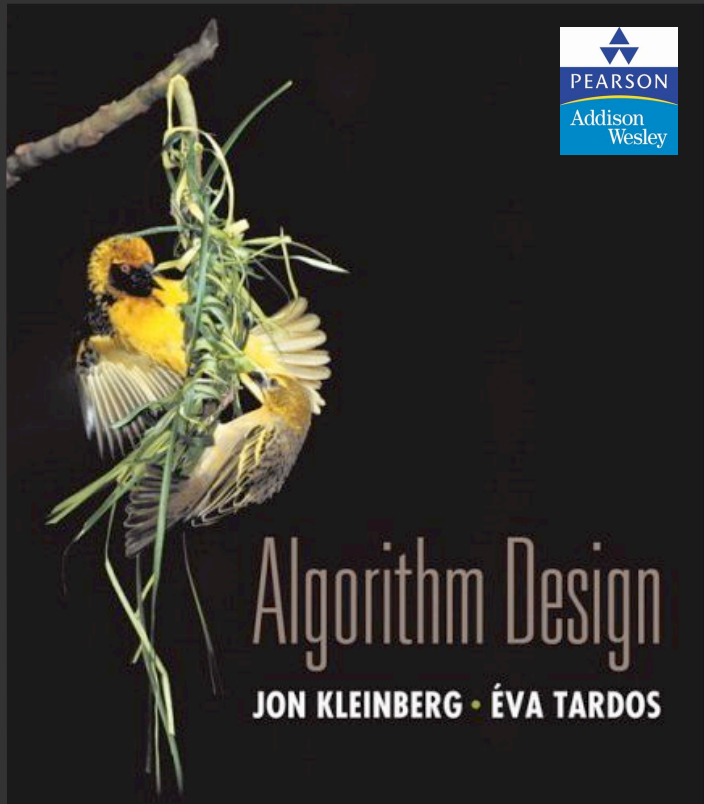




7. NETWORK FLOWS I

- ▶ *Ford-Fulkerson pathological example*

Lecture slides by Kevin Wayne

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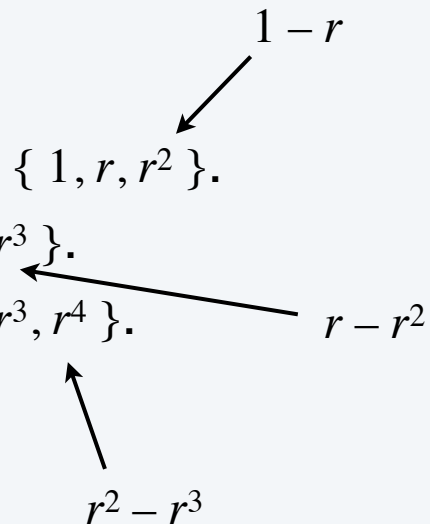
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<http://www.cs.princeton.edu/~wayne/kleinberg-tardos>

Ford-Fulkerson pathological example

Intuition. Let r satisfy $r^2 = 1 - r$.

- Initial capacities are $\{1, r\}$.
- After some augmentation, residual capacities are $\{1, r, r^2\}$.
- After some more, residual capacities are $\{1, r, r^2, r^3\}$.
- After some more, residual capacities are $\{1, r, r^2, r^3, r^4\}$.

