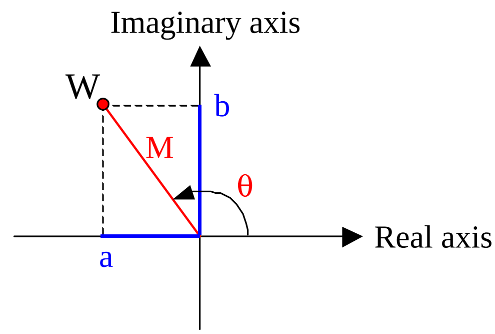
Review of Complex Numbers

A complex number represents a point in a 2D space. The value of the complex number can be represented either by its real part (a) and imaginary part (b), or by its magnitude (M) and its phase angle (θ), as shown in the figure below.

$$\mathbf{W} = a + jb$$
$$= M e^{j\theta} = (M\cos\theta) + j(M\sin\theta)$$



Computations involving two complex numbers: x = a + jb = $M e^{j\theta}$ and y = c + jd = $N e^{j\varphi}$

(a) Addition and subtraction:

$$z = x \pm y = (a + c) + j\,(b + d)$$

(b) Multiplication:

$$z = x\,y = (a + jb)\,(c + jd) = (ac - bd) + j\,(bc + ad)$$

$$z = x\,y = (M e^{j\theta})(N e^{j\varphi}) = (MN)\,e^{j(\theta+\varphi)}$$

Therefore, Magnitude of z = (magnitude of x) · (magnitude of y)
Phase angle of z = (phase angle of x) + (phase angle of y)

(c) Division:

$$z = \frac{x}{y} = \frac{a + jb}{c + jd} = \frac{(a + jb)(c - jd)}{(c + jd)(c - jd)} = \left(\frac{ac + bd}{c^2 + d^2}\right) + j\left(\frac{bc - ad}{c^2 + d^2}\right)$$

$$z = \frac{x}{y} = \frac{M e^{j\theta}}{N e^{j\varphi}} = \left(\frac{M}{N}\right) e^{j(\theta-\varphi)}$$

Therefore, Magnitude of z = (magnitude of x) ÷ (magnitude of y)
Phase angle of z = (phase angle of x) - (phase angle of y)

Power and Voltage Ratios Expressed in Decibels (dB's)

1 Bel = log(Power2 / Power1)

1 decibel = 1 dB = 0.1 Bel, hence 10 dB = 1 Bel

To express a Power Ratio in dB's, use dB = 10 log(Power2 / Power1)

Let Power2 =2 Power1
Power Ratio in dB's = 10 log(2 Power1 / Power1) = 10 log(2) = +3.01

Let Power2 = 0.5 Power1
Power Ratio in dB's = 10 log(0.5 Power1 / Power1) = 10 log(0.5) = -3.01
-3 dB is often expressed as "3 dB Down" which is the half power point (Power2 = 1/2 Power1)

Let Power2 = Power1
Power Ratio in dB's = 10 log(Power1 / Power1) = 10 log(1) 0
dB = 0 does not imply zero power but rather a power ratio of one-to-one
dB = 0 can be used as a zero reference; that is to say, set your reference level to a particular value and then use the dB scale to refer all other values to that reference level.

Examples: Reference Level = 400 watts.
200 watts = -3 dB
800 watts = +3 dB
400 watts = 0 dB
4000 watts = +l0 dB
40 watts = -l0 dB
650 watts = +2.1 dB
65 watts = -7.9 dB
l00 watts= -6 dB
2,500,000 watts = +38 dB

Note: A reference of 1 milliwatts is used for dBm's
1 milliwatts = 10 log(1 / 1) = 0 dBm
5 milliwatts = 10 log(5 / 1) = +7 dBm
500 milliwatts = +27 dBm
0.001 milliwatts = -30 dBm

For Voltage, Power = IE = (E/R)E = E^2/R

To express a Voltage Ratio in dB's, use dB = 10 log($Power_2$ / $Power_1$) = 10 log[(E_2^2/R) / E_1^2/R)]
10 log[(E_2^2/R) / E_1^2/R)] = 10 log(E_2^2 / E_1^2) = 20 log(E_2 / E_1)

For Power Ratio dB = +3, 20 log(E_2 / E_1) = +3
For Power Ratio dB = -3, 20 log(E_2 / E_1) = -3
For Power Ratio db = 0, 20 log(E_2 / E_1) = -0.15 and E_2 / E_1 = 0.707 = SQRT(2) / 2

