

WRIGHT STATE UNIVERSITY
Department of Computer Science and Engineering
CS7200: Algorithm Design and Analysis

Spring 2020

Midterm (50 pts)

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Write the answers only in the space provided. Use the “blank” sides of each page for scratch work. Be concise, precise and legible.

Suggestion: Attempt the problems in the order of your comfort-level and familiarity. If you feel you are stuck, move onto other problems, complete them, and then return so that you attempt all the problems.

NAME	
UID	
EMAIL	

1) [Stable Matching Problem: 4 + 4 pts]

Consider an instance of the stable marriage problem with the priority lists:

Albert	Diane	Emily	Fergie
Bradley	Emily	Diane	Fergie
Charles	Diane	Emily	Fergie

Diane	Albert	Bradley	Charles
Emily	Albert	Bradley	Charles
Fergie	Albert	Bradley	Charles

- (a) Is **{Albert-Diane, Bradley-Emily, Charles-Fergie}** a stable matching? If yes, then provide an argument and show which pairs you tested for, for thorough verification. If no, provide a pair that causes instability.

- (b) Determine man-optimal and woman-optimal solutions.

Man-optimal:

Woman-optimal:

2) [Greedy Algorithm: (3 + 6) + (3 + 5)]

(i) Consider the greedy algorithm for interval scheduling which picks the compatible interval that finishes as soon as possible. This gives an optimal solution for the problem of finding the maximum number of intervals. Does the corresponding algorithm using *latest start time* find an optimal solution? (That is, consecutively schedules compatible jobs in descending order of *latest start time*.)

If YES, provide an informal argument, and discuss if it constructs the same schedule?

If NO, provide a counterexample to show where the heuristics breaks down.

(ii) Is the optimal job schedule obtained for Interval Partitioning problem always unique? If YES, provide an informal argument. If NO, provide a counterexample.

3) [Asymptotic Complexity Notation – Big Oh: 10 pts]

Pre-order the following set of functions in increasing complexity (that is, partition and totally order the equivalence classes induced by Big-Theta definition). For example, for the functions $\{n, kn, n^2, n^2 + 2n, 10^n\}$, you would write $\{n, kn\} < \{n^2, n^2 + 2n\} < \{10^n\}$, where k is a constant. [*Grading guideline*: 10 pts for perfect solution. 7 pts for one or two mistakes, and < 5 pts otherwise.]

2^{10} , 2^n , n^3 , $n^{\frac{1}{2}}$, $\log n$, n^n , $\ln n$, e^n , $2^{(n-1)}$, $(\log n)^2$, $n \log n$, $5\sqrt{n}$, $(\log \log n)$

4) [Divide and Conquer: 3 + 7 pts]

Given a sequence $a_1, a_2, \dots, a_n$ of n distinct numbers, an *inversion* is a pair $i < j$ such that $a_i > a_j$. Note that a sequence has no inversions if and only if it is sorted in ascending order. In class, we discussed a variant of merge sort algorithm for counting inversions as follows.

```
Sort-and-Count(L) {  
  if list L has one element  
    return 0 and the list L  
  Divide the list L into two halves A and B  
   $(r_A, A) \leftarrow \text{Sort-and-Count}(A)$   
   $(r_B, B) \leftarrow \text{Sort-and-Count}(B)$   
   $(r_t, L) \leftarrow \text{Merge-and-Count}(A, B)$   
  return  $r = r_A + r_B + r_t$  and the sorted list L  
}
```

Write the pre-post conditions and pseudo code for *Merge-and-Count*(A, B).

5) [Graph Property: 5 pts]

Let G be a graph on n nodes, such that G is connected and acyclic. Show that G has $(n-1)$ edges. [Your solution will be graded on its clarity.]