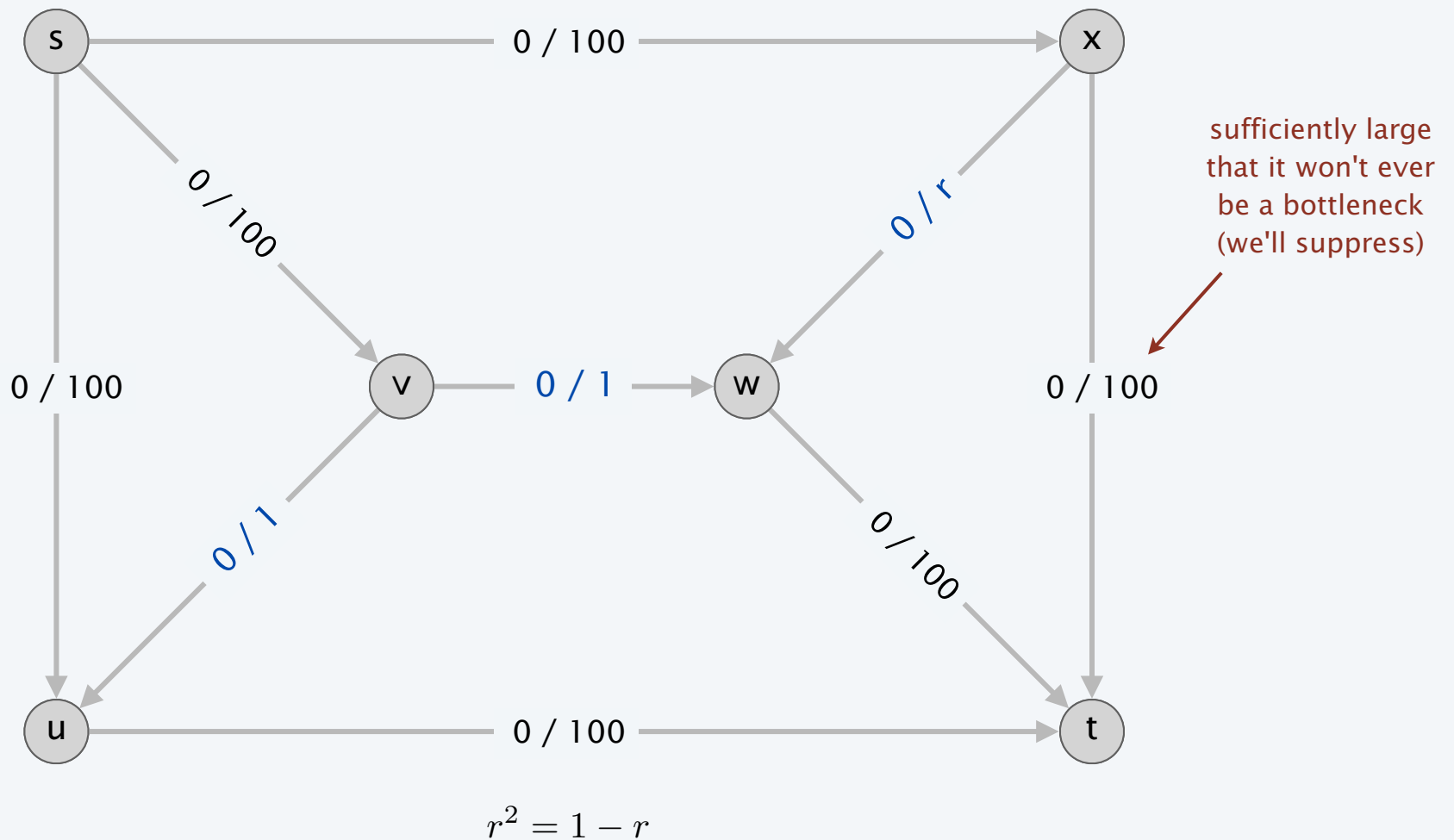


$$r = \frac{\sqrt{5} - 1}{2} \implies r^2 = 1 - r$$



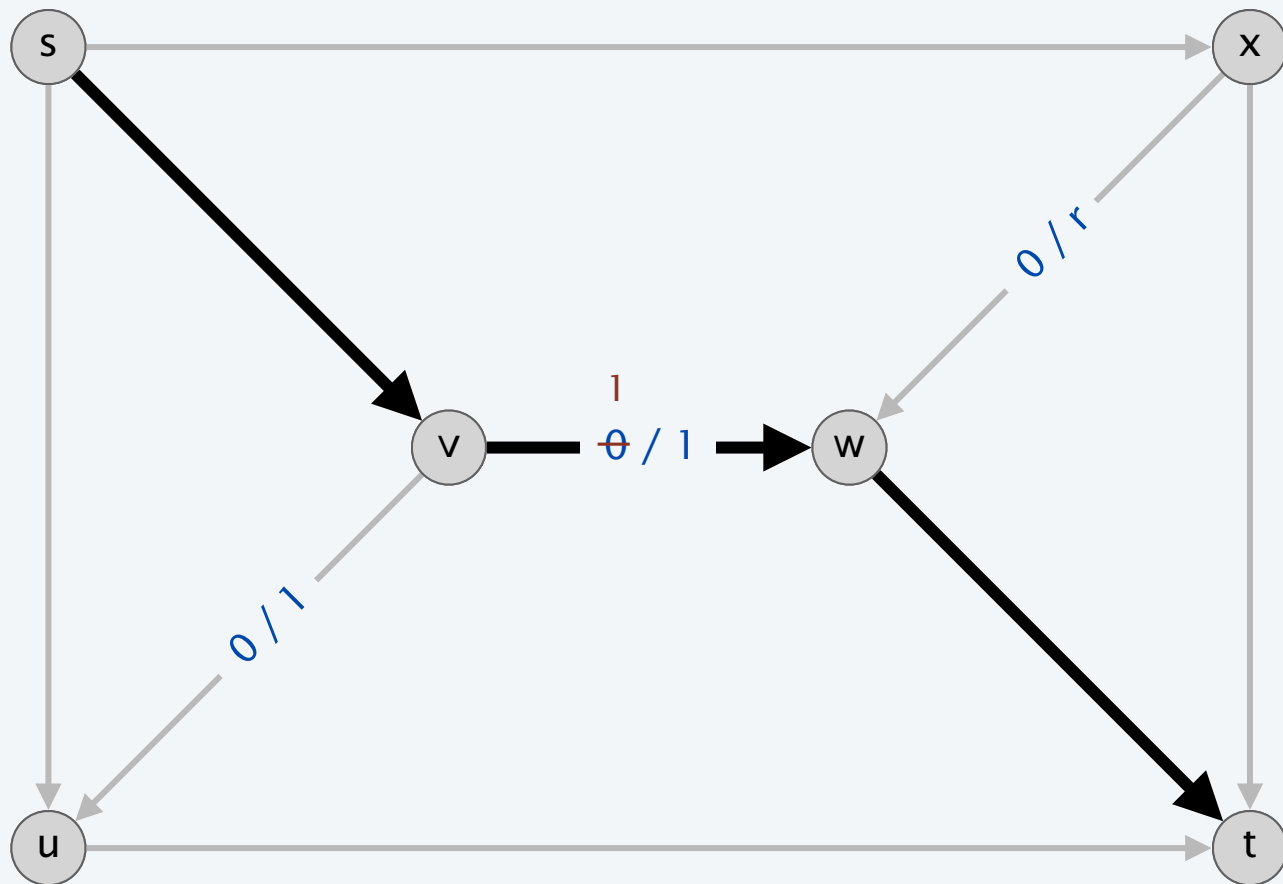
Ford-Fulkerson pathological example

network G



Ford-Fulkerson pathological example

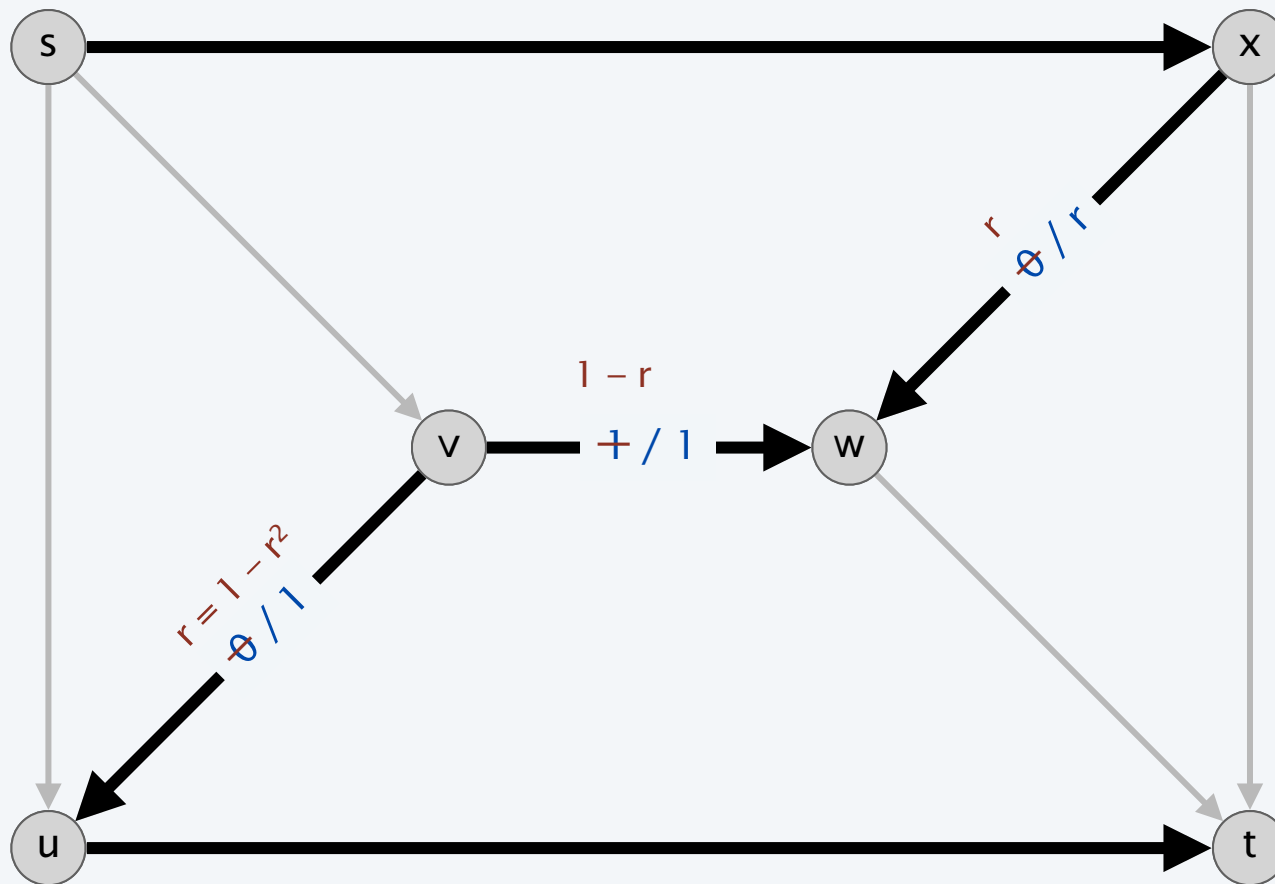
