Notes: 1. Total time allowed is 4 hours. Please indicate on your exam how long you spent on the exam.
2. Make sure you reference figure by figure no.

1. \( u(t) \)

\[
G_c(s) = \frac{10}{s^3 + 10s + 5} \quad \text{Controller}
\]

\[
G_p(s) = \frac{10}{s(s^2 + 1)} \quad \text{Plant}
\]

\[
C(u) \rightarrow
\]

Figure 1

(a) Find \( \omega_n \) and \( \zeta \) of the system compensated by an analog controller shown in Figure 1.

(b) It is required to discretize the system shown in Figure 1, i.e., convert the continuous controller into a digital controller.

(i) Find an appropriate sampling period \( T \) and give a block diagram of the digital control system.

(ii) Convert the continuous controller given in Figure 1 to a digital controller using the following methods:

- Matched \( z \)-transform
- Bilinear transformation with prewarping

(iii) Obtain the response of the digital control system with digital controller obtained in part(ii) and find the \( \omega_n \) and \( \zeta \) of the systems.
Figure 2

Design a PID controller so that the closed-loop system shown in Figure 2 satisfies the following specifications:

(i) Steady-state error to a ramp input ≤ 10%.
(ii) Maximum percentage overshoot to a step input ≤ 10%.
(iii) Settling time ≤ 1 sec.

Design a phase-lead digital controller, D(z), in the z-domain to satisfy the following specs:

(i) $K_L \gg 2$
(ii) $P.M. \geq 50^\circ$
(iii) $G.M. \geq 10$ dB

---

Figure 3