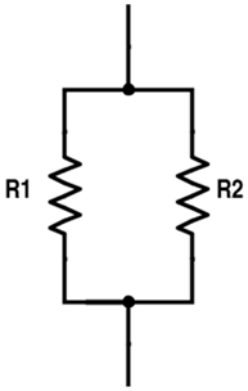
Equivalent Resistance



$$R_{EQ} = \frac{R_1 R_2}{R_1 + R_2}$$

Example 1:

$$R_1 = 100 \qquad R_2 = 300$$

$$R_{EQ} = \frac{100 \times 300}{100 + 300} = \frac{30{,}000}{400} = \frac{300}{4} = 75$$

Example 2:

$$R_1 = 2700 \qquad R_2 = 2700$$

$$R_{EQ} = \frac{2700 \times 2700}{2700 + 2700} = \frac{2700 \times 2700}{2(2700)} = \frac{2700}{2} = 1350$$

Example 3:

$$R_1 = 100 \qquad R_2 = 10\text{M}$$

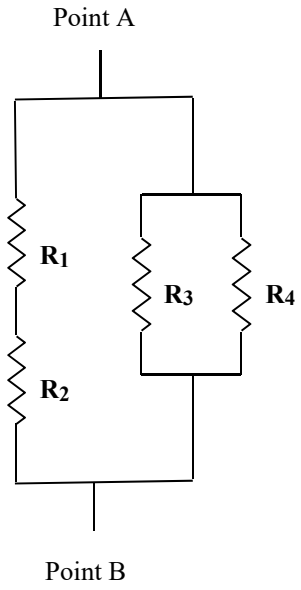
$$R_{EQ} = \frac{100 \times 10^6}{100 + 1{,}000{,}000} = \frac{10^8}{1{,}000{,}100} \approx \frac{10^8}{10^6} = 10^2 = 100$$

Example 4:

$$R_1 = 5000 \qquad R_2 = 0$$

$$R_{EQ} = \frac{5000 \times 0}{5000 + 0} = 0$$

Series and Parallel Resistor Example Problems



Given the configuration above, calculate the equivalent resistance from Point A to Point B.

R1 = 100 Ω R2 = 300 Ω R3 = 100 Ω R4 = 400 Ω

If the current through R4 is 100 mA;
What is the voltage across R2?
What is the current through R1?
What is the total current from Point A to Point B?

Equivalent Resistance from Point A to Point B	66.7 Ω
Voltage across R2	30 V
Current through R1	100 mA
Total current from Point A to Point B	600 mA

What if I_{R4} =100 mA:

Find I_{R3}, V_{R2}, I_{R1}, V_{AB}, Total Current

I_{R3} = 400 mA

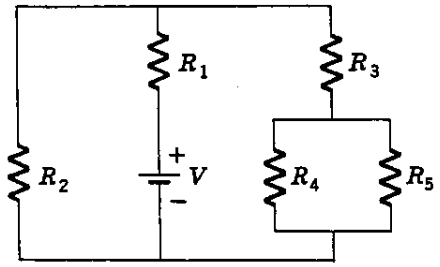
V_{R2} = 30 V

I_{R1} = 100 mA

V_{AB} = 40 V

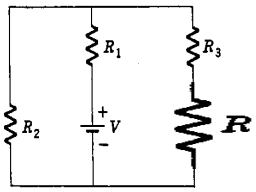
Total Current = 600 mA

Circuit Analysis by Equivalent Resistors

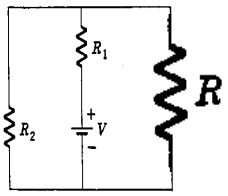


R1 = 270
R2 = 500
R3 = 720
R4 = 400
R5 = 600
V = 9 VDC

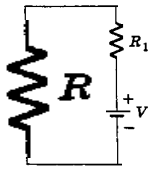
R4 || R5 = (400 x 600) / (400 + 600) = 240000 / 1000 = 240



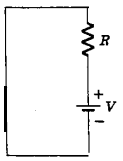
R3 + R4 || R5 = 720 + 240 = 960



R2 || (R3 + R4 || R5) = 500 || 960 = (500 x 960) / (500 + 960) = 330



Req = R1 + 330 = 270 + 330 = 600 Ohms



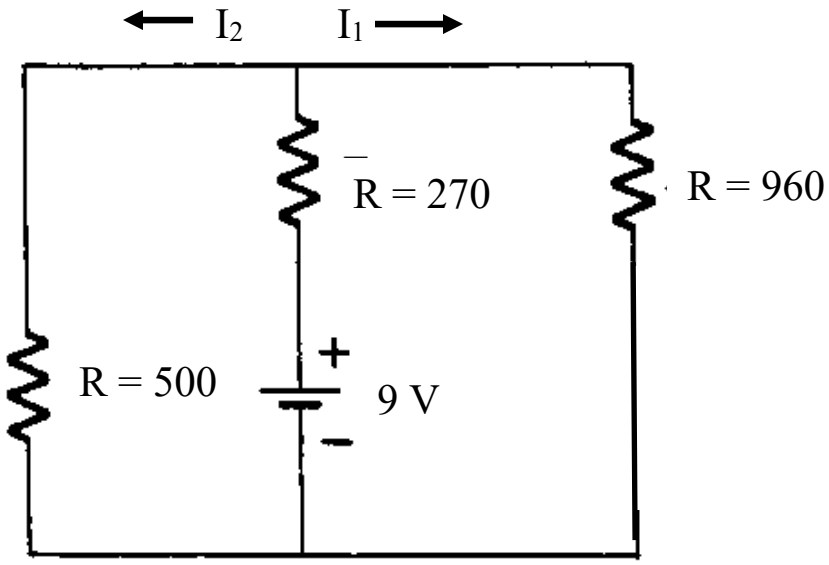
I_{R1} = V / 600 = 9 / 600 = 1.5×10^{-2} = 0.015 A = 15 milliamps