augmenting path 1: $s \rightarrow v \rightarrow w \rightarrow t$ (bottleneck capacity = 1)



$$r^2 = 1 - r$$

Ford-Fulkerson pathological example

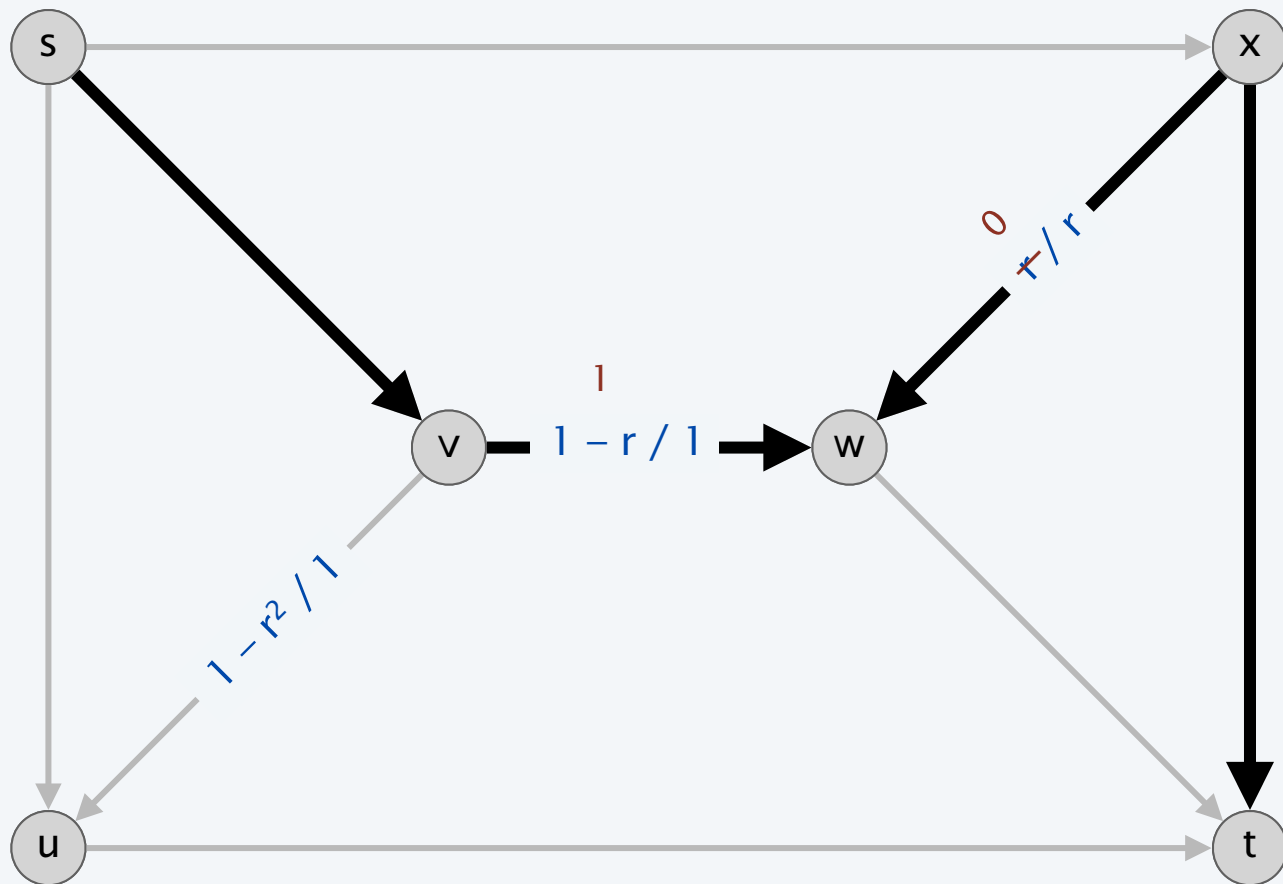
augmenting path 2: $s \rightarrow x \rightarrow w \rightarrow v \rightarrow u \rightarrow t$ (bottleneck capacity = r)



