4. Calculate $I_{Total} = I_{Load} + I_2$

Step 5. Calculate R_1 (Bias Resistor) = $(V_S - V_{Load}) / (I_{Total})$

Example Problem:

Given:
V_S = 12 volts
V_{Load} = 7.5 V @ 50 mA

Find:

R_{Load} = 150 Ohms

Bleed Resistor = 1500 Ohms

Bias Resistor = 82 Ohms

Check Results:
Note - by design
$I_{Total} = 1.1 \times I_{Load}$ = 55 mA
$I_{Bleed} = 0.1 \times I_{Load}$ = 5 mA
$R_{Bleed} = 10 \times R_{Load}$ = 1500 Ohms
$V_{RBleed} = R_{Bleed} \times I_{Bleed} = 1500 \times 5 \times 10^{-3}$ = 7.5 V
Sanity Check: $V_{RBleed} = V_{Load}$ = 7.5 V !!!!!
Sanity Check: $V_{RBias} = V_S - V_{RBleed} = 12 - 7.5$ = 4.5 V
$V_{RBias} = I_{Total} \times R_{Bias} = 55 \times 10^{-3} \times 82$ = 4.5 V !!!!!!!!!

Short Cut Closed Form:
$R_{Bias} = (V_S - V_{Load}) / (1.1 \times I_{Load}) = (12 - 7.5) / (1.1 \times 0.050) = 4.5 / 0.055$ = 82 Ohms
$R_{Bleed} = V_{Load} / (0.1 \times I_{Load}) = 7.5 / (0.1 \times 0.050) = 7.5 / 0.005$ = 1500 Ohms