Obtaining Response Between Consecutive Sampling Instants

- The z-transform analysis only give the response at the sampling instants.

- In ordinary cases this is not serious, because the sampling theorem is satisfied, then the output will not vary much between any two consecutive sampling instants.

- In certain cases however, we may need to find the response between consecutive sampling instants.

If the delay is $\frac{T}{4}$, the output is $C\left(\frac{3T}{4}\right)$, $C\left(T + \frac{3T}{4}\right)$, ...
In general any value of output between two consecutive sampling instants could be computed if the output were delayed by a proper interval of time in making the calculation.

\[
\frac{C(z,m)}{X(z)} = G(z,m)
\]

**Block Diagrams**

1. \(X(t)\) \(\rightarrow\) \(G(s)\) \(\rightarrow\) \(C(t)\)

\[C(z) = G(z)X(z)\]

\[C(z,m) = G(z,m)X(z)\]

2. \(X(t)\) \(\rightarrow\) \(G_1(s)\) \(\rightarrow\) \(G_2(s)\) \(\rightarrow\) \(C(t)\)

\[C(z) = G_1(z)G_2(z)X(z)\]

\[C(z,m) = G_1(z)G_2(z,m)X(z)\]

3. \(X(t)\) \(\rightarrow\) \(G_1(s)\) \(\rightarrow\) \(G_2(s)\) \(\rightarrow\) \(C(t)\)

\[C(z) = \overline{G_1G_2(z)}X(z)\]

\[C(z,m) = \overline{G_1G_2(z,m)}X(z)\]
Closed-loop System

1. \[ C(z) = \frac{G(z)}{1 + G(z)H(z)} \]
   \[ C(z, m) = \frac{G(z, m)}{1 + G(z)H(z)} \]

2. \[ C(z) = \frac{G(z)}{1 + G(z)H(z)} \]
   \[ C(z, m) = \frac{G(z, m)}{1 + G(z)H(z)} \]

3. \[ C(z) = \frac{GR(z)}{1 + GH(z)} \]
   \[ C(z, m) = \frac{GR(z, m)}{1 + GH(z)} \]
Example: Consider the system shown below. Obtain the unit step response at \( t = \frac{T}{2}, T + \frac{T}{2}, \ldots \)

\[
\begin{align*}
C(z, m) &= \frac{G(z, m)}{1 + G(z)} R(z) \\
\Lambda(t) &= u(t) \\
R(z) &= \frac{z}{z-1}
\end{align*}
\]

\[
G(s) = \frac{1 - \frac{sT}{s(s+1)}}{1 - \frac{sT}{s(s+1)}}
\]

\[
G(z) = (1 - z^{-1})^3 \left[ \frac{1}{s(s+1)} \right] = (1 - z^{-1})^3 \left[ \frac{368 z^2 + 264 z^2}{(1 - z^{-1})(1 - 368 z^{-1})} \right]
\]

\[
G(z, m) = (1 - z^{-1})^3 \delta m \left[ \frac{1}{s(s+1)} \right] = (1 - z^{-1})^3 \left[ \frac{1}{(z-1)} + \frac{1}{2(z-1)} + \frac{1}{(z-1)(z-368)} \right]
\]

\[
C(z, m) = \frac{1065 z^2 + 4709 z + 05468}{z^3 - 2 z^2 + 16321 z + 4321}
\]

\[
C(z, 0.5) = -1 \left( 1065 z + 6839 z^2 + 12487 z^3 + 14485 z^4 + 1.2913 z^5 \right) + 1.0078 z^2
\]

\[
C(0.5) = 1065
\]

\[
C(T + \frac{T}{2}) = 6839
\]

\[
C(2T + \frac{T}{2}) = 12487
\]

\[
C(3T + \frac{T}{2}) = 14485
\]