$$r^2 = 1 - r$$

Ford-Fulkerson pathological example

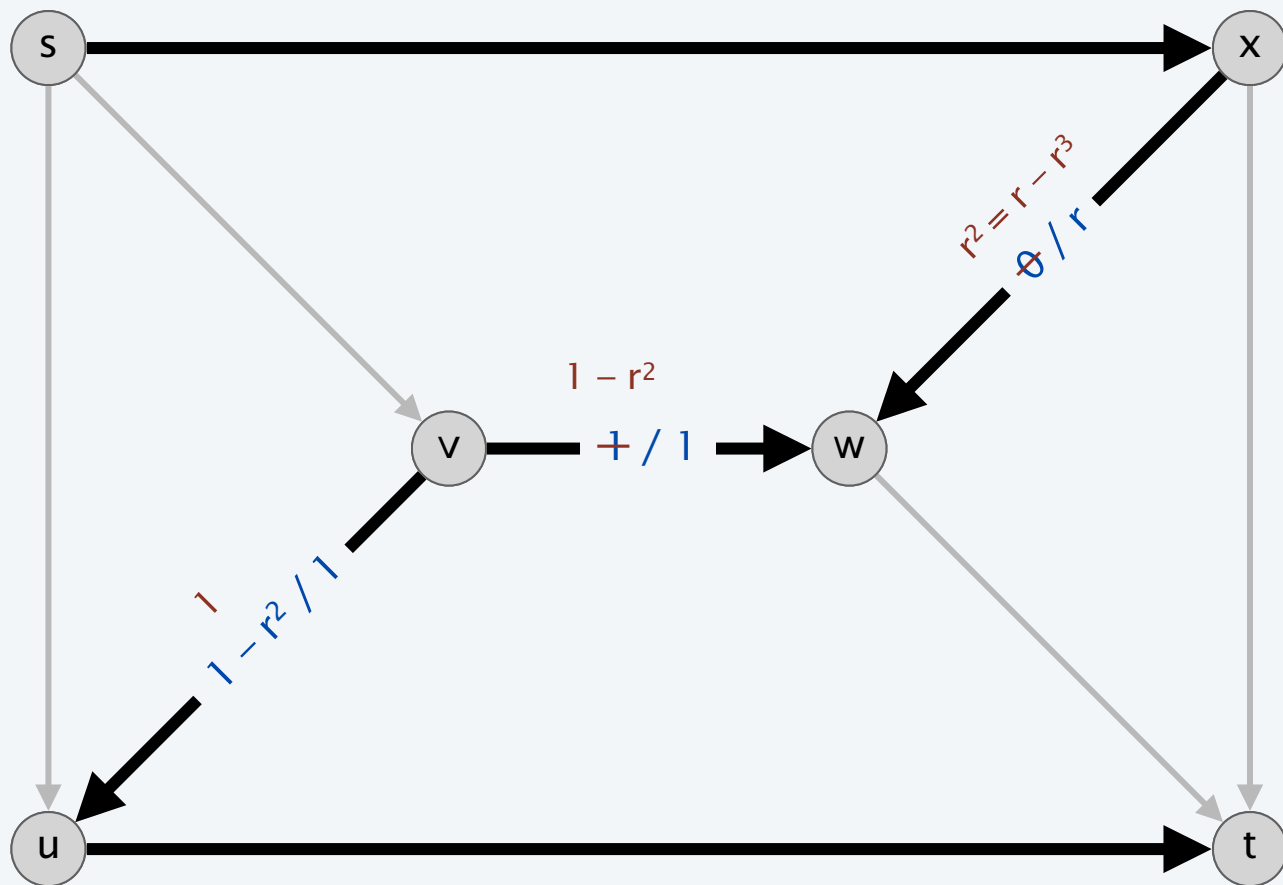
augmenting path 3: $s \rightarrow v \rightarrow w \rightarrow x \rightarrow t$ (bottleneck capacity = r)