Ideal Transformer Relations (Equations)

Definitions:

Primary Winding (input - subscript $_1$)
Secondary Winding (output - subscript $_2$)

Turns Ratio = n_1 / n_2 (number of turns on primary winding / number of turns on secondary winding)

Voltage Ratio: $V_1 / V_2 = n1 / n_2$ (Directly Proportional)

Current Ratio: $I_1 / I_2 = n_2 / n_1$ (Inversely Proportional)

Power Ratio: 1 to 1 (Power Out = Power In) Ideal
Power Out = *e* x Power In where *e* is the Efficiency Factor (*e* < 1)

Impedance Ratio: $Z_1 / Z_2 = (n_1 / n_2)^2$

For additional information, refer to
Practical Electronics for Inventors, 3ed pp 374 - 402

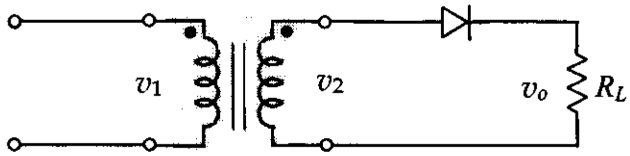
Transformer Problems and Questions

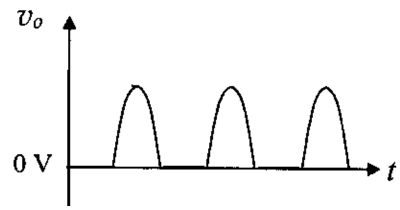
1. Given an ideal transformer with primary turns = 9600 and secondary turns = 480, assume 100% efficiency. For input voltage = 120 VAC and output impedance = 16 ohms;
   a. Calculate output voltage
   b. Calculate output current
   c. Calculate output power
   b. Calculate input current
   c. Calculate input power
   b. Calculate input impedance

2. Determine the turns ratio for an impedance matching transformer where the first stage input impedance is 50 ohms and the second stage output impedance is 8 ohms.

Half-Wave &Full Wave Rectifiers, Filtering, Regulated Power Supply

Half-Wave Rectifier Equivalent DC Output Voltage





Example

Given:
v_{in}(RMS) = 110 V (60 HZ)
Turns Ratio 10:1

Find: v_{out}(DC Effective)

v_{in}(Peak) = 1.414 v_{in}(RMS) = 1.414 x 110 = 155.5 V

v_{out}(Peak) = 1/10 v_{in}(Peak) = 1/10 x 155.5 = 15.6 V

v_{Diode} = 15.6 - 0.7 = 14.9 V

V_{out}(DC Effective) = 0.318 v_{Diode} = 0.318 x 14.9 ≈ 4.7 VDC

Exercise #1

Given: v_{in}(RMS) = 110 V (60 HZ) Turns Ratio 5:1

Find: V_{out}(DC Effective)

Answer: 9.7 VDC

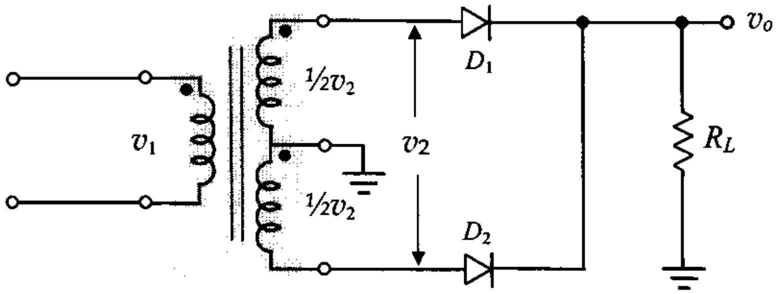
Exercise #2

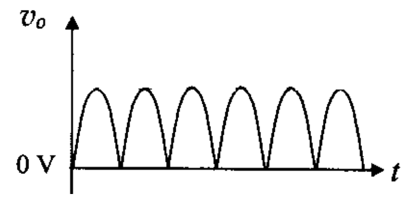
Given: v_{in}(RMS) = 120 V (60 HZ) Turns Ratio 5:1

