## Alternating Current

Capacitive \& Inductive Reactance and Complex Impedance
RC \& RL Circuit Analyses (DC Transients, Time Constants, Steady State)
RC \& RL Passive Filters

## Electrical Theory (Alternating Current)

Ohm's Law for AC $\quad \mathrm{I}_{\mathrm{RMS}}=\mathrm{V}_{\mathrm{RMS}} / \mathrm{Z} \quad$ where Z is the Complex Impedance
$|\mathrm{Z}|=\left[\left(\mathrm{R}^{2}+(\mathrm{XL}-\mathrm{XC})^{2}\right]^{1 / 2}\right.$
$\theta=\tan ^{-1}[(\mathrm{XL}-\mathrm{XC}) / \mathrm{R}]$
Power Factor $\cos \theta=\mathrm{R} / \mathrm{Z}$
Joule's Law Average Power $=1 / 2 \mathrm{~V}_{\text {peak }} \mathrm{I}_{\text {peak }} \cos \theta=\mathrm{V}_{\mathrm{RMS}} \mathrm{I}_{\mathrm{RMS}} \cos \theta \quad$ Watts
Purely Resistive Element $(\theta=0, \cos \theta=1)$ Average Power $=1 / 2 V_{\text {peak }} I_{\text {peak }}=V_{\text {RMS }} I_{\text {RMS }}$ (Watts)
$\begin{array}{llll}\text { ELI the ICE man } & \begin{array}{lll}\text { Component } & \text { Voltage / Current } \\ & \begin{array}{l}\text { Resistor } \\ \text { Capacitor }\end{array} & \text { Lags } \\ & \text { Inductor } & \text { Leads }\end{array}\end{array}$

## Capacitive \& Inductive Reactance and Complex Impedance

$\omega=2 \pi f \quad f=0.159 \omega$
Capacitive Reactance $\mathrm{X}_{\mathrm{C}}=1 / \omega \mathrm{C}=1 /(2 \pi f \mathrm{C})=0.159 / f \mathrm{C}$
Inductive Reactance $\mathrm{X}_{\mathrm{L}}=\omega \mathrm{L}=2 \pi f \mathrm{~L}$
Complex Impedance
R in series with series $\mathrm{CL} \quad \mathrm{Z}=\mathrm{R}+\mathrm{j}(2 \pi f \mathrm{~L}-1 /(2 \pi f \mathrm{C})) \quad$ Impedance is a minimum at resonance
R in series with parallel $\mathrm{CL} \mathrm{Z}=\mathrm{R}+\mathrm{j}\left(2 \pi f \mathrm{~L} /\left(1-(2 \pi f)^{2} \mathrm{LC}\right)\right)$ Impedance is a maximum at resonance

Time Constants
RC Circuit Time Constant $=\mathrm{RC}$
RL Circuit Time Constant $=\mathrm{L} / \mathrm{R}$

## Equations and Relationships

Inductive Reactance

$$
X_{L}=2 \pi f L
$$

Capacitive Reactance

$$
X_{C}=\frac{1}{2 \pi f C}
$$

## RC Circuit

$$
f_{0}=\frac{1}{2 \pi R C}
$$

Cut-off Frequency Resonant Frequency

RL Circuit
$f_{0}=\frac{1}{2 \pi L / R}$
RCL Circuit
$f_{0}=\frac{1}{2 \pi \sqrt{L C}}$
$t=\frac{R \sqrt{C / L}}{2}$

RCL Series Impedance

$$
Z=\sqrt{R^{2}+\left(X_{L}-X_{C}\right)^{2}}
$$

R\&CL Parallel Impedance

$$
Z=\sqrt{R^{2}+\left(\frac{X_{L} X_{C}}{X_{L}-X_{C}}\right)^{2}}
$$

RCL Parallel Impedance

$$
Z=\frac{R X_{L} X_{C}}{X_{L} X_{C}-R\left(X_{L}-X_{C}\right)}
$$

## Common Configuration



Notes:
When $\omega=0, \mathrm{X}_{\mathrm{C}} \rightarrow \infty$, i.e., C appears as an open circuit, so that $V_{\text {out }}=\frac{R_{2}}{R_{1}+R_{2}}$


When $\omega \gg 0, \mathrm{X}_{\mathrm{C}}=0$, i.e., C appears as a short circuit, so that $V_{\text {out }}=0$