$$r^2 = 1 - r$$

Ford-Fulkerson pathological example

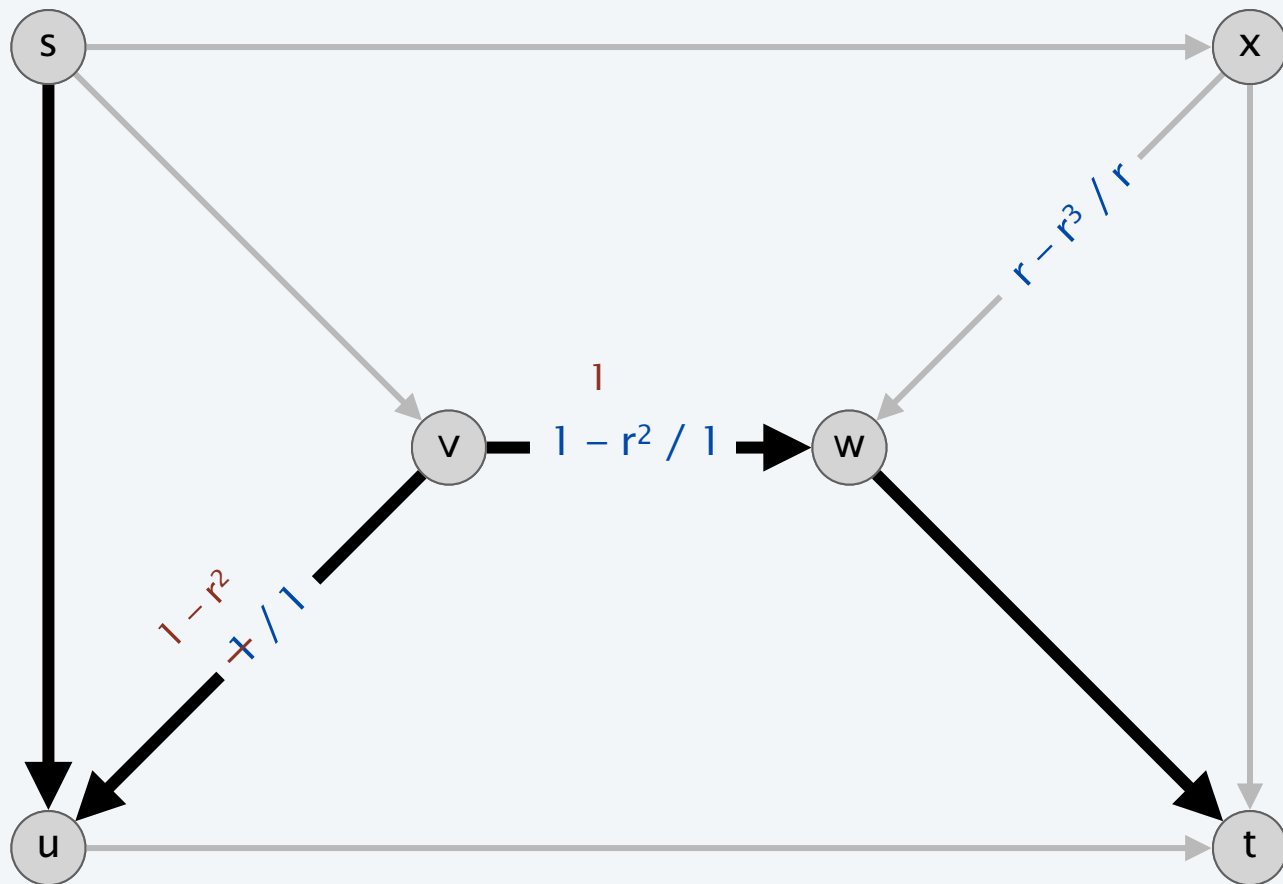
augmenting path 4: $s \rightarrow x \rightarrow w \rightarrow v \rightarrow u \rightarrow t$ (bottleneck capacity = r^2)



$$r^2 = 1 - r$$

Ford-Fulkerson pathological example

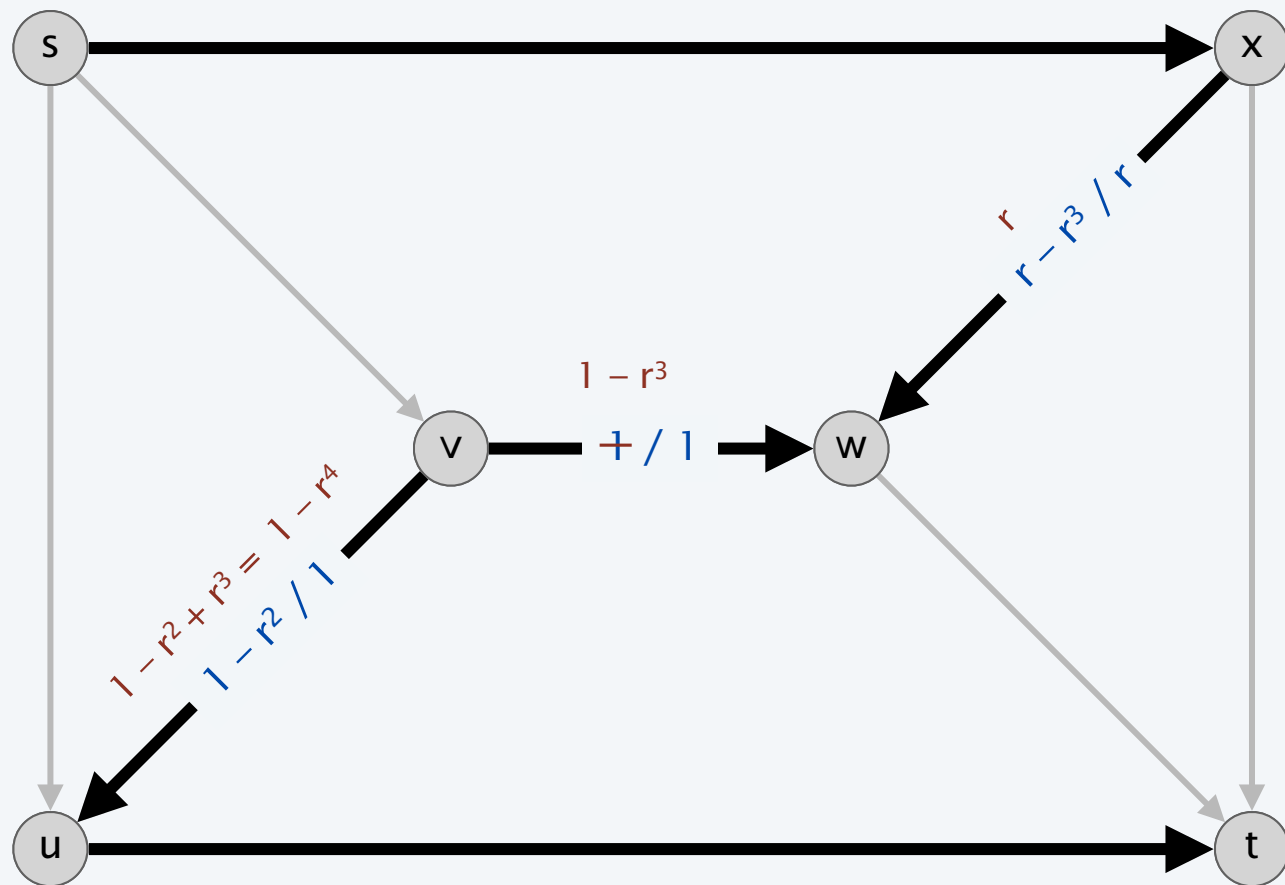
augmenting path 5: $s \rightarrow u \rightarrow v \rightarrow w \rightarrow t$ (bottleneck capacity = r^2)