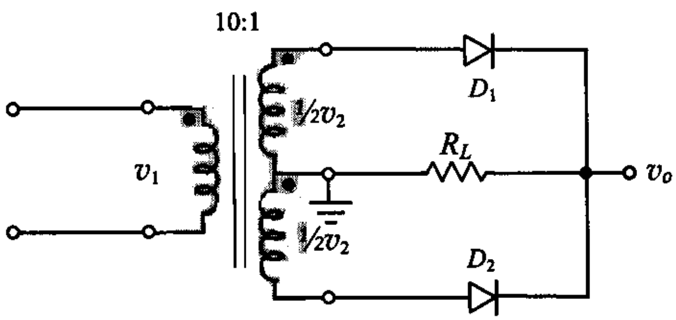
Find: V_{out}(DC Effective)

Answer: 10.6 VDC

Full-Wave Center-Tapped Rectifier Equivalent DC Output Voltage







Example

Given:
v_{in}(RMS) = 110 V (60 HZ)
Turns Ratio 10:1

Find: V_{out}(DC Effective)

v_{in}(Peak Center) = 1.414 v_{in}(RMS) = 1.414 x 110 = 155.5 V

v_{out}(Peak) = (1/2) (1/10) v_{in}(RMS) = 1/20 x 155.5 = 7.8 V

v_{Diode} = 7.8 - 0.7 = 7.1 V

V_{out}(DC Effective) = 0.636 v_{Diode} = 0.636 x 7.1 ≈ 4.5 VDC

Exercise #1

Given: v_{in}(RMS) = 110 V (60 HZ) Turns Ratio 5:1

Find: V_{out}(DC Effective)

Answer: 9.5 VDC

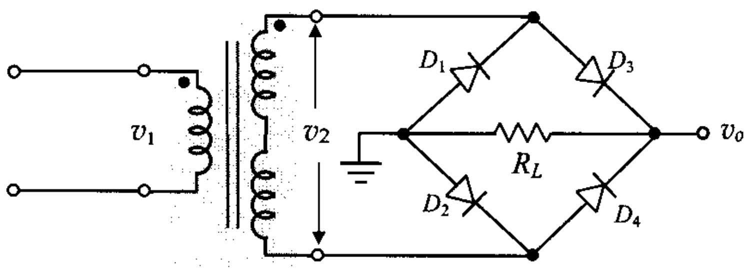
Exercise #2

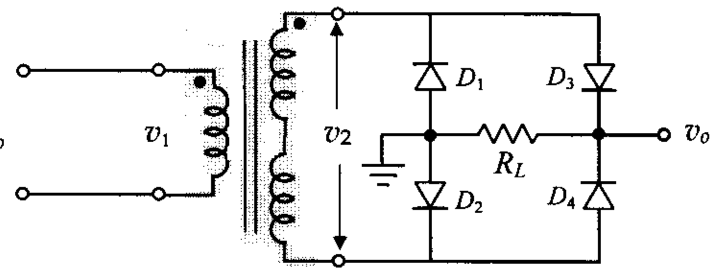
Given: v_{in}(RMS) = 120 V (60 HZ) Turns Ratio 5:1

Find: V_{out}(DC Effective)

Answer: 10.4 VDC

Full-Wave Bridge Rectifier Equivalent DC Output Voltage





Example

Given:

v_{in}(RMS) = 110 V (60 HZ)

Turns Ratio 10:1

Find: V_{out}(DC Effective)

v_{in}(Peak) = 1.414 v_{in}(RMS) = 1.414 x 110 = 155.5 V

v_{out}(Peak) = 1/10 v_{in}(RMS) = 1/10 x 155.5 = 15.6 V

v_{Diode} = 15.6 - 2(0.7) = 14.2 V

v_{out}(DC Effective) = 0.636 v_{Diode} = 0.636 x 14.2 ≈ 9 VDC

Exercise #1

Given: v_{in}(RMS) = 110 V (60 HZ) Turns Ratio 5:1

Find: V_{out}(DC Effective)

