**Step Invariance**

The step-invariance method preserves the step response. That is, the step response of $G_c(s)$ is set equal to the step response of $D(z)$ on a sample-and-hold basis. Then

$$\left(\frac{1}{1-z^{-1}}\right) D(z) = 3 \left[ \frac{1}{s} G_c(s) \right]$$

or

$$D(z) = (1-z^{-1}) 3 \left[ \frac{G_c(s)}{s} \right]$$

**Figure 4.12** $G_c(s)$ preceded by a fictitious sample-and-hold device

**Example 4.3**

Consider the step-invariant integrator

$$G_c(s) = \frac{1}{s}$$

Then

$$D(z) = (1-z^{-1}) 3 \left[ \frac{1}{s^2} \right]$$

$$= (1-z^{-1}) \left[ \frac{T z^{-1}}{(1-z^{-1})^2} \right]$$

$$= \frac{T z^{-1}}{1-z^{-1}} = \frac{T}{z-1}$$

Which yields the same approximation derived earlier for the forward difference.