

WRIGHT STATE UNIVERSITY
Department of Computer Science and Engineering
CS7200: Algorithm Design and Analysis

Fall 2013

Final (40 pts)

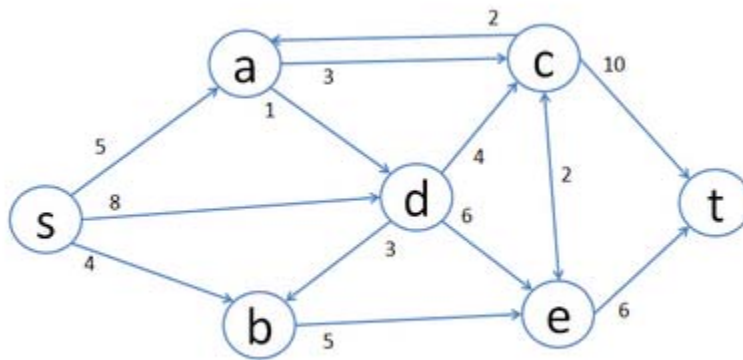
Prasad

NOTE: The exam is open book / lecture notes, but use of computer/Internet is strictly prohibited. You may enter your solutions for the Problems 4 and 5 in the separate sheet provided. Do not forget to include your name on it.

- 1) [4 pts] Consider four graph algorithms with the following worst case running times (where n is the size of the node set and m is the size of the edge set): $n \lg n$, $n \lg m$, $m \lg n$, and $m \lg m$. Using the big-Oh criteria, determine which ones are equivalent, and on that basis, order the equivalence classes.
- 2) [4 pts] Are there values of n for which repeated square rooting ($\sqrt{\dots\sqrt{n}}$) can yield integers? Either argue that such arbitrarily long finite sequence of integers do not exist, or provide a general formula/schema for constructing such sequences. (FYI: Such problems arise while solving recurrences.)
- 3) [2 + 3 + 2 + 3 pts] Provide closed form and/or order-theoretic solutions to the following recurrences. (Assume $T(1) = 1$.)
 - (a) $T(n) = 16 T(n/4) + n$
 - (b) $T(n) = 16 T(n/4) + n^2$
 - (c) $T(n) = 16 T(n/4) + n^3$
 - (d) $T(n) = T(n-1) + n^2$
- 4) [1 + 4 + 2 + 2 pts] Use dynamic programming to find least-cost alignment of the two strings CAT and ACGC under the operation cost model where *insert*, *delete* and *substitute* cost +1 and *match* costs -2.

Clearly state (a) the recurrence relation and then (b) fill the appropriate table. (c) Give an explicit list of operations to align the two strings or display the strings in such a way that the details of the alignment are clear. (d) Is the least-cost alignment unique (or are there multiple ways of achieving it)?

- 5) [2 + 5 pts] (a) Determine the maximum flow from node s to node t in the following network. The labels on the edges indicate their capacity.
- (b) Show the actual flow through each edge of the network so as to realize the maximum flow. (Note that c-e edge goes both ways.)



- 6) [2 + 4 pts] *The Clique problem:* Given a graph $G = (N, E)$ and an integer k , determine if G contains a clique of size k . A *clique* is a set of nodes such that every pair of nodes is connected by an edge.

Prove that the Clique Problem CP is NP-Complete in two steps:

- Select a suitable NP-Complete problem NPC discussed in the class and specify the direction of reduction.
- State and justify the validity of the construction/reduction that relates instances of CP and NPC.