Answer: 18.9 VDC

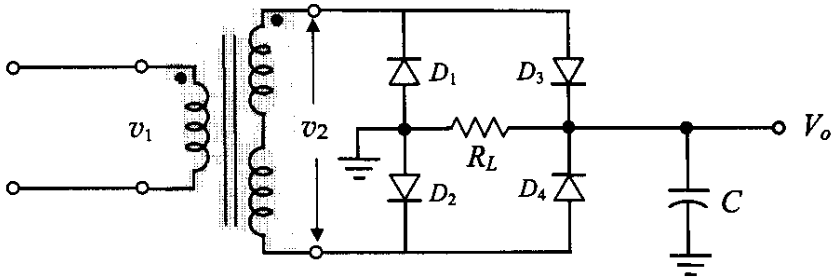
Exercise #2

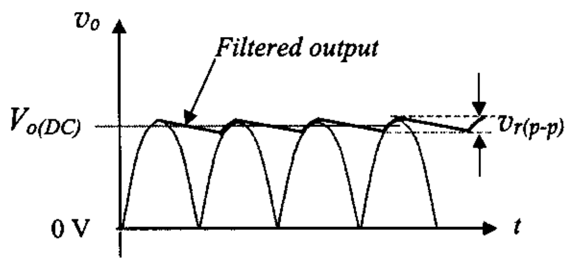
Given: v_{in}(RMS) = 120 V (60 HZ) Turns Ratio 5:1

Find: V_{out}(DC Effective)

Answer: 20.7 VDC

Filtering





$v_{ripple(peak-peak)} = I_{out(DC)} / 2fC$

$I_{out(DC)} = V_{out}(DC) / R_{Load}$

Two Steps

1. Assume $V_{out}(DC)$ (Without filtering, i.e., use peak of the rectified wave , **NOT** the DC average value.)
2. Solve for $I_{out(DC)}$ and $v_{ripple(peak-peak)}$
3. Recalculate $V_{out}(DC)$ Load = $V_{out}(DC)$ (without filtering) - $[v_{ripple(peak-peak)}] / 2$

Example

Given:
$v_{in}(RMS) = 110$ V (60 HZ)
Turns Ratio 10:1
$R_{Load} = 100\ \Omega$ C = 1000 µF f = 60 Hz

Find: $V_{out}(DC)$ Load

From Full-Wave Bridge Rectifier (from Example page 3, above) $V_{out}(DC)$ (without filtering) = 14.2 VDC

$I_{out(DC)} = V_{out}(DC) / R_{Load} = 14.2/100 = 0.142A = 142$ mA

$v_{ripple(peak-peak)} = I_{out(DC)} / 2fC = 0.142 / (2 \times 60 \times 1000 \times 10^{-6}) = 1.18$ V

$V_{out}(DC)$ Load = $V_{out}(DC)$ (without filtering) - $[v_{ripple(peak-peak)}] / 2 = 14.2 - (1.18) / 2 = 13.6$ VDC

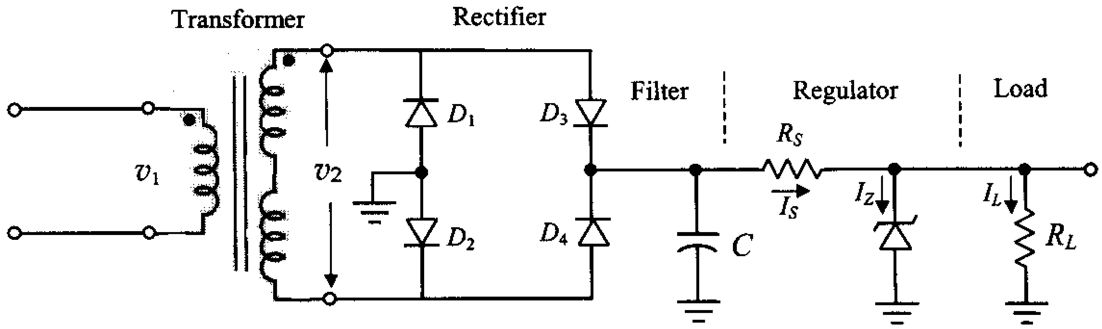
Exercise

Given:
$v_{in}(RMS) = 120$ V (60 HZ) Turns Ratio 5:1
$R_{Load} = 240\ \Omega$ C = 470 µF f = 60 Hz
$V_{out}(DC)$ (without filtering) = 32.5 VDC (from Problem, page 3, above. Note 32.5 not 32.5 x .636 = 20.7)

Find: $V_{out}(DC)$ Load

Answer: $I_{out(DC)} = 136$ mA $v_{ripple(peak-peak)} = 2.4$ V $V_{out}(DC)$ Load = 31.3 VDC

Regulated Power Supply



Scanned Images: Electronic Devices, Ali Aminian & Marian Kazimierczuk, Pearson-Prentice Hall, 2004