Some Useful Signals

Step Function

\[ f(t) = k \ u(t) \]

where

\[ u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases} \]

If \( k = 1 \),
\( f(t) = u(t) \) is called a Unit-Step function.

- For any continuous-time signal \( x(t) \), the product \( x(t) \ u(t) \) is equal to \( x(t) \) for \( t \geq 0 \) and is equal to zero for \( t < 0 \).
- Thus, multiplication of a signal \( x(t) \) with \( u(t) \) eliminates any non-zero values of \( x(t) \) for \( t < 0 \).

Ramp Function:

\[ f(t) = k \ r(t) \]

where

\[ r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases} \]

If \( k = 1 \),
\( f(t) = r(t) \) is called a Unit-Ramp function.

- Note that for \( t \geq 0 \), the slope of \( r(t) \) has "unit slope," which is the reason \( r(t) \) is called the Unit-Ramp function.
• The unit-ramp function $r(t)$ is equal to the integral of unit-step function $u(t)$, i.e.

$$r(t) = \int_{0}^{t} u(\lambda) d\lambda$$

• Conversely, the first derivative of $r(t)$ with respect to $t$ is equal to $u(t)$, except at $t=0$, where the derivative of $r(t)$ is not defined.

**Exponential Function:**

$$f(t) = Ke^{-at} u(t)$$

- At $t = \frac{1}{a} = T$, the value drops to $K \frac{1}{e} = 0.37K$ (about 37% of the initial value).

**Impulse Function:**

• The unit impulse, $S(t)$, also called the delta function or the Dirac distribution, is defined by

$$S(t) = 0, \quad t \neq 0$$

$$\int_{-\infty}^{\infty} S(t) dt = 1$$

• The first condition states that $S(t)$ is zero for all nonzero values of $t$, while the second condition states that area under the impulse is 1, so $S(t)$ has unit area.
- It is important to point out that the value \( S(0) \) of \( S(t) \) at \( t=0 \) is not defined; in particular, \( S(0) \) is not equal to infinity.

- The impulse \( S(t) \) can be approximated by a pulse centered at the origin with amplitude \( A \) and time duration \( \frac{1}{A} \), where \( A \) is very large positive number.

- For any real number \( K \), \( KS(t) \) is the impulse with area \( K \) shown as

- The unit-step function, \( u(t) \), is equal to the integral of the unit impulse \( S(t) \), i.e.

\[
u(t) = \int_{-\infty}^{t} S(\lambda) \, d\lambda, \quad \text{all } t \text{ except } t=0
\]

**Proof:**

1. for \( t < 0 \)
   \[
   \int_{-\infty}^{t} S(\lambda) \, d\lambda = 0, \quad \text{since } S(t) = 0 \text{ for all } t < 0
   \]

2. for \( t > 0 \)
   \[
   \int_{-\infty}^{t} S(\lambda) \, d\lambda = \int_{-\infty}^{0^-} S(\lambda) \, d\lambda + \int_{0^+}^{0} S(\lambda) \, d\lambda + \int_{0^+}^{t} S(\lambda) \, d\lambda
   \]
Sampling Property of Unit Impulse Function

\[
\int_{-\infty}^{\infty} f(t) \delta(t) \, dt = f(0)
\]

\[
\int_{-\infty}^{\infty} f(t) \delta(t-T) \, dt = f(T)
\]