V_{R1} = R1 x I_{R1} = 270 x .015 = 4 V

V_{R2} = V - V_{R1} = 9 - 4 = 5 V

Circuit Analysis by Equivalent Resistors

Additional Analysis



See Page One for equivalent resistances

Loop Currents

Loop #1 $9 = (i_1 + i_2)R_1 + i_1R_2$

Loop #2 $9 = (i_1 + i_2)R_1 + i_1R_{EQ}$

$$R_{EQ} = R_3 + R_4 \,||\, R_5 = 720 + \frac{400 \times 600}{400 + 600} = 720 + 240 = 960$$

From #1 and #2

$$(i_1 + i_2)R_1 + i_1R_2 = (i_1 + i_2)R_1 + i_1R_{EQ}$$

$$i_1R_2 = i_2R_{EQ}$$

$$i_1 = i_2\frac{R_{EQ}}{R_2} = i_2\frac{960}{600} = 1.92i_2$$

From #1

$$9 = (i_1 + i_2)R_1 + i_1R_2$$

$$9 = (1.92i_2 + i_2)270 + 1.92i_2(500)$$

$$9 = 2.92i_2(270) + 1.92i_2(500)$$

$$9 = i_2(2.92 \times 270 + 1.92 \times 500)$$

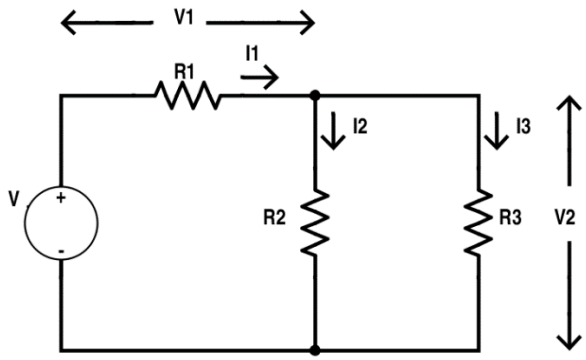
$$i_2 = \frac{9}{788 + 960} = \frac{9}{1748} = .0052$$

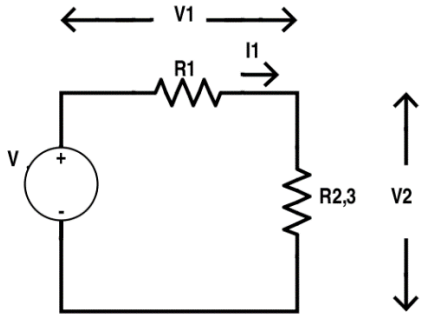
$$i_1 = 1.92 \times .0052 = 0.0099$$

$i_1 = 0.010\text{A}$ $i_2 = 0.005\text{A}$

$I_{R_1} = 0.010 + 0.005 = 0.015\text{A}$

$V_{R_1} = I_{R_1}R_1 = 0.015 \times 270 = 4.05\text{V}$





Find V_1, V_2, I_2, I_3

Given: $V = 10\text{V}$ $R_1 = 22\text{k}\Omega$ $R_2 = 47\text{k}\Omega$ $R_3 = 180\text{k}\Omega$

$$R_{2,3} = \frac{R_2 R_3}{R_2 + R_3} = \frac{47 \times 180}{47 + 180} = 37.26\text{k}\Omega$$

$$V = I_1\left(R_1 + R_{2,3}\right)$$

$$I_1 = \frac{V}{R_1 + R_{2,3}} = \frac{10}{\left(22 + 37.26\right) \times 10^3} = 0.168\text{mA}$$

Use the voltage divider rule

$$V_1 = \left[\frac{R_1}{R_1 + R_{2,3}}\right] V = \left[\frac{22}{22 + 37.26}\right] \times 10 = 3.71\text{V}$$

$$V_2 = \left[\frac{R_{2,3}}{R_{2,3} + R_1}\right] V = \left[\frac{37.26}{37.26 + 22}\right] \times 10 = 6.288\text{V}$$

Use the current divider rule

$$I_2 = \left[\frac{R_3}{R_2 + R_3}\right] I_1 = \left[\frac{180}{47 + 180}\right] \times 0.168 = 0.133\text{mA}$$

$$I_3 = \left[\frac{R_2}{R_3 + R_2}\right] I_1 = \left[\frac{47}{180 + 47}\right] \times 0.168 = 0.034\text{mA}$$