Implicit Shape Representation and Registration Motivated by the Segmentation Problem

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CVPR 2008 Tutorial
Outline

• Problem Statement and Motivation.

• Scalar and Vector Implicit Spaces.

• Shape Registration.

• Conclusion and Future Directions.
Motivations and Problem Statement
Image Segmentation

The process of image segmentation/classification aims to partition the image into regions of interest.

→ It is a key component in image analysis and computer vision.

→ The literature is quite rich; early work started in late 1960’s.

**Example**: Separate/outline the kidney from surrounding tissues in a low resolution MRI scan.
**Image Segmentation Challenges**

- Noise
- Inhomogeneities
- Lack of Strong Edges
- Different Objects may Share Similar Colors or Gray levels
- Cluttered/Occluded/Overlapped Objects
Segmentation Approaches

1. Boundary-Based
   - Edge detectors (e.g., Marr-Hildreth 1980)
   - Active contours/snakes (Kass et al., 1987)
   - Active shape models (T.F. Cootes 1996)

2. Region-Based
   - Deformable templates
   - Statistical/clustering techniques
   - MRF-based techniques (Besag 1974)
   - Active appearance models (T.F. Cootes 1998)
3. Boundary + Region-Based

- Active contours
- Graph-based techniques

Level set methods (model free + knowledge-based)
The Level Set Method

- Represents a curve/surface (∆) by a higher dimensional function.
- Object boundary can be extracted by controlling the propagation of the level set front (∆: zero level).

\[ \Phi \text{ implicitly determines } \Delta \]
\[ \Delta : \Phi(X,t) = 0 \]
Edge-based Evolution “Boundary Driven”

- A curve/surface is initialized inside/outside the object and then it evolves depending on the edge map.
  - Chunming Li et. al 2005.

- It works only when the image has good edges that identify the object. High noise levels affects the results.
Evolution without Edges “Region Driven”

• The evolution mainly depends on the region information (enclose by the contour) not edges.
  • Yezzi-Tsai-Willisky 1999.
  • Baillard et.al 2000.
  • Chan-Vese 2002

Notes:
  • Some of these approaches are designed only for bimodal images.
  • Region description is limited. (Either fixed parameters or just region means)
  • Depend on a big number of parameters which need fine tuning. It makes the application fail in many cases.
Knowledge-based Evolution

• A model that represents shape variations is embedded into the image domain by means of registration.
  • Leventon-Grimson-Faugeras 2000.
  • Tsai et.al 2003.
  • Cremers-Osher 2004.
  • Rousson-Paragios-Deriche 2004.
  • Metaxas et.al. 2004.

Notes:
• It still has the weighting coefficients tuning problem.
• It uses binary representation to align shapes.
• Conventional shape representation using level sets restricts the embedding process to a special case (only homogeneous scaling).
Problem Statement

1. Given an image $I$, we need to mark boundaries of an object inside the image.

2. A moving front $\Gamma$ implicitly represented by a function $\Phi$ as the zero level, is required to evolve to hit the object boundaries based on the image and prior object information.

3. A variational formulation is used by the energy:

$$E = E(I, \nabla I, \Phi, \nabla \Phi, \Phi_{\text{prior}})$$

The problem is solved by minimizing the given energy to generate a system of PDEs to drive the front towards the boundaries.
This Problem Includes

1- Curve/Surface Representation and Evolution (PAMI’07).

2- Adaptive Region Modeling (MICCAI’04).

3- Shape Representation and Registration (ISBI’07, CVPR’07).

4- Model-based Segmentation (ICCV’05, PAMI’07).
Advantages of using the Variational Formulation

- Modeling images in continuous time domain which simplifies the formalism and makes it grid independent.

- Research on numerical analysis makes it possible to achieve high speed, accurate, and stable solutions of the derived PDEs.

- Viscosity solutions provide a framework for dealing with non-smooth signals used with these approaches.

- Formal analysis in this area gives the possibility of providing successful algorithms and useful theoretical results like existence and uniqueness of the solution.
Shape and Its Representation
What is Shape?

• It is a binary image representing the extent of the object.
• It can be thought of as a silhouette of the object.
• And of course many applications exit for the shape and its analysis.
A planar curve can be defined as:

\[ C(p) : \mathbb{R} \rightarrow \mathbb{R}^2 \]

Where curve parameterization is \( p \):

\[ p \in [0,1] \]

In Cartesian coordinates:

\[ C(p) = \begin{bmatrix} x(p) \\ y(p) \end{bmatrix} \]

The curve is defined to be closed if:

\[ C(0) = C(1) \]
A surface can be defined as:

\[ S(u, v) : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \]

where surface parameterization is \((u, v)\):

\[(u, v) \in [0,1] \times [0,1]\]

In Cartesian coordinates:

\[
S(u, v) = \begin{bmatrix}
x(u, v) \\
y(u, v) \\
z(u, v)
\end{bmatrix}
\]
Curve/Surface Implicit Representation

Curves and surfaces can be represented in implicit forms:

\[ C = \{(x, y) : \Phi(x, y) = 0\}, \Phi : \mathbb{R}^2 \rightarrow \mathbb{R} \]

Surface: \[ S = \{(x, y, z) : \Phi(x, y, z) = 0\}, \Phi : \mathbb{R}^3 \rightarrow \mathbb{R} \]
• Topological changes in the evolving front are handled naturally. No parameterization is needed.

