**Specs:**

\[ K_u = 20 \]
\[ \text{PM} \geq 5^\circ \]
\[ \text{GM} > 10 \text{ dB} \]

**Stk:**

\[ G(z) = \left(1 - \frac{1}{z}\right)^3 \left[ \frac{K}{s^2(s+0.5)} \right] \]
\[ = 0.004918K \frac{z+0.9835}{(z-1)(z-0.9512)} \]

\[ K_u = \frac{1}{T} \lim_{z \to 1} (z-1) G(z) \]
\[ = 10 \left(0.004918K\right) \frac{1.9835}{0.488} = 20 \]

\[ \therefore K = 10 \]

For \( K = 10 \),

\[ G(w) = \frac{0.0002(20-w)(w+2404.2)}{w(w+0.5002)} \]

The Bode plot of the uncompensated system is shown in Figure 1. It can be seen from this figure that
\[ \text{GM} = 0.077 \text{ dB} \]
\[ \text{PM} = 0.08^\circ \]

Therefore, we need a controller. The system is marginally stable, therefore, the leg controller would not work well.
Figure 1. Bode plot of the uncompensated system with $K = 10$. 

Bode Diagram

$G_m = 0.077139$ dB (at 3.1765 rad/sec), $P_m = 0.080128$ deg (at 3.1621 rad/sec)
Since, we need a phase of approximately $50^\circ$, lead controller by itself would not work well either. We either need a lead-lead controller or lead-lag controller. Let us design a lead-lead controller. We will first design a phase lead controller for a phase angle of $30^\circ$.

\[ D_1(w) = \frac{1 + a_1 w}{1 + w}, \quad a_1 > 0 \]

\[ a_1 = \frac{1 + 5 \cdot 30^\circ}{1 - 5 \cdot 30^\circ} = 3 \]

\[ -10 \log_{10} 3 = -4.77 \text{ dB} \]

The value of \( w_m \) where the magnitude of the Bode plot is $-4.77 \text{ dB}$ is $4.2. \text{ rad/s}$

\[ \frac{1}{T_1} = \sqrt{a_1 w_m} = 7.27. \text{ rad/s} \]

\[ D_1(w) = 3 \cdot \frac{w + 2.42}{w + 7.27} \]

The Bode plot of the system compensated by a phase lead controller is shown in Figure 2. It can be seen from this figure that the GM is $10.75 \text{ dB}$ and the PM is $25.1^\circ$. Now, we need to design a phase lag part of the controller to get an additional phase of $25^\circ$. To get this phase, we need to design another phase lead controller. Let \( \phi_m = 40^\circ \).
Figure 2. Bode plot of the system with a phase-lead controller.
\[ D_2(W) = \frac{1 + q_2 T_2 W}{1 + T_2 W}, \quad q_2 > 0 \]

\[ Q_2 = 4.8 \]

\[ -10 \log_{10} 3 = -6.02 \text{ dB} \]

\[ \frac{1}{T_2} = \sqrt{\omega_m}^{7^{\text{deg} / 5}} \]

\[ = 15 \]

\[ \therefore D_2(W) = 4.8 \frac{W + 3.26}{W + 15} \]

\[ \therefore D(W) = 13.8 \frac{(W + 2.42)(W + 3.26)}{(W + 7.27)(W + 15)} \]

\[ \therefore D(z) = 7.54 \frac{(z - 0.851)(z - 7.97)}{(z - 4.68)(z - 1469)} \]

\[ \therefore D(z) G(z) = \frac{370.8 z^2 - 1929 z - 3392 z + 2058}{z^4 - 2.5619 z^3 + 2.2075 z^2 - 0.71 z + 0.034} \]

\[ \therefore C(z) = \frac{370.8 z^2 - 1929 z^3 - 3392 z^2 + 2058}{z^4 - 2.19 z^3 + 2.0145 z^2 - 1.0492 z + 2.692} \]

The step response of the system is shown in Figure A.
Figure 3. Bode plot of the system with a lead-lead controller.
Figure 4. Step response of the compensated system