$$r^2 = 1 - r$$

Ford-Fulkerson pathological example

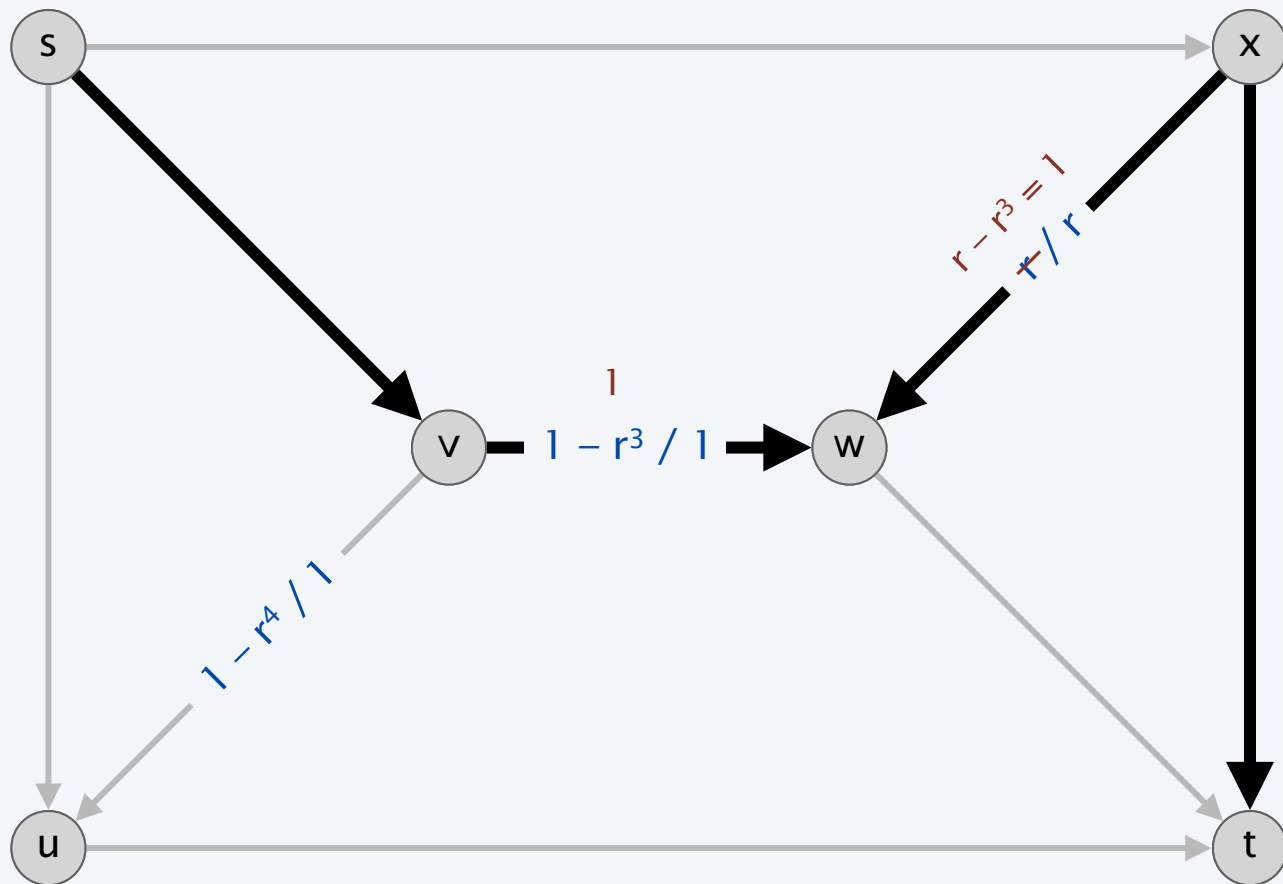
augmenting path 6: $s \rightarrow x \rightarrow w \rightarrow v \rightarrow u \rightarrow t$ (bottleneck capacity = r^3)