• This representation allows merging and breaking of the front as $t$ advances
The Signed Distance Function (SDF)

\[ \Phi(X) = \begin{cases} 
+ D(X) & \text{if inside} \\
0 & \text{on the boundary} \\
- D(X) & \text{if outside} 
\end{cases} \]

- \( D \) is the \textit{min} Euclidean distance between \( X \) and the contour.
- \( \Phi \) is continuous and differentiable around the zero level.
The Vector Distance Function (VDF)

- For the curve shown, we can define the function:

\[
\Phi_{VDF}(X) : R^2 \rightarrow R^2 \\
\Phi_{VDF}(X) = X - X_0
\]

- \(X_0\) is the closest point to \(X\).
- It can represent open/closed boundaries.

- An example for a planar circle is shown below:

Gomes and Faugeras IJCV 2003
The VDF and the SDF Relation

- They have same magnitude.
- The VDF have a direction equal to the gradient of the SDF.
- The following relation holds

\[
\Phi_{VDF} = \Phi_{SDF} \nabla \Phi_{SDF}
\]

where \( \| \nabla \Phi_{SDF} \| = 1 \)
A Variational Framework for the Shape Registration
It is mainly needed for the Shape-based Segmentation Problem
Illustration of the Shape Registration Problem

Several approaches (Cohen 1998, Fitzgibbon 2001, Paragios 2002) are proposed but they have the following problems:

- Scale variations are not handled.
- Dependency on the initialization.
- Local deformations can not be covered efficiently.

This process aims to build point correspondences between the two shapes.
Illustration of the Shape Registration Problem (Cont…)

1- Covering Global Motion:

2- Covering Local Motion (Elastic):
Global Registration of Shapes

• Given two shapes represented by the vector functions $\Phi_1$ and $\Phi_2$.

• A transformation $A$ with scales, rotation and translation is to be calculated to transform the first object to the second.

• The following dissimilarity measure the difference between the vector and the other scaled one:

$$ r = SR\Phi_1(X) - \Phi_2(A) $$

• The following energy is formulated as a sum of squared differences:

$$ E = \int_{\Omega} \delta \in r^T r d\Omega $$

• The delta is an indicator function.

• The gradient descent is used to calculate the parameters.

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Energy Function Convexity

- Transformation parameters are required to minimize the energy function.

- The function is not guaranteed to be convex to all of these parameters.

- An empirical evaluation is performed.
  - A two 2D shapes are registered to each other.
  - The energy is plotted according to the change of only two parameters and fixing the rest.
Energy Function

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Energy Function (Cont…)

• The function is smooth.

• It has global minimum in each case.

• It has nice characteristics because it depends on the Euclidean distance property which is a strong feature.

• The performance is promising.

• Convexity is not guaranteed in full dimensionality.
Examples

The conventional method (Paragios ECCV 2002) uses scalar level sets and can use only homogeneous scales ($s_x = s_y$)
Quantitative Validation

• Two hundreds registration cases are carried out with known ground truth for the parameters. The accuracy of the transformation parameters are shown for shapes of CC and HC:

<table>
<thead>
<tr>
<th></th>
<th>$S_x$</th>
<th>$S_y$</th>
<th>$\theta^o$</th>
<th>$T_x$</th>
<th>$T_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC</td>
<td>$-0.005 \pm 0.009$</td>
<td>$0.003 \pm 0.007$</td>
<td>$0.02 \pm 0.18$</td>
<td>$-0.5 \pm 0.4$</td>
<td>$-0.3 \pm 0.5$</td>
</tr>
<tr>
<td>HC</td>
<td>$0.009 \pm 0.007$</td>
<td>$0.005 \pm 0.004$</td>
<td>$0.01 \pm 0.09$</td>
<td>$0.0 \pm 0.2$</td>
<td>$-0.0 \pm 0.2$</td>
</tr>
</tbody>
</table>

• Another test is considered by measuring the correlation coefficient between the source and target vector representations magnitudes. The experiments show that it is dramatically increases after alignment and almost equals 1.

$$\gamma = \frac{E[(\| \Phi_1 \| - E(\| \Phi_1 \|))(\| \Phi_2 \| - E(\| \Phi_2 \|))]}{\sigma_1 \sigma_2}$$

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Comparison

Ground Truth is:

\( s_x = 2.5, s_y = 3.33 \)

By Paragios et. al. in PAMI 2006

Our Approach

Global Min. is Exactly at:

\( s_x = 2.5, s_y = 3.33 \)

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Shapes Example (11-Shapes). Group#1
Shapes Example (11-Shapes)..Group#2
Shapes Example (11-Shapes)..Group#3
Shapes Example .. Evaluation

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Real Shapes Example

29 Real CCs Before Alignment. After Alignment

Scales variations in each case

\[ S_x \neq S_y \]

Guarantees the need of the Approach
Registering Non-Complete Data
Need for Non-rigid Registration

- Local deformations need to be covered.
Elastic Shape Registration

• Given two globally aligned shapes represented by the vector functions $\Phi_1$ and $\Phi_2$.

