S-Plane Properties of $X^*(s)$

1. $X^*(s)$ is periodic in $s$ with a period of $\omega_s$
   
   $X^*(s) = \sum_{n=0}^{\infty} X(nT)e^{-n\pi s}$
   
   $X^*(s+\omega_s) = \sum_{n=0}^{\infty} X(nT)e^{-n\pi (s+\omega_s)}$
   
   $= \sum_{n=0}^{\infty} X(nT)e^{-n\pi s} e^{-n\pi \omega_s}$
   
   $= \sum_{n=0}^{\infty} X(nT)e^{-n\pi s} e^{2\pi inm}$
   
   $\therefore X^*(s+\omega_s) = X^*(s)$

2. If $X(s)$ has a pole at $s = s_1$, then $X^*(s)$ must have poles at $s = s_1 + \omega_m$, $m = 0, \pm 1, \pm 2, \ldots$

   $X^*(s) = \sum_{m=-\infty}^{\infty} X(s + \omega_m)$
   
   $= \sum_{m=-\infty}^{\infty} [X(s) + X(s+\omega_m) + X(s-\omega_m) + \ldots]$

   If $X(s)$ has a pole at $s = s_1$, then each term of \ref{eq:poles} will contribute a pole at $s = s_1 + \omega_m$, where $m$ is an integer.
Note: If the pole locations are known for $X(s)$, the primary strip then the pole locations in the entire s-plane are known.

However, it is important to note that no equivalent statement can be made concerning the zeros of $X(s)$. For $X(s)$, the zeros of $X^*(s)$ do not uniquely determine the zeros of $X(s)$. 
\[ w_5 = 3 \text{ rad/s} \]
\[ T = \frac{2\pi}{3} \]
\[ w_2 = w_1 + w_5 \]