$$r^2 = 1 - r$$

Ford-Fulkerson pathological example

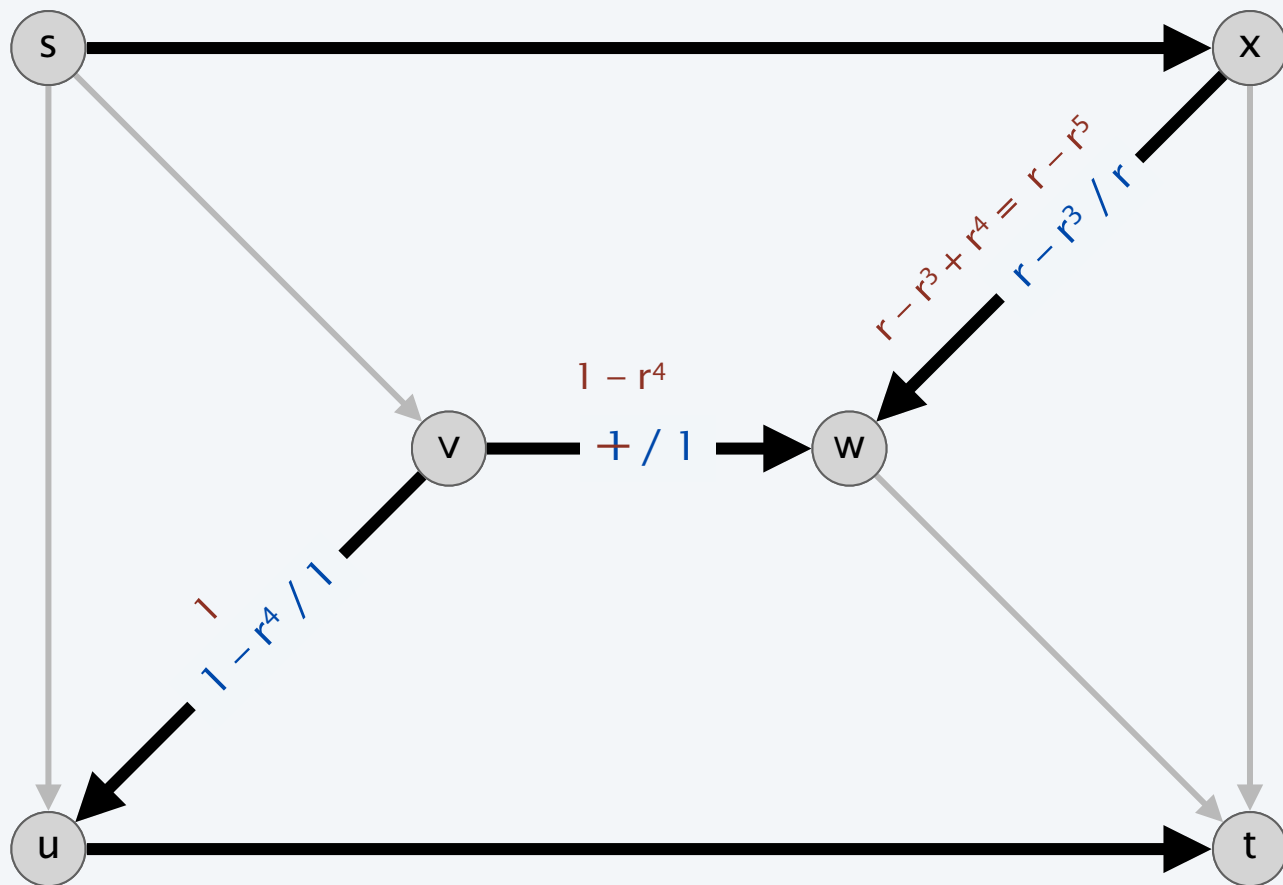
augmenting path 7: $s \rightarrow v \rightarrow w \rightarrow x \rightarrow t$ (bottleneck capacity = r^3)



$$r^2 = 1 - r$$

Ford-Fulkerson pathological example

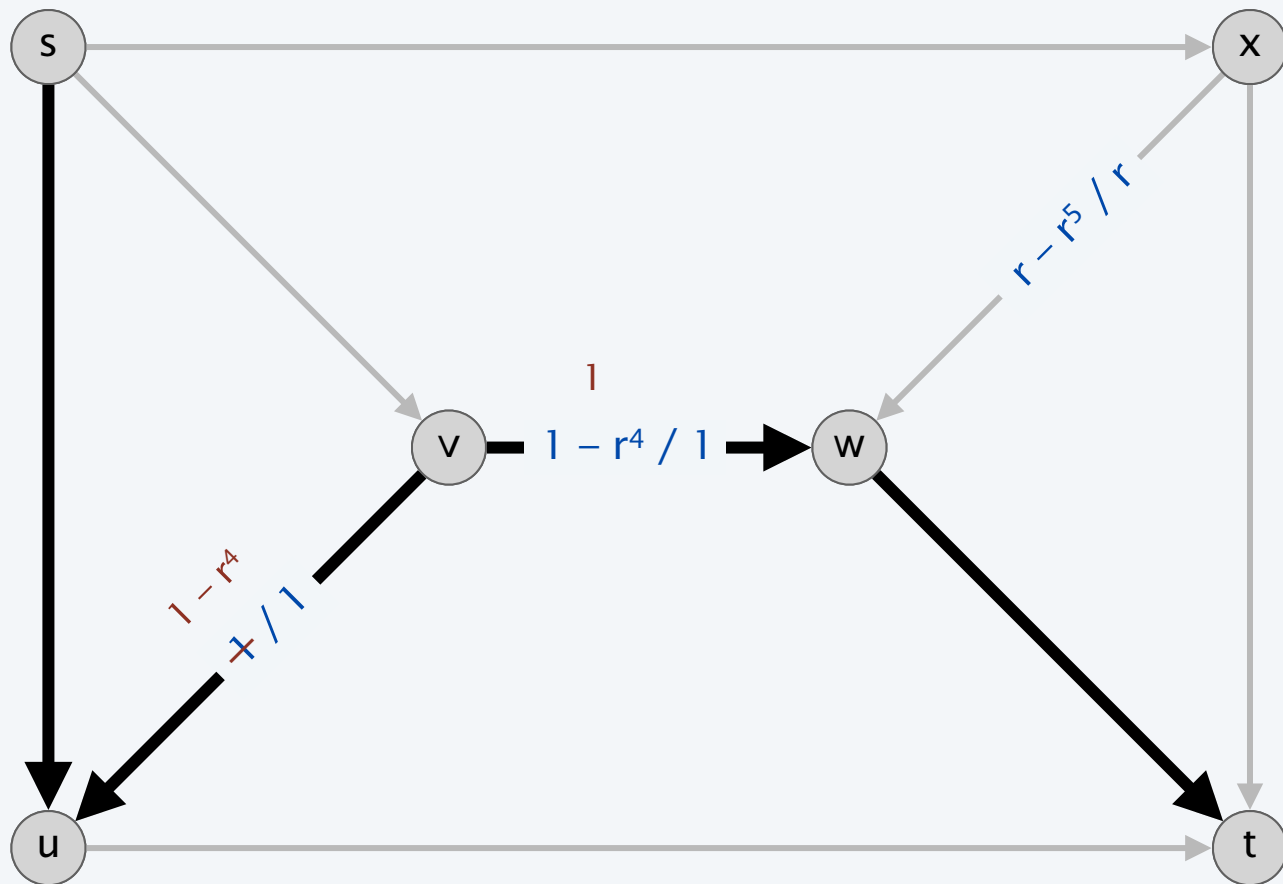
augmenting path 8: $s \rightarrow x \rightarrow w \rightarrow v \rightarrow u \rightarrow t$ (bottleneck capacity = r^4)