• A deformation field $U$ is applied to the source domain to match the target boundaries.

• The following dissimilarity measure the difference between the two vectors:

$$ r = \Phi_1(X) - \Phi_2(X + U) $$

• The following energy is formulated as a sum of squared differences:

$$ E_n = \int_{\Omega} r^T r d\Omega $$

• The deformation field is required to minimize the above function.
Elastic Shape Registration

• Lattice resolution is $M \times N$.

• $P = P_{m,n}$ is the control point where $m = \{1, \ldots, M\}$ and $n = \{1, \ldots, N\}$.

• The deformation field is represented by the deformation of the control lattice: $U(X) = \sum_{l=0}^{3} \sum_{m=0}^{3} B_l(u)B_m(v)\delta P_{i+l,j+m}$

• $(i,j)$ is the index of the nearest node to $X$.

• $(u,v)$ is the relative displacement field from that node.

$x + U(X)$
Elastic Shape Registration

• The deformation field is represented by an FFD lattice:

\[ U(X) = \sum_{l=-3}^{3} \sum_{m=0}^{3} B_l(u) B_m(v) \delta P_{i+l,j+m} \]

• Using Taylor Series: \( \Phi_2(X + U) \approx \Phi_2(X) + \nabla \Phi_2 U \)

• The energy becomes quadratic in terms of the control points deformations. Setting the derivatives to zero, a linear system results:

\[ \Lambda P_c = \Psi \]

• where \( P_c \) is the vector of unknowns.

• The solution of this linear system gives directly the positions of the control lattice.

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Elastic Shape Registration

Why Incremental Free Form Deformations (IFFDs)?

- Parallel grid lines (no folding over).
- Topology is preserved.

Sudden Movement

IFFDs

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Elastic Shape Registration

An Incremental Free Form Deformation

0- Calculate the implicit vector representations $\Phi_1$ and $\Phi_2$.

0- Initialize the lattice size $M=M^0$ and $N=N^0$.

1- Solve for the control lattice deformations and hence calculate their new positions using the given linear system.

2- Update the source shape and its representation $\Phi_1$.

3- Increase the lattice resolution (to get more freedom): $M=M+1$, $N=N+1$.

4- Stop if satisfactory deformation is achieved or a certain resolution is reached. Otherwise go to step 1.
Example 1

Grid Deformations

Correspondences

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Example..2

Grid Deformations

Correspondences

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Comparison with the ICP..1

ICP

OURS
**Comparison..2**

Compared with the approach proposed in PAMI 2006 by Paaragios et. al.:

1. **The vector representation is more general than the distance function.**

2. **The execution time is much less than theirs because of the closed form solution of the control points.**

3. **That allows covering much more complicated deformations.**

4. **In case of open shapes, the implicit vector representation is differentiable around the boundary. The conventional distance map is differentiable in this case.**
More Experiments
Multiple Parts

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More Experiments
Open Structures

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Validation

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Conclusion
1- Curve/Surface Representation and Evolution

• Curve/Surface (front-Γ) is considered to be a shape boundary implicitly represented by a higher dimensional function (Φ).
• New implicit vector representations are used.
• New vector PDEs are derived to control the evolution process.
• Calculus of the new implicit function is derived.
• The definition is maintained using a new technique.
• Speed functions are considered to include intrinsic properties of the front in addition to image information.

2- Shape Modeling and Registration:

• Training curves/surfaces are globally aligned by covering their scales, rotations, and translations w.r.t a mean shape. Implicit functions are used as a matching criteria for a sum of squared differences energy.
• A new global alignment technique is proposed.
• Also a new elastic registration is provided to cover local deformations.

3- Shape-based Segmentation:

Minimize

\[ E = E(I, \Phi, \nabla \Phi, \Phi_{\text{prior}}) \]

By the Gradient Descent Technique:

1. \[ 1 - \Phi_t = - \nabla \Phi E \]
2. \[ 2 - A_t = - \nabla A E \]
3. \[ 3 - w_t = - \nabla w E \]

Initialize:

\[ A, w \text{ (set } w\text{'s to the mean shape-1/N)} \]

Initialize object and background regions parameters using the EM algorithm

Object (π_o, μ_o, σ_o)

Background (π_b, μ_b, σ_b)
Future Directions

- Looking for scale invariant features for shapes.
- Graph cuts and minimization.
- Other applications like tracking and surveillance.
- Faster implementations using GPUs
Selected Publications


3. H. E. Abd El Munim and A. A. Farag, "A Shape-based Segmentation Approach: An Improved Technique using Level Sets," Proc. of International Conference on Computer Vision (ICCV’05), Beijing, China, October 15-21, 2005, pp. 930-935. (Oral Presentation)-(Among about 1200 papers, only 245 could be accepted. Among these, only 45 were accepted as an oral presentation.)


Thank You