$$r^2 = 1 - r$$

Ford-Fulkerson pathological example

augmenting path 9: $s \rightarrow u \rightarrow v \rightarrow w \rightarrow t$ (bottleneck capacity = r^4)



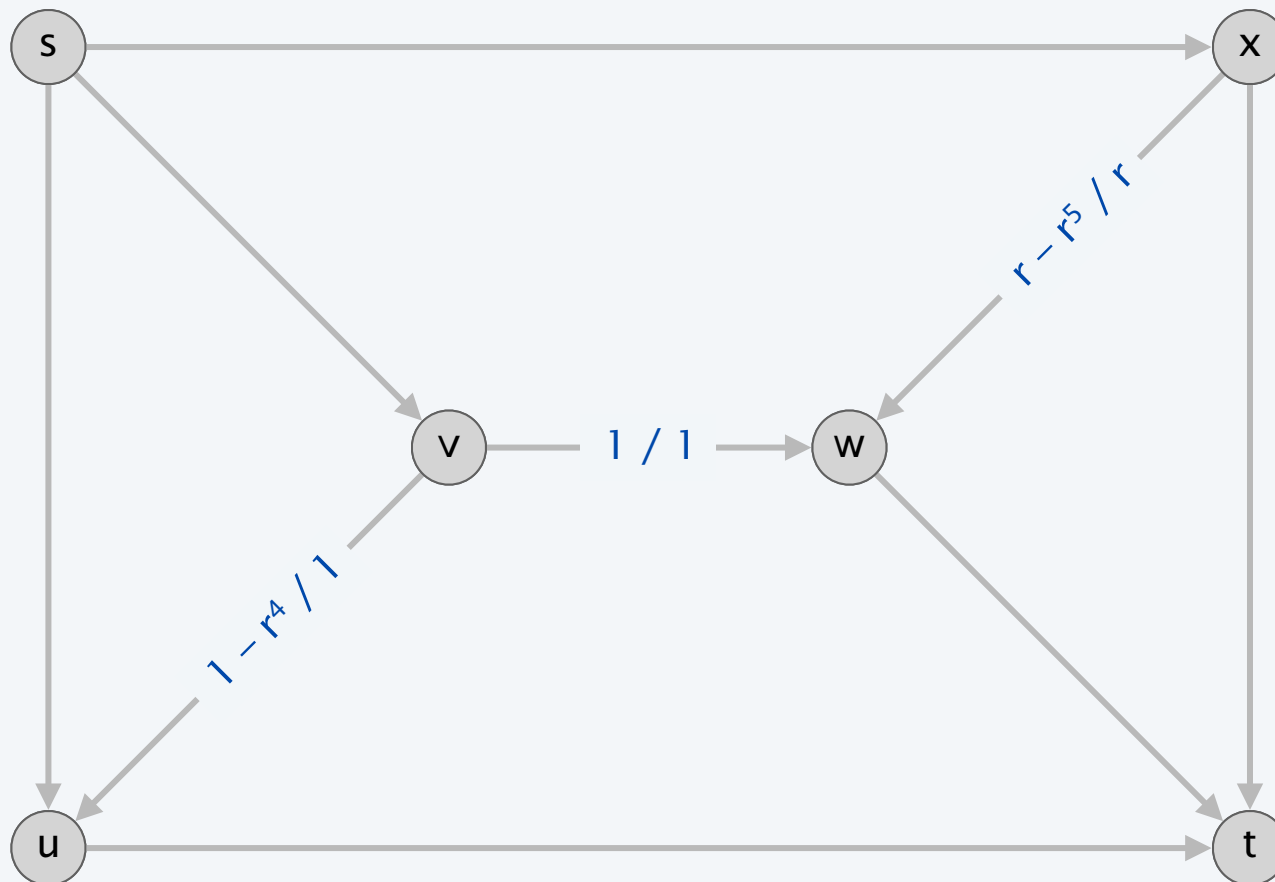
$$r^2 = 1 - r$$

Ford-Fulkerson pathological example

after augmenting path 1: $\{1 - r^0, 1, r - r^1\}$ (flow = 1)

after augmenting path 5: $\{1 - r^2, 1, r - r^3\}$ (flow = $1 + 2r + 2r^2$)

after augmenting path 9: $\{1 - r^4, 1, r - r^5\}$ (flow = $1 + 2r + 2r^2 + 2r^3 + 2r^4$)



$$r^2 = 1 - r$$

Ford-Fulkerson pathological example

Theorem. The Ford-Fulkerson algorithm may not terminate; moreover, it may converge to a value not equal to the value of the maximum flow.

Pf.

- Using the given sequence of augmenting paths, after $(1 + 4k)^{th}$ such path, the value of the flow

$$= 1 + 2 \sum_{i=1}^{2k} r^i$$

$$\leq 1 + 2 \sum_{i=1}^{\infty} r^i$$

$$= 3 + 2r$$

$$< 5$$

$$r = \frac{\sqrt{5} - 1}{2}$$



- Value of maximum flow = $200 + 1$. ■